Efficient Calculation of Transient Flow in Simple Pipe Networks

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Abstract: Extensions to the conventional method of characteristics allow transient conditions in simple pipe networks to be efficiently calculated. In particular, treating both boundary conditions and network topology in a general and comprehensive fashion simplifies the solution of many combinations of hydraulic devices. The algebraic framework presented includes a flexible integration of the friction loss term that reduces to previous linear approximations as special cases. In addition, an explicit algorithm is derived for a general hydraulic element called an external energy dissipator. This boundary condition conveniently represents surge tanks, relief valves, storage reservoirs, valves discharging to the atmosphere, and many other common devices. Significantly, the solution remains explicit even if friction losses and inertia effects are present in both the storage element and a connecting pipe. This comprehensive approach to transient analysis simplifies control logic, encourages accurate reporting of field data, and improves execution times. The procedure is illustrated by analyzing transient conditions in a small network containing a variety of devices.

Introduction

Although water distribution networks have existed for hundreds of years, transients have not always been explicitly considered in design calculations. Early networks were fed by gravity from high-elevation reservoirs or water towers, and nearly steady conditions prevailed in the system. In addition, the number, length, and diameter of pipes were typically small and pressures low. The most significant problem was to predict the equilibrium distribution of flow under assumed demand conditions. Thus, the lack of attention given to transient considerations in these networks was usually well-justified.

Modern water distribution systems, by contrast, are usually fed by numerous pumping stations discharging directly into the system. In addition, flow disturbances are common. The automatic stopping of pumps, the adjustment of control valves and the influence of accidential events, such as power outages, all generate transient conditions. These disturbances are superimposed on a network that has large pipes and high discharge rates. The result is a highly dynamic system that should be analyzed as such (Sharp 1981; Karney and McInnis 1990).

This paper is intended to facilitate calculation of transient conditions in pipe networks. Although a general algebraic and conceptual framework applicable to any network is presented, special attention is given to combinations of simple devices. No attempt is made to treat all boundary conditions comprehensively (an impossible task in a single paper). Rather, the purpose is to show how significant gains in efficiency and accuracy can be achieved for a class of boundary conditions called external energy dissipators. An explicit equation is presented for this composite storage element

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that accounts for losses in a short conduit connecting a reservoir to a junction of any number of pipes. Such a solution is useful in itself and serves as a model for solving more complex boundary conditions. The versatility and efficiency of the approach are demonstrated by analyzing a small test network.

Governing Equations and Their Solution

This section extends the conventional method of characteristics for network applications. The key result is a linear equation that accounts for any number of interacting pipes at a network node. Although a similar relation is developed in Wylie and Streeter (1982), the equation developed here is algebracially simpler while being computationally more flexible, particularly in relation to the linearization of the friction term.

Two equations, a momentum equation and a relation of mass conservation, are used to model transient flow in closed conduits [e.g., Chaudhry (1987), Wylie and Streeter (1982)]. If x is distance long the centerline of the conduit, t is time, and partial derivatives are represented as subscripts, these equations can be written as

$$V_t + gH_x + \frac{f_p V|V|}{2D_p} = 0 \quad (1)$$

$$H_t + \frac{a^2}{g} V_x = 0 \quad (2)$$

in which $H = H(x,t)$ = piezometric head; $V = V(x,t)$ = fluid velocity; D_p = inside pipe diameter; f_p = Darcy-Weisbach friction factor; a = celerity of the shock wave; and g = acceleration due to gravity. To be compatible, x and V must be positive in the same direction. Eqs. (1) and (2) are valid if the flow is one-dimensional, the conduit properties (diameter, wave speed, temperature, etc.) are constant, the convective and slope terms are small, and the friction force can be approximated by the Darcy-Weisbach formula for steady flow. In addition, it is usually assumed that the friction factor f_p is either constant or weakly dependent on Reynolds number.

The popular method of characteristics (MOC) is a simple and numerically efficient way of solving the unsteady flow equations (Wylie and Streeter 1982; Chaudhry 1987). In essence, the MOC combines the momentum and continuity expressions to form the following compatibility equation in discharge Q and head H:

$$dH \pm B\, dQ \pm \frac{R}{\Delta x} Q|Q|\, dx = 0 \quad (3)$$

in which A_p = cross-sectional area of the pipe; $B = a/gA_p$; and $R = f_p \Delta x / 2gD_pA_p^2$. Eq. (3) is valid only along the so-called C^+ and C^- characteristic lines defined by $dx/dt = \pm a$. To satisfy these characteristic relations, the x-t grid is usually chosen to ensure $\Delta x = \pm a\Delta t$ (see Fig. 1).

Once initial conditions and the space-time grid have been specified, (3) can be integrated along AP and BP in Fig. 1. Although the first two terms are easily computed, the third integral requires the variation of Q with x to be known. In practice, the flow component has usually been approximated as either $Q_A|Q_A|$ or $Q_P|Q_A|$ (Wylie 1983; Wylie and Streeter 1982; Chaudhry

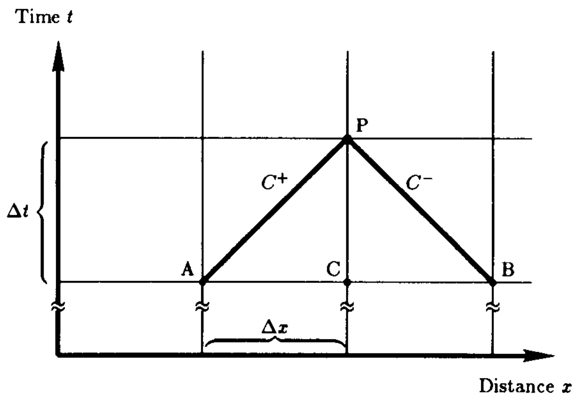

FIG. 1. Grid for Solving Pipe Problems

1987). However, both these forms can be summarized in a single equation by writing the integration as

$$\int_A^P Q|Q|\,dx = [Q_A + \varepsilon(Q_P - Q_A)]|Q_A|\Delta x \quad (4)$$

in which ε = a linearization constant. The form of this expression is motivated by the mean value theorem of integrals (Burden and Faires 1985), which requires an ε between 0 and 1 to exist for monotonic variations of discharge. Note that (4) reduces to the traditional $Q_A|Q_A|$ linearization if $\varepsilon = 0.0$, while it produces the unconditionally stable $Q_P|Q_A|$ form if $\varepsilon = 1.0$. For the approximation to be stable, $|\varepsilon|$ must not exceed one.

The weighting term ε influences the friction approximation without requiring the discretization (i.e., Δx, Δt, or a) to be changed. Hence, (4) provides an excellent way of assessing the sensitivity of a transient simulation to friction values. For example, if two values of ε produce significantly different results, the MOC grid is too coarse, and a smaller time step is required. This property is particularly useful in complex systems, since changing any other discretization parameter (such as time step or wave speed) inevitably influences other numerical approximations, thus, making it difficult to isolate the role of friction. Note also that intermediate values of ε may be more accurate for a given Δt. Preliminary indications are that values near 0.85 are almost optimal for most applications. Space restrictions prohibit a more detailed discussion of the accuracy and stability implications of ε here.

If (3) is integrated along AP and BP as mentioned previously, two equations can be written for the unknowns at P

$$H_P = C_P - B_P Q_P \quad (5)$$

and

$$H_P = C_M + B_M Q_P \quad (6)$$

in which the constants of integration are equal to

$$C_P = H_A + Q_A[B - R|Q_A|(1 - \varepsilon)] \quad (7)$$

$$B_P = B + \varepsilon R|Q_A| \quad (8)$$

$$C_M = H_B - Q_B[B - R|Q_B|(1 - \varepsilon)] \quad (9)$$

$$B_M = B + \varepsilon R|Q_B| \quad (10)$$

In more complex systems, an additional subscript is used to reference particular pipes in the system. Except for the inclusion of ε, these equations are of identical form to those presented by Wylie (1983).

To evaluate the characteristic constants (C_P, B_P, etc.), initial conditions must be known at points A and B. For a complex looped and branched system, this is not a trivial problem. To avoid spurious transients, it is usually best to obtain approximate values of heads and flows from a steady-state program. A transient program can then be used to bring these initial values into final equilibrium.

Once information has been obtained for one time step, the characteristic constants become knowns in the solution. Thus, at a point P internal to a pipeline, H_P can be eliminated from (5) and (6) to obtain

$$Q_P = \frac{C_P - C_M}{B_P + B_M} \quad (11)$$

At the ends of a conduit, however, the solution of the characteristic equations is algebrically complicated by one or more boundary conditions. Only once this auxiliary relation has been specified can the solution be obtained.

Boundary Conditions

The method of characteristics provides a systematic way of calculating transient conditions within (i.e., internal to) a pipeline. However, if this orderly approach is to be extended to the full range of hydraulic devices found in a network, a more powerful framework is required. The first step is to present a systematic classification of simple network devices. The goal here is to avoid arbitrary and unnecessary restrictions that might confuse algorithm development, complicate debugging, or focus undue attention on irrelevant details.

To be more specific, the following sections demonstrate how an entire class of devices can be reduced to the solution of a single equation. This approach has the following advantages over the network formulations presented by others (Fox 1977; Koelle 1982; Wylie and Streeter 1982; Watters 1984): (1) It reduces code size and memory requirements; (2) it improves accuracy, since a more realistic description of device behavior is included in the formulation; (3) it has fewer restrictions on how devices are connected at multipipe junctions; (4) it reduces execution times; (5) it simplifies the structure of the algorithm, thereby making code easier to write, maintain, modify and debug; (6) it explicitly accounts for fluid friction, inertia, and minor losses in all elements; and (7) it encourages more accurate and reliable reporting of field data. To achieve these important benefits, the pipes, reaches, sections, nodes, and boundary conditions that make up a network must be carefully defined.

Definitions

Once a time step Δt has been selected, the method of characteristics divides most conduits in the network into one or more reaches of length Δx. For clarity, the term *pipe* is henceforth restricted to conduits that contain at least one characteristic reach. The end of each reach, where head and flow values must be determined, is called a section. At sections internal to a pipe, the discharge can be obtained from (11). However, at each end of the pipe an auxiliary relation between head and discharge must be specified. Such a head-discharge relation is called a boundary condition.

The term *node* indicates a location where boundary sections meet. The degree of a node indicates the number of pipes (i.e., characteristic sections) connected to it. In most networks, however, various other elements besides pipes are connected to nodes. These other devices might include the discharge flange of a pump or a relief wave discharging to the atmosphere. All nonpipe junctions are labeled external and the number of such connections is called the complexity of the node. A node of complexity zero is called simple, a node of complexity one is called ordinary, and a node of complexity greater than one is called complex. In general, high-complexity nodes are more difficult to solve while the number of pipe connections does not influence the solution procedure. The terminology related to nodes can be extended in a natural way to networks as well (Karney 1984).

Simple and Ordinary One-Node Boundary Conditions

To exploit the developments presented thus far, consider Fig. 2, which depicts a frictionless junction of any number of pipes meeting at a node. Let N_1 be the set of all pipes whose assumed direction is toward the node, let N_2 be those pipes whose assumed direction is away from the junction, and let H_P be the nodal hydraulic grade-line elevation. Also, let one flow be identified as external and governed by an auxiliary relation (a boundary condition). Positive flows are assumed to be from the junction.

For all pipes belonging to the set N_1, (5) holds, while (6) applies for members of N_2. These equations can be rearranged to obtain

$$Q_{P_i} = -\frac{H_P}{B_{P_i}} + \frac{C_{P_i}}{B_{P_i}}, \qquad i \in N_1 \quad \text{(12)}$$

and

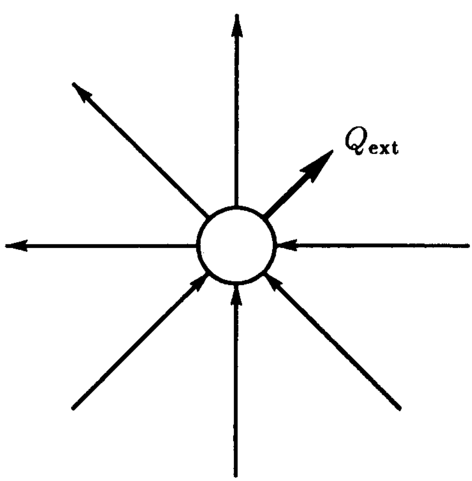

FIG. 2. Generalized Node with One External Flow

$$-Q_{P_j} = -\frac{H_P}{B_{M_j}} + \frac{C_{M_j}}{B_{M_j}}, \qquad j \in N_2 \quad (13)$$

in which the second subscript represents a particular pipe in the set.

The continuity equation for the junction requires the total flow entering the node to equal the flow leaving the node. That is

$$\sum_{i \in N_1} Q_{P_i} - \sum_{j \in N_2} Q_{P_j} - Q_{ext} = 0 \quad (14)$$

Eqs. (12) and (13) can be substituted into (14) to produce an expression for H_P

$$H_P = C_c - B_c Q_{ext} \quad (15)$$

in which

$$B_c = \left(\sum_{i \in N_1} \frac{1}{B_{P_i}} + \sum_{j \in N_2} \frac{1}{B_{M_j}} \right)^{-1} \quad (16)$$

and

$$C_c = B_c \left(\sum_{i \in N_1} \frac{C_{P_i}}{B_{P_i}} + \sum_{j \in N_2} \frac{C_{M_j}}{B_{M_j}} \right) \quad (17)$$

Eq. (15) represents a single relationship between junction head (H_P) and external flow (Q_{ext}) in a multipipe frictionless junction. It is equivalent (except for ε) to the general network equation in Wylie and Streeter (1982).

The form of (15) is equivalent to a single C^+ compatibility equation. Thus, any boundary condition located at an ordinary node of arbitrary degree can be evaluated exactly as if it was at the downstream end of a single pipe. Although this fact makes the general equation trivial to implement within existing code, this convenience does not appear to be generally exploited.

Once a functional relationship representing a particular hydraulic device is substituted into (15), a single equation and unknown results. If this relationship is either linear or quadratic, an explicit formula for the unknown can be obtained. The next section illustrates the procedure for several simple boundary conditions.

Simple Notes and Linear Reservoirs

The simplest boundary conditions occurs when Q_{ext} is either constant or a known function of time (e.g., constant displacement pumps or fixed demands). In this case, the value of Q_{ext} can be substituted into (15) to obtain the junction head. In particular, this equation becomes $H_P = C_c$ when Q_{ext} is zero. This solution for a simple node is algebraically equivalent to (11) if the node has only two pipes.

The continuity equation for a reservoir relates the rate of change of water surface elevation V_r to the external discharge. That is, $Q_{ext} = A_r V_r$, in which A_r = the cross-sectional area of the reservoir. If average values over a time step Δt are used, the result is

$$H_r = H_0 + B_0(Q_e + Q_{ext}) \quad (18)$$

in which H_r = the hydraulic grade-line elevation of the surface of the reservoir; H_0 = the reservoir head at the beginning of the time step; Q_e = the initial external discharge; and $B_0 = \Delta t/2A_r$. Note that if the constant B_0 is zero, the reservoir head is independent of the external discharge. The term H_0 may be either constant or a known function of time (e.g., waves on a reservoir).

If hydraulic losses between the reservoir and the pipe junction are negligible, the nodal head equals the surface elevation of the reservoir. This allows (15) and (18) to be solved simultaneously to produce an expression for the external flow. The result is

$$Q_{ext} = \frac{C_c - H_0 - B_0 Q_e}{B_c + B_0} \quad \text{(19)}$$

Eq. (19) can be used to represent constant head reservoirs, storage tanks, and simple surge tanks. Collectively, these devices are called simple reservoirs.

When the cross-sectional area of the reservoir is small, the analysis should account for both head losses and inertia in the tank. A convenient linear representation is known as a lumped inertia element (Wylie and Streeter 1982) and can be written

$$H_b - H_r = C_1^r + C_2^r Q_{ext} \quad \text{(20)}$$

in which H_b = the head at the base of the tank; H_r = the water surface elevation; and the terms C_1^r and C_2^r = constants related to inertia and friction effects. To be specific, if the improved $Q_{ext}|Q_e|$ integration is used in the inertia element, C_1^r and C_2^r can be expressed as

$$C_1^r = H_0 - H' - \frac{2L_r Q_e}{gA_r \Delta t} \quad \text{(21)}$$

and

$$C_2^r = \frac{2L_r}{gA_r\Delta t} + \frac{f_r L_r}{gD_r A_r^2}|Q_e| \quad \text{(22)}$$

in which H' = the head at the base of the reservoir at the beginning of the time interval; L_r = the initial length of the water column; and D_r = the reservoir diameter. By definition, $H_0 = Z_{bot} + L_r$, in which Z_{bot} = the elevation of the base of the tank.

The reservoir storage relation [(18)] and inertia-head loss relation [(20)] can be combined to produce

$$H_b = C_b + B_b Q_{ext} \quad \text{(23)}$$

in which $C_b = C_1^r + H_0 + B_0 Q_e$; and $B_b = C_2^r + B_0$. Note that this relation is linear and, thus, no more difficult to solve than the simple reservoir model. If friction losses are small, the head at the node H_P equals the head at the base of the reservoir H_b. In this case, solving (15) and (23) produces

$$Q_{ext} = \frac{C_c - C_b}{B_c + B_b} \quad \text{(24)}$$

Note that the simple reservoir relation [(19)] can be recovered from (24) by setting the C_1^r and C_2^r to zero. In addition, if $B_0 = 0$, a constant head reservoir is produced. Thus, (24) represents a general storage element and is called a linear reservoir.

External Flows with Energy Dissipation

If flow passes into a reservoir through a restriction, a general loss and storage expression may be derived. This combination device is called an external energy dissipator (EED) and can be used to represent valves or orifices discharging to linear reservoirs, restricted orifice surge tanks, one-way surge tanks, or pressure relief valves. Having a single solution for all these elements simplifies control logic and the preparation of input data.

Fig. 3 depicts the general energy dissipating boundary condition: A valve discharges through a short connector into a linear reservoir. (Note that the exact location of the orifice element within the connector is irrelevant if the orifice opening is independent of pressure.) The node may be attached to any number of pipes. In addition, head losses and inertia can be accounted for in the tank and the connector. The external flow is related to the head at the junction by the orifice expression

$$Q_{ext} = s\tau E_s\sqrt{s(H_P - H_p^c)} \quad (25)$$

in which H_p^c = the head at the node side of the connector; and s takes the sign of the external flow (i.e., $s = \text{sign}(Q_{ext}) = \pm 1$). The terms τ and E_s in (25) are valve or orifice parameters; by convention, $\tau = 1.0$ implies a fully open valve, while a value of zero requires the valve to be closed. Opening and closing valves can be represented if τ is a function of time. The valve scaling constant E_s actually represents two values: E_+ for flow

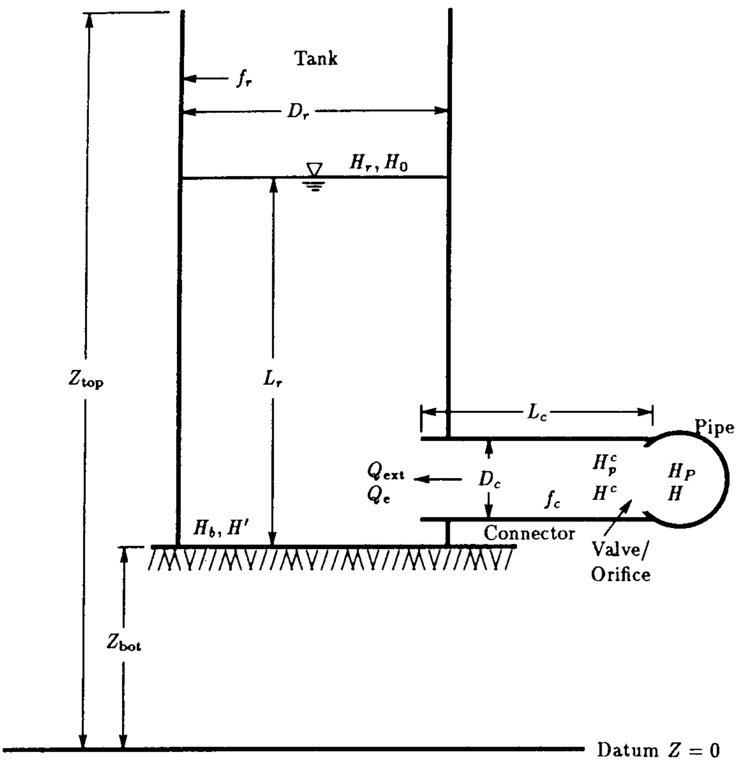

FIG. 3. Generalized External Dissipator and Input Variables

from the network when $s = +1$ and E_- for negative flow. The values of these terms are determined by knowing H_P, H_p^c, H_p^c, τ, and Q_{ext} for one positive and one negative flow. Setting either E_+ or E_- to zero produces instantaneous check valve action in the associated direction.

As for the reservoir, the connector can be modeled as a lumped inertia element. Referring to Fig. 3 for appropriate notation, the equation can be written

$$H_p^c - H_b = C_1^c + C_2^c Q_{ext} \quad (26)$$

in which

$$C_1^c = H' - H^c - \frac{2L_c}{gA_c\Delta t} Q_e \quad (27)$$

and

$$C_2^c = \frac{2L_c}{gA_c\Delta t} + \frac{f_c L_c}{gD_c A_c^2} |Q_e| \quad (28)$$

If the connector is not present, the constants C_1^c and C_2^c can be set to zero.

Frequently, the hydraulic grade-line elevation H_b is constant (say the valve or orifice discharges to the atmosphere or to a constant head reservoir). Nevertheless, few complications arise by assuming H_b is governed by the linear reservoir equation. The solution is obtained by squaring (25) and appropriately substituting equations (15), (23), and (26). The resulting quadratic in Q_{ext} can be written

$$Q_{ext}^2 + 2mQ_{ext} + n = 0 \quad (29)$$

in which

$$m = \frac{(\tau E_s)^2}{2} \cdot s(B_b + B_c + C_2^c) \quad (30)$$

and

$$n = (\tau E_s)^2 \cdot s(C_b + C_1^c - C_c) \quad (31)$$

Consideration of the flow direction in (25) leads to

$$Q_{ext} = -m + s\sqrt{m^2 - n} \quad (32)$$

in which $s = \text{sign}(C_c - C_b - C_1^c)$.

Eq. (32) explicitly determines the discharge through an energy dissipating contraction into a linear reservoir from an ordinary network node. The storage element in (32) may represent any receiving body or structure. Thus, it includes constant head reservoirs, storage tanks, simple reservoirs, linear reservoirs, as well as the soil or the atmosphere. The head loss may occur through a control valve or orifice, and check valve action is permitted. Any of these options can be selected by simply choosing appropriate values of the formulation constants.

It is emphasized that (32) handles all previously mentioned boundary conditions as special cases. This power is better appreciated if the solution presented here is compared with conventional approaches. For example, more than 12 individual boundary conditions are used in Watters (1984) to represent some of the behavior summarized by the EED. Similar numbers apply to both Wylie and Streeter (1984) and Chaudhry (1987). Despite this,

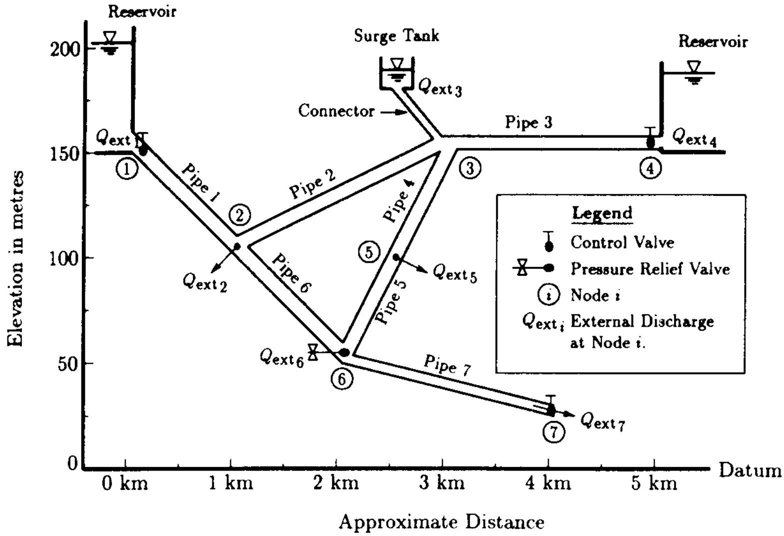

FIG. 4. Definition Diagram for Simple Network

none of the solutions presented in standard references are as general as those presented here. In particular, inertia and friction effects in the tank or the connecting pipe are usually not included in general elements.

Example Network

To illustrate the preceding formulation, a simple network consisting of seven pipes and seven nodes (Fig. 4) is solved. Five boundary conditions (EEDs) are depicted in the drawing as well as two nodes with a fixed demand. No additional difficulty arises if the demands at nodes 2 and 5 are specified functions of time.

In essence, the solution of this network only requires five calls to the external dissipator subroutine at each time step. The EED routine computes the current value of the external flow for each device and allows the nodal characteristic equation [(15)] to calculate the head. The procedure is summarized in pseudocode in Fig. 5. The simulations were performed with a computer program incorporating an extended version of the simple network model presented in this paper.

Data Specification

The information required to describe the system can be split into three groups—node data, pipe data, and boundary condition data. Each group is characterized both by physical parameters and by an initial condition.

Table 1 describes the topology of the network, the initial pressure head, and the external flow at each node. The last column describes the type of device and is included for reference only. Positive demands indicate flow out of the system, while negative flows are supplied to the network.

The connectivity of the system and the initial discharge for each pipe are described in Table 2. The pipe parameters include the pipe length, diameter, wave celerity, and friction factor. A positive discharge means that the flow direction coincides with that implied by the nodal designations.

1. **Initialize Steady State**
2. **While $t \leq t_{\text{final}}$**
   - For $i = 1, 3, 4, 6$ and 7, **CALL EXTERNAL(Q_{ext_i})**
     - ◇ Compute MOC constants C_c and B_c using Eqns. 16 and 17.
     - ◇ Compute inertia constants C_1^r, C_2^r, C_1^c and C_2^c using Eqns. 21, 22, 27 and 28.
     - ◇ Compute linear reservoir constants C_b and B_b (in Eqn. 23).
     - ◇ Compute valve opening τ (in Eqn. 25).
     - ◇ Compute quadratic constants m and n using Eqns. 30 and 31.
     - ◇ Compute sign of external flow s (in Eqn. 32).
     - ◇ **Compute external discharge** Q_{ext_i} using Eqn. 32.
     - ◇ Back calculate updated head valuess H_p, H_b, H_r, and H_p^c using Eqns. 15, 23, 20 and 25, respectively.
   - For $i = 2$ and 5
     - ◇ Compute C_c and B_c using Eqns. 16 and 17.
     - ◇ Calculate $H_{p_i} = C_c - B_c \cdot Q_{\text{ext}_i}$
   - For all i
     - ◇ For all pipes attached to Node i

       Solve C^+ or C^- for Q_{p_j} (Eqn. 5 and 6).
   - Set $t = t + \Delta t$
3. **STOP**

FIG. 5. Pseudocode for External Dissipator

TABLE 1. Nodal Steady-State Data

Node number (1)	Elevation (m) (2)	Hyraulic grade-line elevation (m) (3)	Q_{ext_i} (m^3/s) (4)	Device description (5)
1	150	200.0	−6.211	constant head reservoir
2	100	195.0	+2.000	constant demand
3	150	188.8	+0.000	surge tank
4	150	175.0	+1.183	constant head reservoir
5	100	183.4	+1.000	constant demand
6	50	187.9	+0.000	pressure relief valve
7	25	151.9	+2.028	control valve to atmosphere

The data in Table 3 completely specify the physical characteristics of the five EED boundary conditions. Note that the data for the constant head reservoirs and the control valves (at nodes 6 and 7) are almost identical. The distinguishing feature is that one coefficient (E_-) is zero for valves discharging to the atmosphere. Reservoir valves (orifices) usually permit flow in both directions. Otherwise, a valve discharging to the atmosphere is equivalent to an infinite area reservoir.

All valves (orifices) are considered fully open ($\tau = 1$), except the control valve at node 7 and the pressure relief valve. The relief valve is initially closed, but it is set to open if the nodal head exceeds 210 m. In this case,

TABLE 2. Pipe Physical Data and Initial Flows

Pipe number (1)	From node (2)	To node (3)	Initial flow (m^3/s) (4)	Length (m) (5)	Diameter (m) (6)	Wave speed (m/s) (7)	Darcy friction (8)
1	1	2	6.212	1,001.2	1.500	996.3	0.012
2	2	3	1.708	2,000.0	1.000	995.3	0.013
3	3	4	1.183	2,000.0	0.750	995.0	0.014
4	3	5	0.524	502.5	0.500	1,000.0	0.015
5	6	5	0.476	502.5	0.500	1,000.0	0.015
6	2	6	2.503	1,001.2	1.000	996.3	0.014
7	6	7	2.028	2,000.2	0.750	995.1	0.013

TABLE 3. Data Specification for External Dissipative Devices

External flow device (1)	Valve/Orifice		Tank/Reservoir				Connector		
	E_- ($m^{5/2}/s$) (2)	E_+ ($m^{5/2}/s$) (3)	Z_{bot} (m) (4)	Z_{top} (m) (5)	A_r (m^2) (6)	f_r (7)	L_c (m) (8)	D_c (m) (9)	f_c (10)
Q_{ext_1}	5.0	5.0	150	201.5	∞	0	0	>0	0
Q_{ext_3}	3.0	3.0	180	195.0	5.0	0.020	30	0.500	0.020
Q_{ext_4}	1.0	1.0	150	173.6	∞	0	0	>0	0
Q_{ext_6}	0.0	0.049	50	50.0	∞	0	0	>0	0
Q_{ext_7}	0.0	0.300	25	25.0	∞	0	0	>0	0

the valve opens linearly in 3 s and then closes linearly in 60 s. The control valve at node 7 is partially closed ($\tau = 0.6$ initially), while its behavior is specified differently for each test case:

- Case 1. The control valve opening τ decreases linearly from $\tau = 0.6$ to $\tau = 0.2$ in 10 s.
- Case 2. The control valve opening τ decreases linearly from $\tau = 0.6$ to $\tau = 0.2$ in 10 s. This valve setting is maintained for 15 s. The valve then reopens linearly to its initial value of $\tau = 0.6$ in 5 s.

The transient behavior of the system caused by operating the control valve at node 7 was simulated during 60 s for both cases. The time step selected for the computer simulations was approximately 0.1 s. The pipe system was discretized into 90 pipe reaches, each about 100 m in length, using the method of wave-speed adjustments (Wylie and Streeter 1982). Initial conditions were obtained by operating the transient model with fixed valve settings until the solution reached steady flow.

Simulation Results

Table 4 summarizes the maximum and minumum hydraulic grade-line elevations obtained for case 1 and case 2 over the length of the simulation. Because the valve operation is the same over the first 10 s of simulation, the peak heads are identical for both cases. The minimum heads are somewhat lower for case 2, since the reopening of the control valve produces a