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A practical overview of unsteady pipe flow modeling: from physics to numerical solutions

Bong Seog Junga and Bryan Karneyb

aTebodin, Abu Dhabi, UAE; bDepartment of Civil Engineering, University of Toronto, Toronto, ON, Canada

ABSTRACT
Various unsteady or transient models have evolved in order to help engineers achieve economy of analysis, design, construction, operation and maintenance. The specific usage of each model is strongly dependent on the level of unsteadiness in the system and on the accuracy, assumptions and limitations of the applied mathematical model and its numerical solution. Although the research literature is quite clear on these issues, there is often much confusion in practice. In this paper, the key practical differences and advantage for the four transient models — water hammer models, rigid water column analysis, quasi-steady analysis and so-called Joukowski approach — are compared and contrasted with respect to three criteria: their physical attributes, the hydraulic predictions they lead to, and the related numerical considerations of stability and accuracy. A useful guideline for determining the degree of unsteadiness is presented and then linked to an appropriate unsteady model.

Introduction
Since all flows will eventually be altered, either suddenly or gradually, over a system’s life, all pipeline systems inevitably experience transient effects. Indeed, all engineered systems are at some point started, switched off, or undergo changes associated with routine operational adjustments, human error, equipment breakdowns, earthquakes, or other disturbances. Yet, from either a performance or modeling perspective, a complete consideration of all transient states is clearly impractical. Inevitably attention is restricted to specific and targeted design regimes that reasonably capture the key system performance issues and threats in terms of cost effectiveness, hydraulic capacity, and structural strength, in addition to overall system reliability, robustness, vulnerability, and resilience. Perhaps the four most relevant practical questions of transient pipe flow modeling are the following: (i) How important is the unsteadiness in flow conditions to understand the system response? (ii) What mathematical models are most appropriate to capture this unsteadiness? (iii) What spatial and/or temporal discretization is needed to accurately simulate the selected model? and (iv) How sensitive is the system’s response to uncertainty or variation in key system parameters? A number of intriguing considerations arise from these questions that are connected to issues such as purpose of the analysis, the decisions or choices under consideration, the physical and economic consequences of both conservatism and risk, and the experiences and biases of both the client or owner and the analyst.

This paper briefly explores the related practical questions of modeling intent, physical and numerical approximations, and how these issues inform and influence both an understanding of the system and its physical behavior. Several specific modeling options are overviewed including the rather simplistic but still encountered “Joukowski” approach (Jung et al. 2007a), nearly or quasi-steady approaches (often called extended period analysis), so-called rigid water column models (which assume an incompressible fluid, so neglecting the convective acceleration, but explicitly include inertia effects), and full water hammer models (i.e., which, like Joukowski, also including small compressibility effects). This paper provides a brief summary of these considerations and sets out some guidance into how these questions of unsteadiness need to be framed or posed. There are clearly issues of physics at stake, in terms of whether certain phenomena are dominant or trivial, but even this evaluation is contingent on the context of the modeling exercise and what specific questions are being answered or what operational and design decisions are being made.

Unsteady pipe flow modeling
Analysts are continuously caught in a tension between two opposing but individually desirably objectives in selecting and using modeling tools: simplicity of use versus accuracy of prediction. Transient modeling of water systems has often navigated these tensions in one of four ways (Axworthy 1997). Perhaps most simply, one can use the so-called Joukowski approach which superimposes, often as a confirmation check, a simple formula between sudden changes in velocity and pressure changes — expressed as $dH = \pm a/g \cdot dV$, where $dH =$ pressure...
change, $dV = \text{velocity change}$, $a = \text{wave speed}$ and $g = \text{gravity acceleration}$ — onto an otherwise steady model. Alternatively, extended period (EP) or quasi steady models allow for gradual changes in demand or water level, such as allowing water accumulations or depletions at tanks, and by permitting water demands and device settings (e.g., for pumps and valves) to be functions of time and/or system state. However, most EP models effectively make instantaneous changes at the end of each time step, with no physical consideration for the inertial or mass effects that real water possesses.

Thirdly, and probably most rarely in current practice, if inertia effects are included but compressibility effects are excluded, the so-called rigid water column family of models is created (Wylie and Streeter 1993). Such models represent flow in each pipe as an ordinary differential equation. Yet, interestingly rapid flow changes are not realistically represented in rigid models, which are often overly conservative about pressure changes. In fact, as device operational times decrease, peak pressure heads in a rigid model tend to increase without bound; what actually limits these magnitudes is the invocation of small compressibility effects. Profoundly, it is inertia effects that really threaten systems with compressibility effects usually limiting, to some extent, the severity of transient pressures. Finally, water hammer models include the inertia effects of rigid models but the (generally) head-change-limiting role of compressibility effects (Wylie and Streeter 1993). However, water hammer models are governed by a pair of partial differential equations with a finite water hammer wave speed. Depending typically on model dimensionality and representation of friction, a whole set of water hammer and unsteady friction models are possible. The exploration of this area has been active for some time, and a host of researchers have made contributions (e.g., Brunone et al. 2000, Vitkovsky 2001, Ghidaoui et al. 2005, Vitkovsky et al. 2006, Duan et al. 2012).

The mapping between the intrinsic accuracy of the physical representation, and the simplicity of the model, can be displayed for unsteady models approximately as shown in Figure 1(a). Another representation is shown by the Venn diagram of Figure 1(b). As the quasi-steady equations are special cases of the rigid column approach, all such analysis could have performed, albeit at great computational expense, with a rigid water model.

Similarly, all systems that can be modeled with either rigid or quasi steady approaches can technically be modeled with the water hammer or elastic model. The converse, however, is in general not true, since quasi-steady models misrepresent true transient models. Yet, in fairness, the number of actual models in use in engineering firms or utilities is greatest for the quasi-steady approach which is used extensively for planning the basic components of system operation, design, and expansion.

**Selecting an unsteady model**

Many papers have discussed where to position the boundaries between the unsteady models (Karney and Ruus 1985, Karney 1990, Rogalla and Walters 1994, Axworthy 1997, Islam and Chaudhry 1998, Abreu et al. 1999, Ghidaoui et al. 2005, Duan et al. 2012). Traditionally, the question has been answered by comparing the actual duration of, say, a valve motion to the wave travel time in the pipe system. For example, a typical criterion for a linear valve closure to be considered “rapid” if the closing time takes place in five “wave travel times” (i.e., 10 L/a) or less, while a longer closure can be reasonably modeled by assuming an incompressible fluid (Karney and Ruus 1985). Karney and Russ, however, showed that the appropriate limiting value depends not only on the duration of valve closure, but also on the head loss due to friction and on various pipe parameters (e.g., nondimensional Allievi parameter, usually written as $\frac{aV}{gH^0}$, in which the subscript 0 indicates the initial steady-state values). Karney (1990) introduced an “energy expression” providing the kinetic energy of the fluid, the internal energy associated with the fluid compressibility and pipeline elasticity effects, the energy dissipated by friction, and the work done at the ends of the conduit and then found that the compressibility effect is negligible when the ratio of the change in internal energy to the change in kinetic energy is much less than one. Rogalla and Walters (1994) also compared the boundary between rigid water column model and water hammer models while Islam and Chaudhry (1998) compared quasi-steady model to rigid water column models under slowly varying unsteady flow conditions. Axworthy (1997) compared all three unsteady models and developed boundary conditions interfacing with each of the unsteady models. Axworthy found, in his numerical studies, that slow valve closure and pump speed change examples demonstrated the suitability and reliability of the rigid water column model for slow transient events but a pump power failure example showed the water hammer model was accurate for rapid transient events. Abreu et al. (1999) also compared three models and then used dimensionless parameters to define the range of their applicability for simple hydraulic systems. Abreu et al. investigated that the effects of resistance, inertia and elasticity varied in relative importance under different circumstances. Ghidaoui et al. (2005) presented the overview of historic developments of water hammer theory and modeling practice. Ghidaoui et al. discussed mass and momentum equations for one and two dimensional flows, their numerical solutions, turbulence models, boundary conditions, transient analysis software, and future practical and research needs in the water hammer model. Recently, Duan et al. (2012) investigated the impact of pipe system scale (i.e., pipe length and diameter) on unsteady friction in a pipe transient. They found that

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**Figure 1.** Conceptual representation of unsteady flow modeling approaches.
unsteady friction damping has less effect on the damping rate of the transient envelope as the ratio of the wave travel timescale to the radial diffusion timescale increases and the product of the initial friction factor and Reynolds number increases.

The selection of unsteady model is strongly dependent on the characteristics of system and its dynamic operation. Interestingly, rigid water column models, which are intermediate between quasi-steady and water hammer approaches, are particularly helpful for assisting in the process of selection. Rigid water column models are based on the following equation:

\[H_0 - H_U = \frac{fL}{gD} \left( \frac{dV}{dt} \right) + \left( \frac{L}{g} \right) \frac{dV}{dt} = 0\]  

where \(H_0\) = downstream piezometric head; \(H_U\) = upstream piezometric head; \(f\) = Darcy-Weisbach friction factor; \(L\) = length of pipe; \(V\) = fluid velocity; \(g\) = acceleration due to gravity; \(D\) = inside pipe diameter; and \(t\) = time. In this paper, a simple and preliminary but insightful guideline for determining the degree of unsteadiness is presented that is based on the rigid column model, but then linked to an appropriate unsteady model.

To this end, Figure 2(a) gives a rough indication of inertia effect in a single pipeline and provides a general sense of how to select a reasonable unsteady model. Assuming a moderate system velocity change \(dv\) of 1 m/s, the inertia head \((L/g\,dv/dt)\) in Equation (1) can be calculated for different pipe lengths \(L\) and dimensionless accelerations \((dv/dt)/g\) in \(g\) (the so-called “gee” in dynamics work). The dimensionless acceleration associated with the physical action, and the length of the physical system, thus determine the “acceleration head” associated with the system unsteadiness. As the acceleration decreases and/or the pipe length decreases, the inertia head become negligible so a quasi-steady model represents a reasonably accurate and simple model. For example, the combination of the pipe length of 10 m and the dimensionless acceleration of 0.01 \((dv/dt \approx 10\ s)\) produces the inertia head of 0.1 m, which is almost invariably insignificant. Yet, as the acceleration increases and/or the pipe length increases, the inertia progressively dominates, and at least a rigid water column model is required. A larger pipe length of 100 m and a larger dimensionless acceleration of 0.1 \((dv/dt \approx 1\ s)\) produce an inertia head of 10 m, which is almost certainly large enough to justify consideration of inertia effects. However, as the acceleration head and the pipe length continue to increase, the rigid relationship progressively provides a significant over-estimate of the true value of head change, because the limiting effects of compressibility of fluid and deformability of the pipe become important. A much larger pipe length of 1000 m and a much larger dimensionless acceleration of 1 \((dv/dt \approx 0.1\ s)\) produce an inertia head of 1000 m, which, obviously, would never occur. This unrealistically exaggeration of the inertia effect arises due to the excluded compressibility effects. Thus, either very rapid changes in quite a short system, or much slower changes in long systems, or any acceleration more severe, require the use of a water hammer model.

To illustrate the practical application of this table, consider Figure 2(b) shows the maximum head results of sixteen transient water hammer runs for a pipeline system with a velocity of 1 m/s, a length ranging from 10 m to 10 km and a time change ranging from 0.1 s to 100 s (so the corresponding dimensionless acceleration is ranged from 0.001 to 1). As the acceleration decreases and/or the pipe length decreases, the inertia head becomes negligible (i.e., less than one) so the maximum transient head is insignificant (i.e., less than 2.3 m). This condition can reasonably be presented by a quasi-steady model. Yet, as the acceleration and/or the pipe length increases, the inertia head becomes significant so the maximum transient head can reach more than 100 m. This condition, clearly, requires a water hammer model to capture both the fluid compressibility and fluid inertia. The intermediate range (i.e., the inertial head of about 10 m) can be practically represented by a simpler transient model—rigid water column model that doesn’t consider fluid compressibility. Figure 3(a) represents the transient head profiles with different time changes \(dt\) at the same pipe lengths \((L = 1000\ m)\). Similarly, Figure 3(b) presents the transient head profiles with different pipe lengths, \(L\), at the same time change \((dt = 1\ s)\) (note that the role of vacuum pressures and/or vapor cavity formation is ignored in these calculations).

Model selection should not only consider the level of unsteadiness and the assumptions of the numerical model, but should also consider the degree of system skeletonization. Jung et al. (2007b) and even the specific representation of the water demand model (Jung et al. 2009). Model skeletonization is still widely applied in the context of complicated distribution networks; however, most guidelines used for skeletonizing hydraulic network models (e.g., using a “hydraulically equivalent pipe” with the same flow capacity or trimming dead end mains) work poorly for the water hammer model. Similarly, assuming that water demands are independent of pressure might be justified under quasi-steady state analysis, but inappropriate under transient conditions. Therefore, a pressure-sensitive demand representation with a more detailed model (e.g., including dead ends) is often needed to accurately estimate transient pressures in water systems.
and reducing the maximum pressure, but also that it is the overall system characteristics, and the purpose of the modeler, that largely shape the selection of model type. As the area of the surge tank is progressively reduced, though, particularly for the case of instantaneous closure, progressively more inertia and compressibility effects become evident even in the response of the tank. Figure 5(b) shows the water level with a tank that is about the same diameter as the pipe. Now the rigid water curve has moved considerably above the quasi-steady curve as acceleration head effects have become significant. Moreover, the water hammer curve is not smooth, but has peaks and valleys due to the wave propagation characteristics of the system. Clearly, the water hammer model has more detail than that captured by the rigid water column model, which captures much more than the quasi-steady model. Significantly, only the water hammer model captures the constructive and destructive interplay of the pressure waves as they reflect back and forth between the reservoir and the valve.

**Case studies**

The following case studies show how and when the various models can be applied properly. For simplicity, only so-called steady friction water hammer models are used.

**Cases study #1: single pipeline system with surge tank**

The preliminary case study in Figure 4 shows a simple pipeline system with a tank. The system comprises two identical pipes where the length, diameter, Darcy-Weisbach friction factor, initial flow rate and wave speed of the pipes are 1500 m, 0.5 m, 0.02, 0.2 m$^3$/s, and 1000 m/s, respectively. The pipe is connected to an upstream reservoir with the elevation of 100 m, a tank in the middle and an initially fully open downstream valve. The influence of changing the cross-sectional area of the tank and the valve closing time is to be explored. Quasi-steady, rigid water column and water hammer models are applied and compared with the same computation time step of a tenth of a second (0.1 s).

If the downstream valve is closed instantaneously, the system is obviously and dominantly unsteady so a conventional water hammer model captures the oscillation in the system. However, if the surge tank is sufficiently large (say 10 m$^2$), the response for all three models in terms of tank levels now closely approximates each other, and the difference, for most modeling purposes, may almost be ignored (Figure 5(a)). This clearly demonstrates the effectiveness of a surge tank in attenuating the pressure peaks and reducing the maximum pressure, but also that it is the overall system characteristics, and the purpose of the modeler, that largely shape the selection of model type. As the area of the surge tank is progressively reduced, though, particularly for the case of instantaneous closure, progressively more inertia and compressibility effects become evident even in the response of the tank. Figure 5(b) shows the water level with a tank that is about the same diameter as the pipe. Now the rigid water curve has moved considerably above the quasi-steady curve as acceleration head effects have become significant. Moreover, the water hammer curve is not smooth, but has peaks and valleys due to the wave propagation characteristics of the system. Clearly, the water hammer wave has more detail than that captured by the rigid water column model, which captures much more than the quasi-steady model. Significantly, only the water hammer model captures the constructive and destructive interplay of the pressure waves as they reflect back and forth between the reservoir and the valve.
model. Interestingly, the simple Joukowski approach — \( dH = \pm a/g\cdot dV = \pm 67 \text{ m} \), where \( dV = 0.6 \text{ m} \) and \( a = 1100 \text{ m/s} \) — is in good agreement with the maximum and minimum of the water hammer model. This system happens to be almost ideal for the Joukowski approach; the pump operation time is short enough to produce the potential surge and the measurement location is at the pump so the initial surge pressure is not attenuated along the pipeline. Thus, even the simplest transient analysis can be both powerful and accurate when the required conditions are met.

As the pump operation time is extended to 10 s (Figure 9(b)), the inertia and compressibility effects become less

The rigid water column is a simplistic approximation of the situation and thus represents the water level as smooth undulations of high and low water levels. The quasi steady model, in this case, effectively misses these effects.

As the initiating event is made less abrupt, the appearance of inertial and compressibility effects in any system is delayed. As Figure 6 shows, the behavior of the models, as the closure time is extended, becomes much more similar even for quite a small surge tank. There are small fluctuations in the water hammer and rigid water column model, but these are likely small enough to be ignored for most design and operational purposes. The one generalization that is obvious yet important in practice: as system disturbances become less rapid, the importance of both water hammer and inertial effects decrease. The sensitivity plot in Figure 7 shows that, for rapid closures of the valve, the water hammer model predicts high surge pressures while the rigid water column and the quasi-steady models don’t capture this detail. However as the time of closure is increased, the maximum pressures predicted by all three models gradually converge. Moreover, as more and more storage with a free surface is distributed in the system, there are more locations for water hammer waves to attenuate and fade. Interestingly, as demonstrated by the pressure sensitive demands by Jung et al. (2009), leaking pipes provide some degree of this relief function, though this is certainly a poor reason to avoid repairs. Water hammer protection is best designed, not merely an inadvertence.

**Cases study #2: pipe network**

The methods were applied to a larger and more complex system (Figure 8). This water system consists of 40 pipes, 35 junctions, 1 supply pump, and 1 tank and, for simplicity, was taken from the EPAnET user’s manual (Rossman 1993). In this example a transient is introduced from a pump shutdown causing a linear reduction of pumping discharge. Quasi-steady and water hammer models and the Joukowski approach are applied and compared with various pump closing times.

First, the linear reduction of pumping discharge in 1 s is considered. Figure 9(a) presents the head profile of quasi-steady, water hammer and Joukowski (with maximum and minimum) approaches. Since the closing time is fast enough to be classified as a rapid transient, the pressures predicted by the water hammer model are significantly different to that of the quasi-steady model. Interestingly, the simple Joukowski approach — \( dH = \pm a/g\cdot dV = \pm 67 \text{ m} \), where \( dV = 0.6 \text{ m} \) and \( a = 1100 \text{ m/s} \) — is in good agreement with the maximum and minimum of the water hammer model. This system happens to be almost ideal for the Joukowski approach; the pump operation time is short enough to produce the potential surge and the measurement location is at the pump so the initial surge pressure is not attenuated along the pipeline. Thus, even the simplest transient analysis can be both powerful and accurate when the required conditions are met.

As the pump operation time is extended to 10 s (Figure 9(b)), the inertia and compressibility effects become less
increases (e.g., 100 s), the inertia head becomes negligible — less than 0.5 m — so the quasi-steady model represents a reasonably accurate and simple model. By contrast, as the pump operation time decreases (e.g., 1 s), the inertia dominates — reaching close to 50 m — so the water hammer model is a suitable model to capture the rapid transient flow.

**Summary and conclusion**

Real pipe systems are subject to many forms of unsteadiness, and the relative importance of these changes is a function of the properties of the system, the purpose of the analysis, and the initiating source of the particular event in question. At one extreme, transient events create quiet undulations in the system that may be only a curiosity to the analyst; at the other end of the spectrum, transient events can create sudden changes in flow conditions and consequently produce high system stresses. If these forces exceed the strength of the conduit, failure can occur, or important water quality events can be initiated.

This paper reviews four numerical models of simulating transient pipe flow. The Joukowski approach is the easiest and simplest analysis that superimposes sudden changes onto a steady model. Under a set of highly restricted circumstances, the Joukowski approach can usefully estimate maximum and minimum transient pressures but these restrictions are often unrealistic. A quasi-steady model is one of the simplest ways to represent gradual dynamics in a system. Since it ignores the inertial or mass effects that real water possesses, the approaches can be applied to a limited condition that the changes in pipe velocity take place extremely slowly. The rigid water column model, although mathematically more complicated than the quasi-steady model, includes inertia effects. However, the rigid water column model excludes the compressibility effect so very rapid changes in rigid models are not realistically represented and rigid models are often overly conservative of pressure changes. Water hammer models include both the inertia effects of rigid models and the head-change-limiting role of compressibility effects. The addition of both compressibility and inertial effects comes at a price, though it causes the most complicated mathematical form that each pipe is now governed by a pair of partial differential equations. An introductory guideline for determining the degree of unsteadiness — comparing different pipe lengths $L$ and dimensionless accelerations $(dV/dt)/g$ in $g$ — is presented and then linked to an appropriate unsteady model.
The two case studies demonstrate that the specific usage of each transient model is strongly dependent on the transient and system characteristics as well as the assumption and limitation of the applied mathematical model. Real systems have many details and many challenges, in an infinite and sliding scale of complexity and challenge. Not all the details of the system are relevant to all modeling questions, and the analyst must know their purpose, not just the unstated desire for accuracy for its own sake. However, whenever it is important to capture not only the magnitude but the rate of their decay, much more complex models might be justified. Thus, the one dimensional steady friction water hammer model presented in both case studies can be further extended to an unsteady friction water hammer model or two dimensional one for the same problems; the advanced models are likely closer to reality, but the question is whether this additional accuracy is worth the computational price. Indeed, this is always the question.

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