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Technical note

Rigid-plug elastic-water model for transient pipe flow with entrapped air pocket

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ABSTRACT
Pressure transients in a rapidly-filling pipe with an entrapped air pocket are investigated analytically. A rigid-plug elastic-water model is developed by applying elastic-water hammer to the majority of the water columns while applying rigid-water analysis to a small portion near the air–water interface. The proposed model is validated by the full elastic-water model and experimental data. It effectively avoids the interpolation error of the method of characteristics reducing its complexity when tracking the air–water interface. Moreover, the current model has the same accuracy as the full elastic-water model.

Keywords: Air pocket, entrapped air, mathematical model, pipe system, pressure surge, transient flow

1 Introduction

Air pockets are often entrapped in water pipe systems in power stations, urban water supply and drainage sewer systems. Problems of pipe filling arise if the entrapped air is not completely expelled from the system (Wylie and Streeter 1993, Pozos *et al.* 2010a,b). A sudden transient of the entrapped air pocket in a dead-end pipe may induce high-pressure surges leading to deformations or even pipe failure (Zhou *et al.* 2004).

Most of the existing mathematical models on the numerical simulation of hydraulic transients involving an air pocket are one dimensional (1D), including either an elastic-water model or a rigid-water column model. Martin (1976), Cabrera *et al.* (1992), Izquierdo *et al.* (1999) and Zhou (2000) used the rigid model. Wylie and Streeter (1993), Lee and Martin (1999), Lee (2005) and Zhou *et al.* (2011b) included water compressibility and solved the governing equations by the method of characteristics (MOC), thereby establishing the elastic model. From a 2D and 3D viewpoint, Zhou *et al.* (2011a) introduced the volume of fluid (VOF) method to simulate transient flow during the filling process in a confined pipe system.

Lee and Martin (1999) studied the pressurization of an air pocket trapped at a dead end of a horizontal pipe. The experimental results were compared with predictions from both a rigid model and an elastic model. It was found that the rigid column theory breaks down for systems with small entrapped air pockets, while the water hammer theory shows fairly good overall agreement with the tests. The rigid model is fast and efficient, avoiding interpolation problems of tracking the air–water interface.

The purpose of this study is to develop a simple and effective model to investigate flow transients (Fig. 1). Based on the elastic-water and rigid-water theories, a rigid-plug elastic-water model

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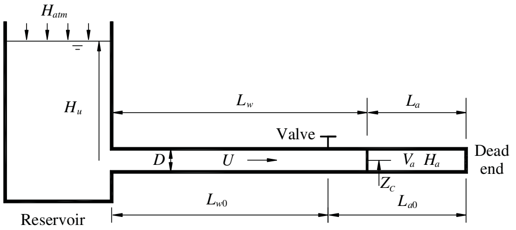

Figure 1 Schematic of water filling pipe system with entrapped air

was developed by applying elastic-water hammer to the majority of the water columns while using the rigid-water column analysis to a small portion near the air–water interface. This model was validated with both results of a full elastic-water model and test data of Lee and Martin (1999), Zhou (2000) and Zhou *et al.* (2004).

2 Governing equations

2.1 *Governing assumptions*

The following assumptions were made to develop the 1D mathematical model (Cabrera *et al.* 1992): (1) the air–water interface is perpendicular to the pipe axis, and the air–water mixture and the dissolution and release of air in water are negligible; (2) the pipe is fully closed and neither water nor air is leaked or released; (3) the wall friction factor for steady flow is applied under unsteady flow conditions; and (4) a polytropic law applies for the air phase using a constant polytropic exponent m; the inertia of the air pocket is negligible, implying a constant pressure within the air pocket.

2.2 *Equation for air phase*

The equation for the air phase is

$$H_a V_a^m = H_{a0} V_{a0}^m \quad \text{or} \quad H_a L_a^m = H_{a0} L_{a0}^m, \tag{1}$$

where H_a, V_a and L_a are the absolute (subscript a) air pressure, air volume and air length, with H_{a0}, V_{a0} and L_{a0} as the initial (subscript 0) conditions, respectively (Fig. 1).

2.3 *Equations for air–water interface*

For the air–water interface, the continuity equation and pressure balance equation can be written as

$$\frac{\mathrm{d}V_a}{\mathrm{d}t} = -Q \text{ or } \frac{\mathrm{d}L_a}{\mathrm{d}t} = -U \tag{2}$$

$$H_D = H_a + Z_c, \tag{3}$$

where Q and U are the discharge and velocity at the air–water interface, t is the time, and H_D and Z_c are the head and the elevation of the horizontal pipe axis at the air–water interface, so that $Z_c = 0$ (Fig. 1).

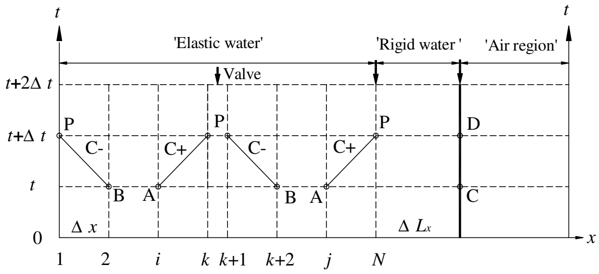

Figure 2 Definition sketch in the $x - t$ plane with fixed grid

2.4 *Equations for water phase*

The continuity and momentum equations are applied to the water phase along with the standard orifice equation to represent the valve boundary condition (Wylie and Streeter 1993). The air–water interface moves under the pressure difference between two phases. Applying the fixed-grid MOC to solve the moving interface boundary inevitably causes interpolation problems. As the elasticity of the water phase is far less than that of the air pocket, the compressibility of a small water column ΔL_x near the air–water interface is neglected. Length ΔL_x is continually changed with the movement of air–water interface, but its extent is related to the MOC grid length. As shown in Fig. 2, as long as ΔL_x is sufficiently small, this model generates nearly the same calculated numerical results as the full elastic-water model while avoiding interpolations. However, an infinitesimal ΔL_x leads to running mistakes of the simulation programme. Therefore, a rational length extent of ΔL_x is crucial to ensure the calculation accuracy and the performance of the numerical model, so that $\Delta x \le \Delta L_x < 2\Delta x$ is adopted here.

As the rigid-water column is short, representing a "rigid plug", the rigid-plug elastic-water model is based on the following equations:

$$Q_{Pk} = Q_R \tag{4}$$

$$\frac{\Delta L_x}{gA} \frac{\mathrm{d}Q_R}{\mathrm{d}t} = H_{Pk} - H_D - \frac{f \Delta L_x}{2gDA^2} Q_R \left| Q_R \right|, \tag{5}$$

where Q_R is the discharge of the rigid (subscript R) water section, and H_{Pk} and Q_{Pk} are the transient piezometric (subscript P) head and discharge at the elastic–rigid (subscript k) water interface, respectively.

2.5 *Numerical scheme*

The initial conditions consistent with the assumption of uniform initial pressure and no fluid motion at $t = 0$ consist of

$$H_P(x, 0) = H_u, Q_P(x, 0) = 0, \text{ for } x \text{ in } (0, L_{w0}) \tag{6}$$

$$V_a = V_{a0}, H_a = H_{a0}, \text{ for } x \text{ in } (L_{w0}, L). \tag{7}$$

The water discharge and the head at the elastic-water region are obtained using MOC (Wylie and Streeter 1993). The calculations at the rigid-water region consist of the elastic–rigid interface, the rigid-water region and the air–water interface. The values of Q_R and V_a result from combining Eqs. (1)–(5) and the C^+ equation, which are solved numerically using the fourth-order Runge–Kutta technique (Ayyub and McCuen 1996). The time-step increment in calculating the rigid-water region has to be identical to that of MOC for the elastic-water region, $\Delta t = \Delta x/a$, where a is the wave speed. Based on a sensitivity analysis, the numerical scheme is stable if the $\Delta t < 1 \times 10^{-4}$ s. For the small initial air pocket, a smaller time step within the elastic–rigid water regions is required. Then, H_a, Q_{Pk} and H_{Pk} follow by substituting Q_D and V_a into Eqs. (1)–(5).

As shown in Fig. 2, either $2\Delta x \leq \Delta L_x$ or $\Delta L_x < \Delta x$ may occur due to compression or expansion of the air pocket. To validate the assumption $\Delta x \leq \Delta L_x < 2\Delta x$, a computed node must be added or deleted, estimating the variables at the newly added node by linear interpolation between the nearby nodes.

3 Results

3.1 *Proposed model accuracy against elastic model*

Compared with the full elastic-water model, the current model avoids interpolations. The length range of ΔL_x is the key parameter, affecting the calculation accuracy. Lee (2005) developed a full elastic-water model with an instantaneous valve opening for a horizontal pipe ("Lee's model"). Table 1 compares the results of the proposed model with different length ranges of the "rigid plug" and Lee's elastic model. The parameters used are as follows: pipe diameter $D = 0.09$ m, pipe length $L = 100$ m, water density $\rho = 1000\,\text{kg/m}^3$, wave speed $a = 1000$ m/s, $f = 0.02$, $m = 1.4$, $H_{a0} = 10.33$ m, $H_u = 120.33$ m and $\Delta x = 0.02$ m. The proposed model ranged within $N\Delta x \leq \Delta L_x < (N+1)\Delta x$, with $N =$ positive integer. From Table 1, the rigid-plug length is important, especially for small air pockets. If the entrapped air pocket is relatively long, ΔL_x has a small effect on the model accuracy. As long as $\Delta x \leq \Delta L_x < 2\Delta x$ or $\Delta L_x < \Delta x$, the model generates nearly the same calculated numerical results as the full elastic-water model while avoiding interpolations. However, an infinitesimal ΔL_x leads to running mistakes. Therefore, a rational length extent of ΔL_x is crucial to ensure the calculation accuracy. The model with $\Delta x \leq \Delta L_x < 2\Delta x$ results in air pressures consistent with Lee's elastic model.

Table 1 Accuracy of the model for different length ranges of rigid plug

	Elastic model	Proposed model ($\Delta x = 0.02$ m)	
L_{a0} (m)	$H_{a\max}$ (m)/t_m (s)	N	$H_{a\max}$ (m)/t_m (s)
15	492.084/2.222	1	492.089/2.2243
		10	492.287/2.2242
		100	494.240/2.2232
		1000	515.299/2.2101
		4000	546.481/2.1993
80	176.255/6.920	1	176.250/6.9191
		100	176.268/6.9185
		800	176.380/6.9150

Notes: $H_{a\max}$, maximum air pressure; t_m, corresponding time.

3.2 *Experimental verification*

Zhou (2000) and Zhou *et al.* (2004) investigated experimentally the effects of trapped air on pressure surges in both horizontal and vertical pipes. The parameters used for the horizontal pipe are those of Zhou (2000), namely $D = 0.035$ m, $L = 10$ m, initial water column length $L_{w0} = 5$ m, relative inlet pressure $H_r = 275$ kPa, initial air pocket pressure $H_{a0} = 9.794$ m and $\rho = 1000\,\text{kg/m}^3$. The parameters of the vertical pipe part include in addition the following: $D = 0.035$ m, vertical pipe length $L_s = 0.6$ m and $L_{w0} = 0.48$ m. The calculations include the friction factor f between 0.032 and 0.035, $a = 200$–1400 m/s, exponent $m = 1.4$ and valve opening time at $t = 0$ s. Figure 3 compares the calculated and measured pressure patterns for $f = 0.035$ and $a = 1000$ m/s, indicating that the pressure oscillations from the proposed model are similar to the experiments in both pipe setups, especially for the first pressure peak.

Lee and Martin (1999) conducted experiments on the pressure transients in a horizontal acrylic plastic pipe with a dead end. They studied the effects of initial void fraction of the air pocket and the inlet pressure on the air pressure surge. The parameters used were as follows: $D = 0.026$ m, $L = 10.4$ m, $\rho = 1000\,\text{kg/m}^3$, $a = 600$ m/s, $m = 1.4$ and $H_{a0} = 1.0 \times 10^5$ Pa (10.2 m H_2O), yet f was not reported. The measured friction factor of $f = 0.025$ was suggested by Lee (2005). Moreover, the valve opening was instantaneous. Lee and Martin (1999) investigated the pressure oscillation pattern during the filling process (Fig. 4a) and the effects of air fraction and inlet pressure on the maximum air pressure (Fig. 4b). The proposed model agrees well with the measurement.

Comparisons of calculations against both data of Zhou (2000, 2004) and Lee and Martin (1999) indicate that the numerical data of the second and the following pressure peaks lag behind the experimental data. The main reason for this is that the 1D model is based on assumptions including (1) the air pocket occupies the entire cross-section and (2) the air–water interface is always perpendicular to the pipe axis. Before the first pressure peak arrives, these assumptions are reasonable because the air pocket is uniformly compressed. Then, the air pocket is separated into several portions by the water filling process, however. The proposed model predicts effectively the maximum pressure as the most important factor to ensure the safety of a pipeline system.

3.3 *Validity of assuming instantaneous valve opening*

The most dangerous case is a rapid pipe filling process with an entrapped air pocket assuming that the valve is instantaneously opened despite operating valves needing a certain amount of

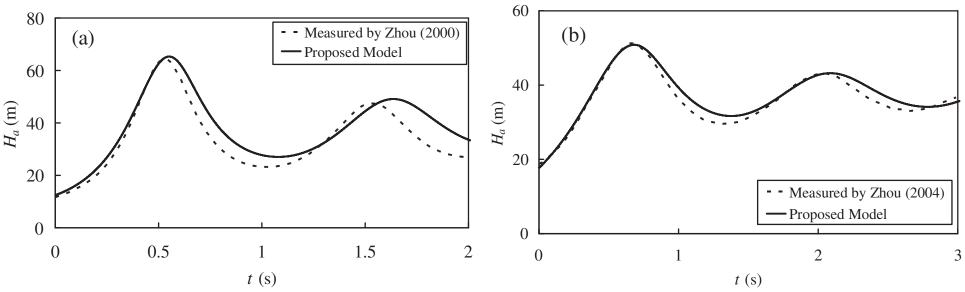

Figure 3 Comparison of air pocket pressure $H_a(t)$ predicted with the proposed model against experimental data of Zhou (2000) and Zhou *et al.* (2004) (a) horizontal pipe and (b) horizontal and vertical pipes

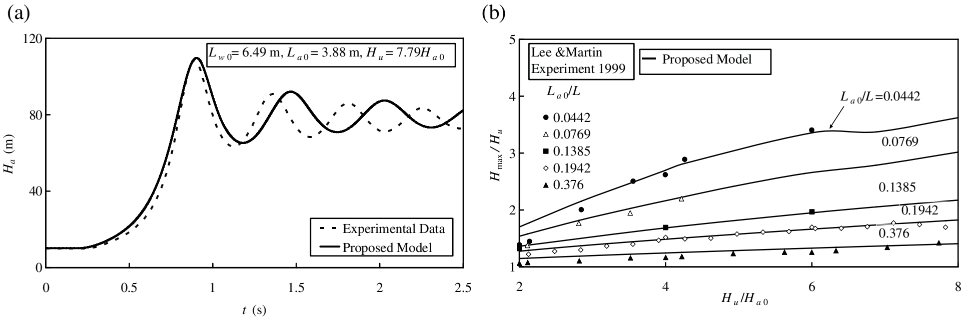

Figure 4 Comparison of air pocket pressure $H_a(t)$ predicted with the proposed model against experimental data of Lee and Martin (1999) (a) pressure oscillation pattern and (b) variation of air pocket volume and inlet pressure

Table 2 Results of various L_w cases for $L_{a0} = 15$ m and $H_u = 41.33$ m

	$H_{a\max}$ (m)/t_m (s) of the proposed model		
L_w (m)	$T_s = 0$ s	$T_s = 0.1$ s	$T_s = 1.0$ s
3	120.14/1.121	116.45/1.142	106.75/1.265
5	124.46/1.241	121.48/1.257	114.15/1.343
10	131.04/1.499	129.19/1.509	125.87/1.547
15	134.80/1.714	133.54/1.721	131.96/1.745
20	137.20/1.904	136.25/1.910	135.50/1.928
30	140.21/2.238	139.65/2.242	139.30/2.249

Notes: $D = 0.09$ m, $\rho = 1000$ kg/m^3, $a = 1000$ m/s, $f = 0.02$, $m = 1.4$.

time. Zhou (2000) studied experimentally valve opening times from fully-closed to fully-opened ranging between 0.06 and 0.08 s. From Table 2, the valve opening time, which is less than the occurrence time of peak pressure, has only a small effect on the peak air pressure, yet a long opening time reduces the peak pressure.

4 Conclusions

A closed pipe system with an entrapped air pocket due to rapidly-filled water may cause high-pressure surges, higher than those in a system without entrapped air. It is significant to predict the pressure in such a pipe system containing an air pocket to effectively avoid abnormal pressures. A rigid-plug elastic-water model is herein developed, in which only the small water column near the air–water interface is assumed as rigid, whereas the remaining water phase is treated as elastic. A comparison with the full elastic-water model indicates that the current model has the same accuracy as the full elastic-water model. Moreover, the proposed model effectively avoids interpolation errors of the complete model when tracking the air–water interface. Results of the proposed model are also verified against experimental data, confirming the above findings.

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Notation

A = cross-sectional area of pipe section (m^2)
a = wave speed (m/s)
D = pipe diameter (m)
f = wall friction coefficient (−)
g = gravitational acceleration (m/s^2)
H_a = absolute pressure of entrapped air pocket (m)
H_{a0} = initial condition of H_a (m)
$H_{a\max}$ = maximum air pressure (m)
H_D = pressure at air–water interface (m)
H_{Pk} = transient pressure at elastic–rigid water interface (m)
H_r = upstream water head (relative pressure) (m)

L_a = air pocket length (m)
L_{a0} = initial condition of L_a (m)
L_{w0} = initial water portion length (m)
L_x = length of filling water column (m)
m = polytropic exponent (−)
Q_D = discharge of air–water interface (m^3/s)
Q_{Pk} = discharge of elastic–rigid water interface (m^3/s)
Q_R = discharge of rigid-water section (m^3/s)
t_m = occurrence time of maximum air pressure (s)
V_a = volume of entrapped air pocket (m^3)
V_{a0} = initial condition of V_a (m^3)
Z_c = elevation of pipe axis at air–water interface (m)
ΔL_x = length of rigid water (m)
Δx = node point distance along pipeline (m)
Δt = time-step increment (s)

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