Efficient Inverse Transient Analysis in Series Pipe Systems

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Abstract: Measurement of the changes in heads initiated by a valve closure are used to calibrate the friction factors in a hydraulic system consisting of a series-connected pipeline and a downstream valve joining two reservoirs. The calibration is performed by minimizing the inverse least-squares problem representing the difference between the measured and predicted transient response in the network. The hydraulic transient is modeled using the method of characteristics. Unlike previous approaches, the sensitivity information required for the minimization or error is obtained by direct differentiation of the method of characteristic equations and the valve equation with respect to the friction factors. The sensitivity of the calibration results to the amount of data included in the analysis, the initial friction factor estimates, and modeling and simulated measurement errors is discussed.

INTRODUCTION

Inverse transient analysis (Liggett 1993; Liggett and Chen 1994a,b) allows multiple operating conditions to be implicitly considered during calibration. By approaching calibration from the transient viewpoint, the need to locate a series of approximately steady states in the available head and flow data is largely eliminated. When inverse transient analysis is used, calibration is better when the head and flow vary widely in a short time span than for an approximately steady state (Liggett and Chen 1994b).

To highlight the simplicity of the approach to calibration proposed here, this work uses numerical experiments in a trivial hydraulic system consisting of two reservoirs joined by series-connected pipes with a valve located at the downstream voir (Fig. 1). This system is used primarily to investigate several extensions to the previously published implementations of inverse transient analysis. The valve is the key dynamic element, providing both a means of assessing the applicability of inverse transient analysis and a means of initiating transient events. Computational advantages of the method of characteristics (MOC) are explicitly exploited in a "direct-equation" implementation of inverse transient analysis, which differs from the previously reported matrix-based (or adjoint) approach of Liggett and Chen (1994a,b). This new formulation calculates, by differentiating the characteristic and valve equations, the sensitivity information required by the optimization algorithm. Data requirements and the sensitivity of results to modeling and measurement errors are discussed. Extensions of the new approach to more complex systems are simple and straightforward.

DIRECT-EQUATION APPROACH

The inverse transient analysis presented by Liggett and Chen (1994a,b) is a matrix-based formulation. The direct-equation approach introduced here is motivated by a desire to use the MOC to maximum advantage. The MOC propagates information along characteristic lines that join points on a space-time grid. The state variables at a given grid point can be calculated independent of those at all other grid points for the given time step. Unlike the adjoint approach, the direct-equation method clearly indicates this relationship and properly preserves the domain of dependence at each point in the solution. Moreover, the direct-equation approach may execute an order of magnitude more quickly.

MOC EQUATIONS

Transient flow is governed by two quasi-linear partial differential equations, typically solved using the MOC (Chaudhry 1987; Wylie and Streeter 1993). To use this approach, a time step Δt is selected and an x-t grid is selected so that $\Delta x_j = a_j \Delta t$, where a_j is the wave speed of the pipe and Δx_j is the spatial grid size for pipe j. The following points are defined on the grid: P has coordinates (x_P, t_P); L has coordinates ($x_P - \Delta x$, $t_P - \Delta t$); and R has coordinates ($x_P + \Delta x$, $t_P - \Delta t$). Linearizing the friction term after Karney and McInnis (1992), the relationship between the state variables (H and Q) at the grid points is given as

$$H_P = C_P - B_P Q_P; \quad H_P = C_M + B_M Q_P \qquad (1a,b)$$

The characteristic constants are given by $C_P = H_L + Q_L[B_j - R_j|Q_L|(1 - \varepsilon)]$, $B_P = B_j + \varepsilon R_j|Q_L|$, $C_M = H_R - Q_R[B_j - R_j|Q_R|(1 - \varepsilon)]$, and $B_M = B_j + \varepsilon R_j|Q_R|$, where $B_j = a_j/gA_j$; g = acceleration due to gravity; A_j = cross-sectional area of the pipe; $R_j = f_j\Delta x_j/2gD_jA_j^2$; f_j = Darcy-Weisbach friction factor; D_j = pipe diameter; and the friction linearization term ε is constrained by $0 \le \varepsilon \le 1$.

FORWARD CALCULATIONS

When system parameters are known, (1) allows the forward problem—the calculation of H and Q throughout the system—to be solved, provided that a relationship between H_P and Q_P is specified at each boundary location. For common appurtenances the nature of the relationship can be found in standard references [e.g., Chaudhry (1987), Wylie and Streeter (1993)]. In the adjoint method (Liggett and Chen 1994a,b), two compatibility equations are written for each interior calculation section, and at each boundary location one characteristic equation and one boundary condition are specified. This system is solved simultaneously for the values of H and Q at each section in the system. In the direct-equation approach, the two equations are combined to yield one equation in one unknown. When appropriate boundary (and initial) conditions are known, one of the two state variables can be obtained directly and the remaining variable found through back substitution. Although the specifics of the MOC solution for the system shown in Fig. 1 are well known, the required equations are summarized here to facilitate computation of the required sensitivity information.

Because the head at the upstream reservoir H_1 is specified, the flow is found using the negative compatibility equation

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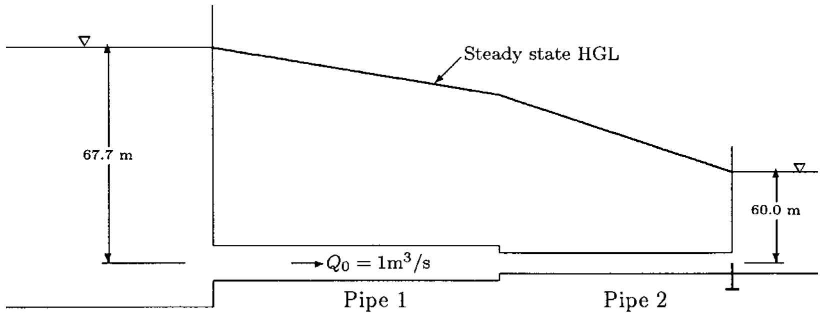

FIG. 1. Sketch of Test System Showing Initial Conditions

passing through this point. The state variables at interior points are calculated using (1), which can be combined to give the unknown flow Q_P

$$Q_P = \frac{C_P - C_M}{B_P + B_M} \tag{2}$$

and the unknown head H_P

$$H_P = \frac{C_P B_M + B_P C_M}{B_P + B_M} \tag{3}$$

At the flow control valve connecting the downstream reservoir, the applicable relations are the positive characteristic equation

$$H_{N+1} = C_P - B_P Q_{N+1} \tag{4}$$

and the valve equation

$$Q_{N+1} = sE_s\tau\sqrt{s(H_{N+1} - H_{DS})} \tag{5}$$

where $s = \text{sign}(H_{N+1} - H_{DS})$; E_s = known valve constant that accounts for the orifice loss and the cross-sectional area; τ = relative effective gate opening; H_{DS} = downstream reservoir head; and the subscript $N+1$ represents the valve location. Combining (4) and (5), and solving gives

$$Q_{N+1} = \frac{-sE_s^2\tau^2 B_P + \sqrt{(sE_s^2\tau^2 B_P)^2 - 4sE_s^2\tau^2(H_{DS} - C_P)}}{2} \tag{6}$$

The head at the valve location, H_{N+1}, can be found by using the result of (6) in (4).

INVERSE CALCULATIONS

The estimation of friction factors (or any other input parameter) based on measured system variables—heads in this case—is termed the inverse problem. The inverse calculations often proceed by minimizing a least-squares merit function using the Levenberg-Marquardt method (Liggett and Chen 1994a), although other minimization approaches (e.g., genetic algorithms) may have application as well. The primary difference in the current work is that the sensitivity information is extracted directly from the MOC equations for the upstream and downstream reservoirs and the interior sections. Separate calculations are provided for the initial steady state and for transient conditions.

For the initial steady-state conditions it is assumed that $\partial Q_P/\partial f_i = 0$ at all sections. At the upstream reservoir, $\partial H_1/\partial f_i = 0$, where the subscript 1 indicates the start of the first reach. The sensitivity of the remaining steady-state heads to the friction factor in pipe i is calculated as

$$\frac{\partial H_P}{\partial f_i} = \frac{\partial H_{P-1}}{\partial f_i} - Q_0^2 \frac{\Delta x_k}{2gD_k A_k^2}\frac{\partial f_k}{\partial f_i} \tag{7}$$

where $(P - 1)$ denotes the point with coordinates $(x_P - \Delta x, t_P)$, which is one reach upstream of P, the point with coordinates (x_P, t_P); k represents the pipe upon which P is located; and $\partial f_k/\partial f_i = 1$ if $i = k$, and is equal to zero otherwise.

Under transient conditions, the sensitivity of the upstream reservoir head to the friction factor in pipe i, $\partial H_1/\partial f_i$, is equal to zero for all pipes. The sensitivity of the flow at the upstream reservoir to the friction factor in pipe i is calculated from the negative compatibility equation as

$$\frac{\partial Q_1}{\partial f_i} = -\frac{1}{B_M}\left[\frac{\partial C_M}{\partial f_i} + \frac{(H_1 - C_M)}{B_M}\cdot\frac{\partial B_M}{\partial f_i}\right] \tag{8}$$

Flow and head sensitivities at interior sections are calculated from (2) and (3), respectively, as

$$\frac{\partial Q_P}{\partial f_i} = \frac{1}{B_P + B_M}\left[\frac{C_M - C_P}{B_P + B_M}\left(\frac{\partial B_P}{\partial f_i} + \frac{\partial B_M}{\partial f_i}\right) + \left(\frac{\partial C_P}{\partial f_i} - \frac{\partial C_M}{\partial f_i}\right)\right] \tag{9}$$

$$\frac{\partial H_P}{\partial f_i} = \frac{1}{B_P + B_M}\left[\frac{\partial C_P}{\partial f_i}B_M + \frac{\partial B_M}{\partial f_i}C_P + \frac{\partial B_P}{\partial f_i}C_M + \frac{\partial C_M}{\partial f_i}B_P - \frac{C_P B_M + B_P C_M}{B_P + B_M}\left(\frac{\partial B_M}{\partial f_i} + \frac{\partial B_P}{\partial f_i}\right)\right] \tag{10}$$

Similarly, the sensitivities at the downstream reservoir are calculated from (4) and (6), respectively, as

$$\frac{\partial H_{N+1}}{\partial f_i} = \frac{\partial C_P}{\partial f_i} - B_P\cdot\frac{\partial Q_{N+1}}{\partial f_i} - Q_{N+1}\cdot\frac{\partial B_P}{\partial f_i} \tag{11}$$

$$\frac{\partial Q_{N+1}}{\partial f_i} = \frac{1}{2}\left\{-sE_s^2\tau^2\frac{\partial B_P}{\partial f_i} + \frac{1}{2}\left[(sE_s^2\tau^2 B_P)^2 - 4sE_s^2\tau^2(H_{DS} - C_P)\right]^{-1/2}\cdot\left[2(sE_s^2\tau^2)^2 B_P\frac{\partial B_P}{\partial f_i} + 4sE_s^2\tau^2\frac{\partial C_P}{\partial f_i}\right]\right\} \tag{12}$$

where $\partial H_{N+1}/\partial f_i$ is given by (11).

The quantities $\partial C_P/\partial f_i$, $\partial B_P/\partial f_i$, $\partial C_M/\partial f_i$, and $\partial B_M/\partial f_i$ are readily calculated from directly differentiating the characteristic constants. For example, the derivatives of the positive characteristic constants are

$$\frac{\partial C_P}{\partial f_i} = \frac{\partial H_L}{\partial f_i} + \frac{\partial Q_L}{\partial f_i}\left[\frac{a_j}{gA_j} - \frac{f_j\Delta x_j}{2gD_jA_j^2}|Q_L|(1 - \varepsilon)\right] + Q_L\left[\frac{(\varepsilon - 1)\Delta x_j}{2gD_jA_j^2}\left(|Q_L|\frac{\partial f_j}{\partial f_i} + f_j\frac{\partial|Q_L|}{\partial f_i}\right)\right] \tag{13}$$

$$\frac{\partial B_P}{\partial f_i} = \frac{\varepsilon\Delta x_j}{2gD_jA_j^2}\left(\frac{\partial f_j}{\partial f_i}|Q_L| + f_j\frac{\partial|Q_L|}{\partial f_i}\right) \tag{14}$$

The interpretation of these relations is suggestive and physically satisfying. In addition to the state variables (Q_L, H_L, Q_R, and H_R), Jacobian terms ($\partial Q_L/\partial f_i$, $\partial H_L/\partial f_i$, $\partial Q_R/\partial f_i$, and $\partial H_R/\partial f_i$) from the previous time step appear in $\partial C_P/\partial f_i$, $\partial B_P/\partial f_i$, $\partial C_M/\partial f_i$, and $\partial B_M/\partial f_i$. These terms allow the current grid point P to inherit the sensitivities from the previous time step (grid points L and R) and from the current time step. Thus, sensitivity information is propagated through the system in a transient event along with the state variables. The accumulated sensitivity information at the end of each simulation becomes input to the Levenberg-Marquardt method (Press et al. 1992), and new estimates of the friction factors are obtained. After this, the forward problem is resolved using the newly estimated friction factors. The cycle continues until convergence.

SENSITIVITY ANALYSIS USING PERFECT DATA

In this section, the system shown in Fig. 1 is calibrated using the direct-equation approach and the Levenberg-Marquardt minimization algorithm. The data used to drive the inverse problem are generated by a run of the forward algorithm. The relevant parameters are L = 550 m, D = 750 mm, a = 1,100 m/s, and f = 0.010 for pipe 1, L = 450 m, D = 600 mm, a = 900 m/s, and f = 0.012 for pipe 2; E_s = 4.11 $m^{5/2}$/s, and the time for a wave to travel from one reservoir ($\Sigma L/a$) to the other is 1 s. The valve is assumed to close linearly in either 2 or 6 s. Each pipe is divided into two reaches, so that no discretization error is produced.

The time steps used in simulating hydraulic transient behavior are short, allowing a large amount of data to be quickly generated. However an interesting question arises: Should all of the available head data be used for calibration? The answer is not immediately obvious. If, for example, an accurate calibration could be performed using only a portion of the available head record, using the entire record would be unnecessary and inefficient. As the simulation duration increases, the execution time for each iteration also increases; yet, the accuracy of the calibration should also increase because the least-squares merit function is summed over all measurement points for all time steps. Conversely, if only a few time steps are used after the onset of the transient event, each iteration will require less time, but accuracy will be lowered. Clearly the preferred range should exist somewhere between these two extremes, a situation that is likely common to inverse applications.

Fig. 2 explores this relationship by showing the accuracy of the converged results as the amount of data included in the analysis is varied by changing the simulation time. In this example, the inverse problem is driven by the head measured at the junction between the two pipes. The valve is closed in 6 s; thus, the information that the valve has fully closed reaches the junction at t = 6.5 s. The benefit of including this initial pressure surge in the analysis is clear—accurate results are obtained as long as this initial surge reaches the measurement point. It was also found that beginning at t = 6.5 s there is a local minimum in program execution time that extends for approximately ~2 s ($2\ \Sigma\ L/a$). These results, and results from other examples, suggest that the optimal time period for the analysis should be in the interval ending at least two wave travel times ($2\ \Sigma\ L/a$) following the initial pressure surge.

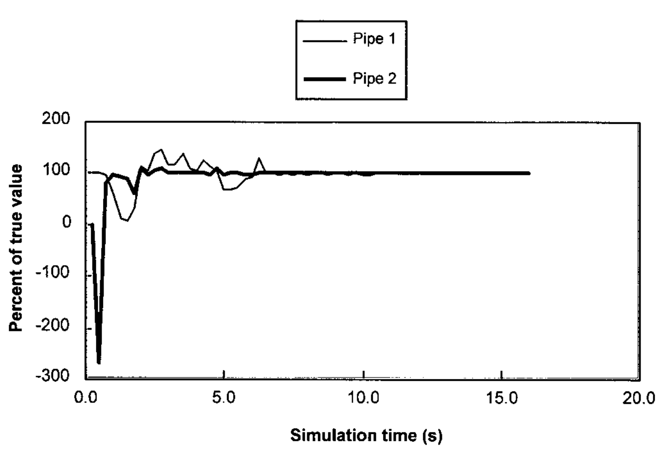

FIG. 2. Accuracy of Calibration for Perfect Data

Sensitivity of the results to initial friction factor estimates was also examined. A valve closure time of 2 s was assumed, and the head used as input to the inverse problem is from the middle of the downstream pipe. The simulation time chosen was 2 s, and the initial pressure surge arrives at the measurement point at t = 1.25 s. Eight different initial values in each pipe were used for the friction factors, varying from a 1000th to 100 times the true value. The friction factors recognized at convergence were at most 1% different than the true value—an encouraging result given the extreme range of values tested. In addition, it was found that the results were insensitive to the values of the Levenberg-Marquardt parameters. In fact, changing these parameters by several orders of magnitude did not significantly affect the results, although very large increases did produce less accuracy at convergence for small simulation times. The values suggested by Press et al. (1992) were used for all remaining tests.

SENSITIVITY ANALYSIS USING NONPERFECT DATA

Errors are introduced to the analysis through both the modeling process and the data. Because the calibration process depends on the minimization of a least-squares function, both sources of error affect the accuracy of the converged results. This section briefly considers two sources of modeling error and simulated data collection error.

The MOC compatibility equations include the friction linearization constant ε (Karney and McInnis 1992). The head and flow values calculated during a solution of the forward problem change as this value is altered. Error was introduced into the analysis by generating data to be used as input to the inverse problem using $\varepsilon = 0$, and then specifying that a different value be used during the analysis. When error was introduced in this manner, it was found that the accuracy at convergence recognition could be improved by including more data in the analysis. Not surprisingly, the closer the assumed value of ε is to the value used to generate the data, the more accurate the results. Thus, for a field application, a value of ε that most accurately accounts for the physical processes should be used. This could be obtained by letting the optimization search for the best value of ε in exactly the same way as it is searching for value of the friction factor. The required sensitivity information for ε can also be obtained from differentiation of the MOC equations.

Error is also introduced during the discretization process. Extending the length of the second pipe in the system by 10 m created a discretization problem that was resolved here using the wavespeed adjustment method (Chaudhry 1987; Wylie and Streeter 1993). Input data were generated by specifying that the hydraulically shortest pipe in the system be divided into 10 reaches. These data were then used when it was specified that the shortest pipe should be divided into either two, or five reaches. The size of the time step decreases as the number of reaches on the hydraulically shortest pipe increases. For a given simulation time, the error remaining at convergence recognition was lower for the case of five reaches, indicating the benefits of incorporating more data into the inverse analysis.

Finally, field data were simulated by adding a normally distributed error to the data generated by a solution of the forward problem. The mean of the error term was taken as zero and the standard deviation was specified as indicated in Fig. 3, which shows the accuracy of the Pipe 1 calibration when the head is measured at the middle of Pipe 2. The plots are averaged over five runs of the algorithm. As for the cases where error was introduced through modeling, the benefit of including more data in the analysis can be seen.

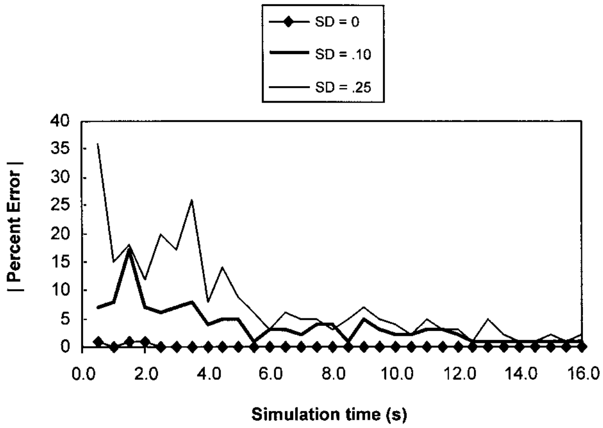

FIG. 3. Accuracy of Calibration for Simulated Field Data (SD = Standard Deviation of Head Measurements)

CONCLUSIONS

Widely varying heads generated during a transient event can be used to calibrate for the unsteady friction factors in a system. This study considers a direct-equation implementation of inverse transient analysis using both perfect data and data with modeling and simulated measurement errors. Accuracy depends on both of these errors through a least-squares merit function. Sensitivity information was supplied by direct differentiation of the characteristic and valve equations, a procedure that is shown to be fast, physical, and numerically robust.

The ultimate test of the method will involve the use of field collected data. Data of this type will provide a more rigorous test of the assumptions central to the solution of the forward problem. In particular, it is expected that it will not be possible to continuously improve the results by extending the simulation time due to the increasing error caused by the standard assumption of quasi-steady friction.

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APPENDIX I. REFERENCES

Chaudhry, M. H. (1987). *Applied hydraulic transients*. Van Nostrand Reinhold, New York.

Karney, B. W., and McInnis, D. A. (1992). ''Efficient calculation of transient flow in simple networks.'' *J. Hydr. Engrg.*, ASCE, 118(7), 1014–1030.

Liggett, J. A. (1993). ''Network monitoring and the algorithmic location of leaks under steady and unsteady conditions.'' *Water supply systems: State of the art and future trends*, E. Cabrera and F. Martinez, eds., Computational Mathematics Publications, Southampton, England.

Liggett, J. A., and Chen, L.-C. (1994a). ''Inverse transient analysis in pipe networks.'' *J. Hydr. Engrg.*, ASCE, 120(8), 934–955.

Liggett, J. A., and Chen, L.-C. (1994b). ''Monitoring water distribution systems: The inverse method as a tool for calibration and leak detection.'' *Improving efficiency and reliability in water distribution systems*, E. Cabrera and Antonio F. Vela, eds., Kluwer Academic, Boston, 107–134.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P. (1992). *Numerical recipes in C, second edition*. Cambridge University Press, Cambridge, England.

Wylie, E. B., and Streeter, V. L. (1993). *Fluid transients in systems*. Prentice-Hall, Englewood Cliffs, N.J.

APPENDIX II. NOTATION

The following symbols are used in this paper:

A = pipe cross-sectional area;
a = wave propagation speed;
B = pipeline characteristic impedance;
B_M, B_P = known constants in MOC equations;
C_M, C_P = known constants in MOC equations;
D = inside pipe diameter;
E_s = valve constant;
f = Darcy-Weisbach friction factor;
g = acceleration due to gravity;
H = piezometric head;
L = pipe length along centerline;
Q = volumetric flow rate;
R = resistance coefficient;
s = sign;
t = time;
x = distance along pipe centerline;
Δt = time step;
Δx = spatial step;
ε = friction linearization constant; and
τ = relative valve opening.

Subscripts

DS = condition at downstream reservoir;
L = condition at foot of positive characteristic;
$N+1$ = condition at most downstream node;
P = unknown condition at grid point;
$P-1$ = condition one reach upstream of P;
R = condition at foot of negative characteristic;
0 = steady-state condition; and
1 = condition at upstream reservoir.