

The authors compared the formulation and computational performance of two numerical methods for modeling hydraulic transients in water distribution systems. One method is Eulerian-based, and the other is Lagrangian-based. The Eulerian approach explicitly solves the hyperbolic partial differential equations of continuity and momentum and updates the hydraulic state of the system in fixed grid points as time is advanced in uniform increments. The Lagrangian approach tracks the movement and transformation of pressure waves and updates the hydraulic state of the system at fixed or variable time intervals at times when a change actually occurs. Each method was encoded into an existing hydraulic simulation model that gave initial pressure and flow distribution and was tested on networks of varying size and complexity under equal accuracy tolerance. Results indicated that the accuracy of the methods was comparable, but that the Lagrangian method was more computationally efficient for analysis of large water distribution systems.

# Numerical methods for modeling transient flow in distribution systems

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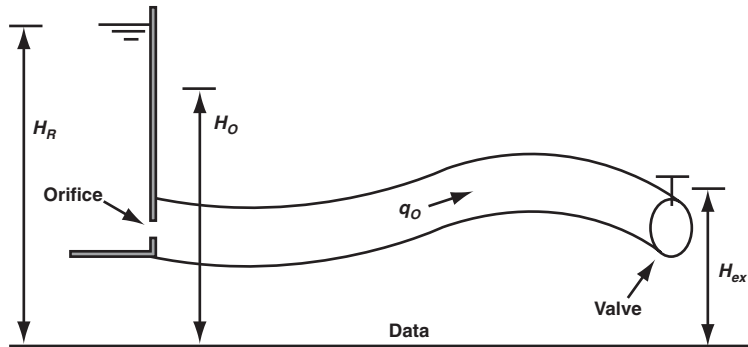
**C**omputer models for simulating the hydraulic and water quality behavior of water distribution systems have been available for many years. Recently these models have been extended to analyze hydraulic transients as well. In the past, most transient analyses were performed only on large transmission mains using highly skeletonized models for engineering design; proprietary transient computer programs were largely limited to specialist consulting companies, research organizations, and universities.

In addition to improved design and operation of water distribution systems, a driving force behind the trend toward increased analysis of hydraulic transients has been the growing awareness that hydraulic transients can create unexpected opportunities for pathogens present in the external environment to intrude into the distribution system with disastrous consequences to public health. Modern management of water distribution systems requires simulation models that are able to accurately predict transient flow and pressure variations within the distribution system environment.

## PRESSURE TRANSIENTS

**Effect on water quality.** It is well-recognized that pressure transients may adversely affect the quality of treated water. Pressure transients in water distribution systems result from an abrupt change in the flow velocity and can be caused by main breaks, sudden changes in demand, uncontrolled pump starting

**FIGURE 1** Pipeline and data for comparison of MOC, WCM, and exact solution



$H_{ex}$ —hydraulic grade at pipe exit,  $H_o$ —hydraulic grade outside the orifice,  $H_R$  reservoir hydraulic grade, MOC—method of characteristics,  $q_o$ —initial flow rate through the pipeline, WCM—wave characteristic method

Case 1 (no orifice):  $H_R = H_o = 45$  ft (13.7 m),  $H_{ex} = 0$ ; case 2 (orifice):  $H_R = 135$  ft (41.2 m),  $H_o = 45$  ft (13.7 m),  $H_{ex} = 0$ ; length = 3,600 ft (1,098 m), diameter = 12 in. (300 mm), capacity = 3,600 fps (1,098 m/s),  $q_o = 3$  cfs (85 L/s).

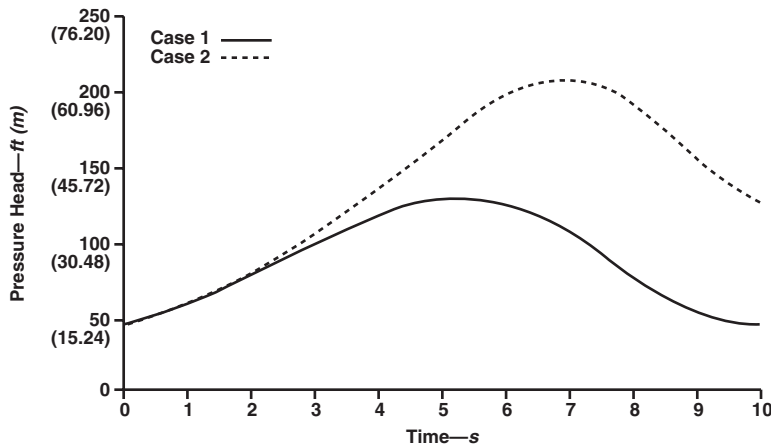
**TABLE 1** Case 1: Single pipe leading from a reservoir to a valve

Time $s$	Ratio $\tau^*$	Exact		MOC		WCM	
		Hydraulic Grade ft (m)	Volumetric Flow Rate cfs (L/s)	Pressure Head ft (m)	Volumetric Flow Rate cfs (L/s)	Pressure Head ft (m)	Volumetric Flow Rate cfs (L/s)
0	1.00	45.00 (13.72)	3.00 (84.90)	45.00 (13.72)	3.00 (84.90)	45.00 (13.72)	3.00 (84.90)
1	0.84	59.13 (18.02)	2.90 (82.07)	59.13 (18.02)	2.90 (82.07)	59.13 (18.02)	2.90 (82.07)
2	0.69	79.60 (24.26)	2.76 (78.11)	79.60 (24.26)	2.76 (78.11)	79.60 (24.26)	2.76 (78.11)
3	0.55	98.62 (30.06)	2.42 (68.49)	98.62 (30.06)	2.42 (68.49)	98.62 (30.06)	2.42 (68.49)
4	0.41	117.91 (35.94)	2.00 (56.60)	117.91 (35.94)	2.00 (56.60)	117.91 (35.94)	2.00 (56.60)
5	0.29	126.69 (38.62)	1.47 (41.60)	126.69 (38.62)	1.47 (41.60)	126.69 (38.62)	1.47 (41.60)
6	0.19	122.48 (37.33)	0.95 (26.89)	122.48 (37.33)	0.95 (26.89)	122.48 (37.33)	0.95 (26.89)
7	0.11	102.82 (31.34)	0.49 (13.87)	102.82 (31.34)	0.49 (13.87)	102.82 (31.34)	0.49 (13.87)
8	0.05	75.08 (22.88)	0.19 (5.38)	75.08 (22.88)	0.19 (5.38)	75.08 (22.88)	0.19 (5.38)
9	0.10	51.89 (15.82)	0.04 (1.13)	51.89 (15.82)	0.04 (1.13)	51.89 (15.82)	0.04 (1.13)
10	0.00	41.92 (12.78)	0.00 (0.00)	41.92 (12.78)	0.00 (0.00)	41.92 (12.78)	0.00 (0.00)

MOC—method of characteristics, WCM—wave characteristic method

\* $\tau$ —ratio of the effective flow area to the fully open flow area for the butterfly valve

**FIGURE 2** Pressure head variations at valve



rine residual loss and possible increases in the concentration of microorganisms (Clark et al, 1996). Pathogens can also enter the distribution system during construction, repair, cross-connections, and conditions and activities in which the system is open to the atmosphere or the environment (Karim et al, 2003). The effects of pressure transients on distribution system water quality degradation have been extensively reviewed (Wood et al, 2005; Boulos et al, 2004; Karim et al, 2003; LeChevallier et al, 2003; Kirmeyer et al, 2001; Funk et al, 1999).

**MODELING APPROACHES**

**Approaches to modeling pressure transients.** Several approaches have been taken to numerically model the movement and transformation of pressure waves in water distribution systems and can be classified as either Eulerian or Lagrangian. The Eulerian approach reformulates the governing transient flow equations (discussed subsequently) into total differential equations, which are then expressed in a finite difference form. The Lagrangian approach tracks changes in pressure waves (having both positive and negative amplitude) as they travel through the pipe network and updates the state of the network only at times when a change actually occurs, such as when a pressure wave reaches the end node of a pipe.

The computational accuracy and performance of these hydraulic transient modeling approaches have not been comprehensively compared using an objective testing protocol applied to reasonably sized networks. Certainly, well-known numerical challenges in solving the transient flow equations exist, for example avoiding numerical dispersion and attenuation and eliminating unnecessary distortion of either the physical pipe system or its boundaries. Yet even with the continuous advances in computing speed and capacity, development of efficient network-modeling methods has lagged for several reasons.

**TABLE 2** Case 2: Single pipe leading from a reservoir with an orifice to a valve

Time s	Ratio $\tau^*$	Exact		MOC		WCM	
		Hydraulic Grade ft (m)	Volumetric Flow Rate cfs (L/s)	Pressure Head ft (m)	Volumetric Flow Rate cfs (L/s)	Pressure Head ft (m)	Volumetric Flow Rate cfs (L/s)
0	1.00	45.00 (13.72)	3.00 (84.90)	45.00 (13.72)	3.00 (84.90)	45.00 (13.72)	3.00 (84.90)
1	0.84	59.13 (18.02)	2.90 (82.07)	59.13 (18.02)	2.90 (82.07)	59.13 (18.02)	2.90 (82.07)
2	0.69	79.61 (24.27)	2.76 (78.11)	79.61 (24.27)	2.76 (78.11)	79.61 (24.27)	2.76 (78.11)
3	0.55	104.68 (31.91)	2.50 (70.75)	104.68 (31.91)	2.50 (70.75)	104.68 (31.91)	2.50 (70.75)
4	0.41	136.09 (41.48)	2.15 (60.85)	136.09 (41.48)	2.15 (60.85)	136.09 (41.48)	2.15 (60.85)
5	0.29	169.59 (51.69)	1.71 (48.39)	169.59 (51.69)	1.71 (48.39)	169.59 (51.69)	1.71 (48.39)
6	0.19	197.60 (60.23)	1.20 (33.86)	197.60 (60.23)	1.20 (33.86)	197.60 (60.23)	1.20 (33.86)
7	0.11	207.54 (63.26)	0.70 (19.81)	207.54 (63.26)	0.70 (19.81)	207.54 (63.26)	0.70 (19.81)
8	0.05	189.91 (57.88)	0.30 (8.49)	189.91 (57.88)	0.30 (8.49)	189.91 (57.88)	0.30 (8.49)
9	0.10	152.07 (46.35)	0.07 (1.98)	152.07 (46.35)	0.07 (1.98)	152.07 (46.35)	0.07 (1.98)
10	0.00	123.20 (37.55)	0.00 (0.00)	123.20 (37.55)	0.00 (0.00)	123.20 (37.55)	0.00 (0.00)

MOC—method of characteristics, WCM—wave characteristic method

\* $\tau$ —ratio of the effective flow area to the fully open flow area for the butterfly valve

**Model criteria.** First, the trend to include dead ends for transient consideration and to develop all pipe models for improved water quality characterization increases the size of the model to be solved. Similarly, the need to directly interface hydraulic network models with computer-assisted design and geographical information system applications (e.g., work-order and maintenance-management systems) requires compatibility with large data sets and the evaluation of an all-pipe model. Finally, the trend toward linking real-time supervisory control and data acquisition systems, data loggers, and graphical user interfaces to network models with interactive analysis and graphical display of results demands that solutions to network models be obtained as quickly and accurately as possible.

This research reviewed and compared the two types of transient models for pipe networks, i.e., Eulerian-based and Lagrangian-based. The models were contrasted with respect to how closely their results matched analytical solutions, how closely the results matched one another, how long the models took to execute, and how many calculations were required.

### EULERIAN AND LAGRANGIAN APPROACHES TO TRANSIENT FLOW ANALYSIS

**Governing equations.** Rapidly varying pressure and flow conditions in pipe networks are characterized by variations that are dependent on both position,  $x$ , and time,  $t$ . These conditions are described by the continuity equation

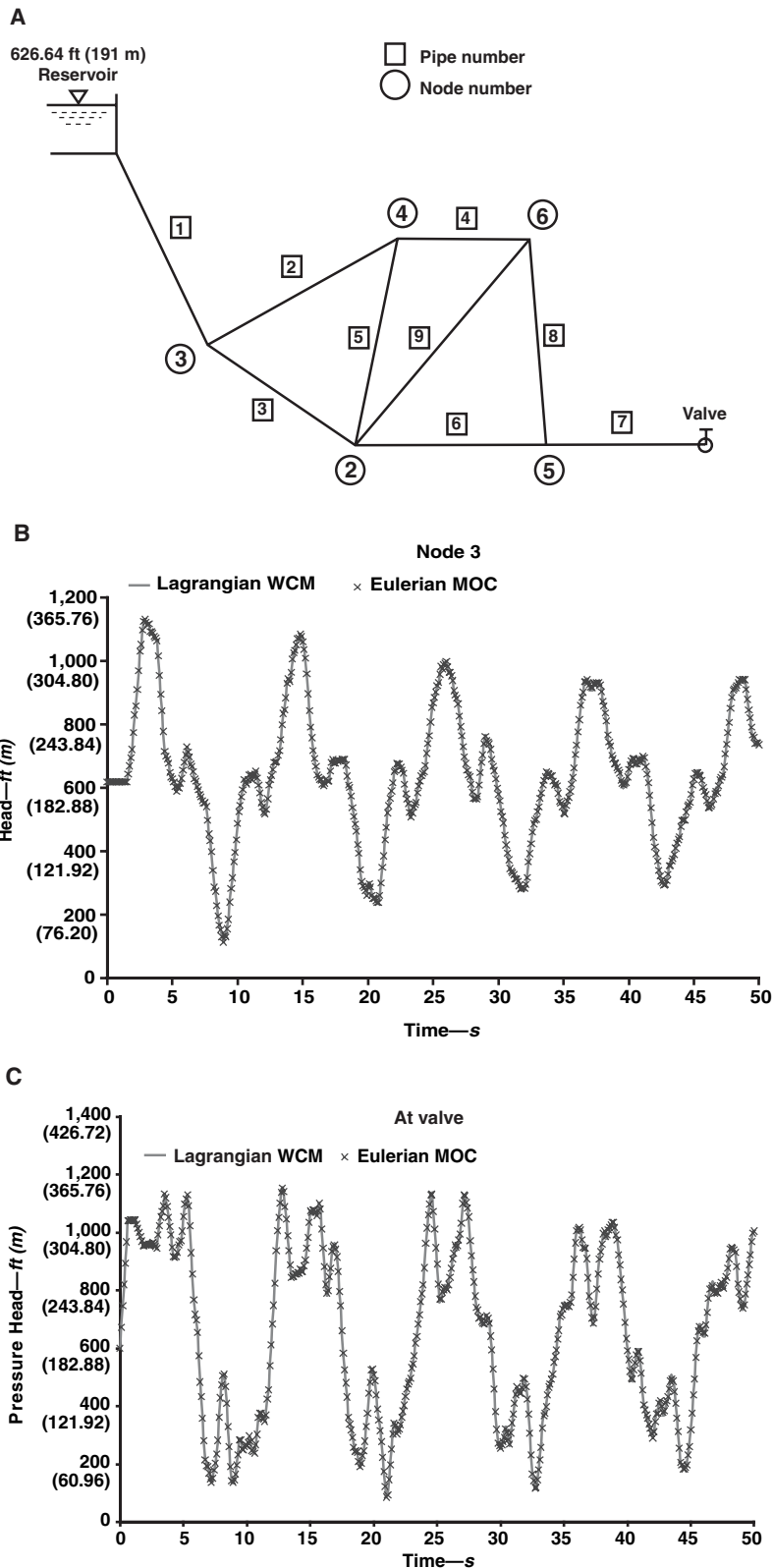
$$\frac{\partial H}{\partial t} = -\frac{c^2}{gA} \left( \frac{\partial Q}{\partial x} \right) \quad (1)$$

and the momentum (Newton's second law) equation

$$\frac{\partial H}{\partial x} = -\frac{1}{gA} \left( \frac{\partial Q}{\partial t} \right) + f(Q) \quad (2)$$

in which  $H$  is the pressure head (pressure/specific weight),  $Q$  is the volumetric flow rate,  $c$  is the sonic wave speed in the pipe,  $A$  is the cross-sectional area,

**FIGURE 3** Pipe network schematic and pressure head at node 3 and valve for Example 2



MOC—method of characteristics, WCM—wave characteristic method

Valve closure = 0.6 s

**TABLE 3 Pipe characteristics for example 2**

Pipe Number	Length ft (m)	Diameter in. (mm)	Roughness	Minor Loss
1	2,000 (610)	36 (900)	92	0
2	3,000 (914)	30 (750)	107	0
3	2,000 (610)	24 (600)	98	0
4	1,500 (457)	18 (450)	105	0
5	1,800 (549)	18 (450)	100	0
6	2,200 (671)	30 (750)	93	0
7	2,000 (610)	36 (900)	105	0
8	1,500 (457)	24 (600)	105	0
9	1,600 (488)	18 (450)	140	0

$g$  is the gravitational acceleration, and  $f(Q)$  represents a pipe-resistance term that is a nonlinear function of flow rate. Eqs 1 and 2 have been simplified by considering changes only along the pipe axis (one-dimensional flow) and discarding terms that can be shown to be of minor significance. A transient flow solution is obtained by solving Eqs 1 and 2 along with the appropriate initial and boundary conditions. However, except for very simple applications that neglect or greatly simplify the boundary

conditions, the transient flow equations are solved using numerical techniques (Axworthy et al, 1999; Karney & McInnis, 1990).

**Lagrangian approach.** The Lagrangian approach solves the transient flow equations in an event-oriented system-simulation environment. In this environment, the pressure wave propagation process is driven by the distribution system activities. The wave characteristic method (WCM) is an example of such an approach (Wood et al, 2005;

methods. For pipe network (closed conduit) applications, the most well-known and widely used of these techniques is the MOC (Boulos et al, 2004). The MOC is considered the most accurate of the Eulerian methods in its representation of the governing equations but requires numerous steps or calculations to solve a typical transient pipe-flow problem. As the pipe system becomes more complex, the number of required calculations increases, and a computer program is required for practical applications. This method has been summarized by other researchers (Boulos et al, 2004; Larock et al, 1999; Chaudhry, 1987; Watters,

## Modern management of water distribution systems requires simulation models that are able to accurately predict transient flow and pressure variations within the distribution system environment.

conditions and the pipe-resistance term, it is not possible to obtain a direct solution. When pipe junctions, pumps, surge tanks, air vessels, and other components that routinely need to be considered are included, the basic equations are further complicated, and it is necessary to utilize numerical techniques. Accurate transient analysis of large pipe networks requires computationally efficient and accurate solution techniques.

Both Eulerian and Lagrangian solution schemes are commonly used to approximate the solution of the governing equations. Eulerian methods update the hydraulic state of the system in fixed grid points as time is advanced in uniform increments. Lagrangian methods update the hydraulic state of the system at fixed or variable time intervals at times when a change actually occurs. Each approach assumes that a steady-state hydraulic equilibrium solution is available that gives initial flow and pressure distributions throughout the system.

**Eulerian approach.** Eulerian methods consist of the explicit method of characteristics (MOC), explicit and implicit finite difference techniques, and finite element

Boulos et al, 2004) and was first described in the literature as the wave plan method (Wood et al, 1966). The method tracks the movement and transformation of pressure waves as they propagate throughout the system and computes new conditions either at fixed time intervals or at times when a change actually occurs (variable time intervals). The effect of line friction on a pressure wave is accounted for by modifying the pressure wave using a nonlinear characteristic relationship describing the corresponding pressure head change as a function of the line's flow rate. Although it is true that some approximation errors will be introduced using this approach, these errors can be minimized using a distributed-friction profile (piecewise linearized scheme).

However, this approach normally requires orders of magnitude fewer pressure and flow calculations, which allows very large systems to be solved in an expeditious manner, and has the additional advantage of using a simple physical model as the basis for its development. Because the WCM is continuous in both time and space, the method is also less sensitive to the structure of the

network and to the length of the simulation process, resulting in improved computational efficiency. This technique produces solutions for a simple pipe system that are virtually identical to those obtained from exact solutions (Boulos et al, 1990). A similar comparison of exact and numerical results is presented in this article.

**MOC strategy.** In the strategy used by the MOC, the governing partial differential equations are converted to ordinary differential equations and then to a different form for solution by a numerical method. The equations express the head and flow for small time steps ( $\Delta t$ ) at numerous locations along the pipe sections. Calculations during the transient analysis must begin with a known initial steady state and boundary conditions. In other words, head and flow at time  $t = 0$  will be known along with head and/or flows at the boundaries at all times. To handle the wave characteristics of the transient flow, head and flow values at time  $t + \Delta t$  at interior locations are calculated making use of known values of head and flow at the previous time step at adjacent locations using the ordinary differential equations expressed in different form.

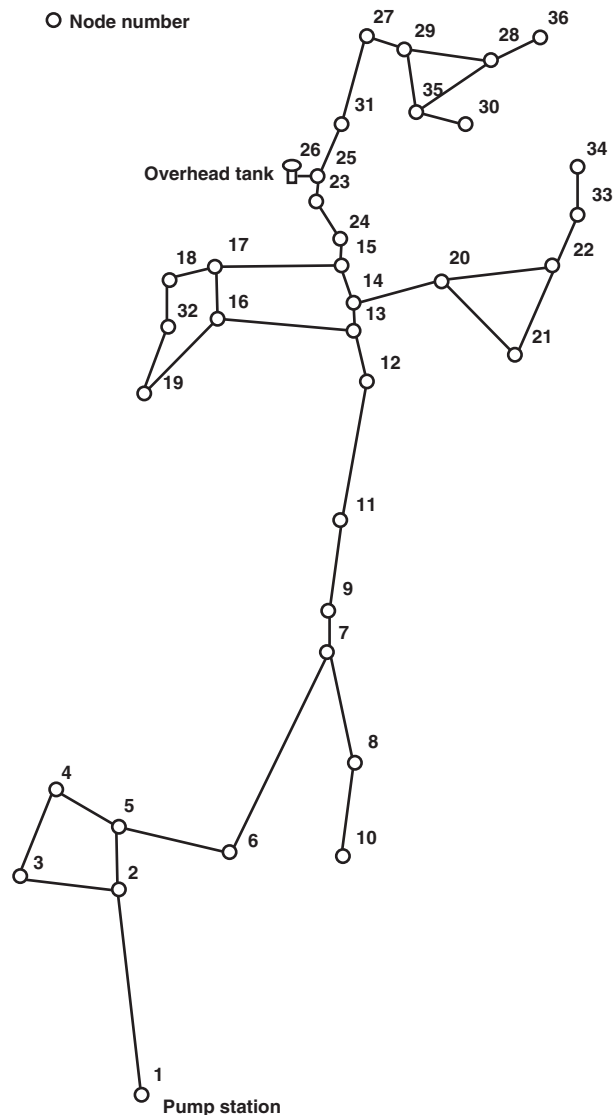
**Calculating WCM concept.** The WCM is based on the concept that transient pipe flow results from the generation and propagation of pressure waves that occur because of a disturbance in the pipe system (e.g., valve closure, pump trip). The wave characteristics are handled using pressure waves, which represent rapid pressure and associated flow changes that travel at sonic velocity through the liquid-pipe medium. A pressure wave is partially transmitted and reflected at all discontinuities in the pipe system (e.g., pipe junctions, pumps, open or closed ends, surge tanks). The pressure wave will also be modified by pipe wall resistance. This description is one that closely represents the actual mechanism of transient pipe flow (Wood et al, 2005; Boulos et al, 2004; Thorley, 1991).

**Differences in the two approaches.** Both the MOC and WCM obtain solutions at intervals of  $\Delta t$  at all junctions and components. However, the MOC also requires solutions at all interior points for each time step. This requirement basically handles the effects of pipe wall friction and the wave propagation characteristics of the solutions. The WCM handles these effects by using the pressure wave characteristics. The waves propagate through pipes at sonic speed and are modified for the effects of friction by a single calculation for each pipe section.

Both the Eulerian MOC and the Lagrangian WCM will virtually always produce the same results when the same data and model are used to the same accuracy. The main difference is in the number of calculations; the Lagrangian approach has an advantage.

**Objectives.** The primary objectives of this research were to (1) evaluate the ability of the MOC and the WCM to solve the basic partial differential transient pipe flow equations for pipe systems of varying degrees of complexity and (2) compare the solution accuracy and computational efficiency of the two methods.

**FIGURE 4** Pipe network schematic for Example 3



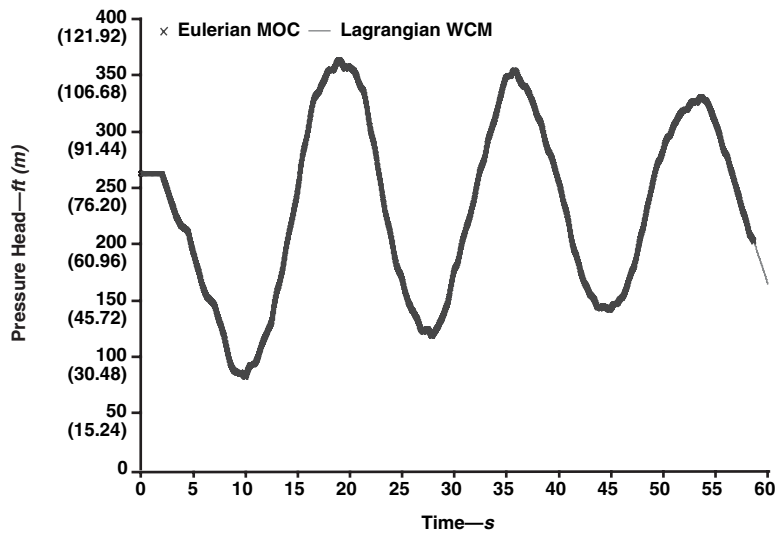
**COMPUTER MODELS FOR TRANSIENT FLOW ANALYSIS**

Both the MOC and WCM techniques were encoded in the Fortran 90 programming language and implemented for pipe system transient analysis in general-purpose computer models that use the same steady-state network flow hydraulics. This ensured that the implementation of each solution method was as consistent as possible. For all examples tested, the results were verified using an MOC-based computer model (Axworthy et al, 1999) and a WCM-based computer model (Boulos et al, 2003; Wood & Funk 1996). The comparisons discussed here were made using these modeling programs.

For both the MOC and WCM solutions, it is necessary to determine a computational time interval such that the



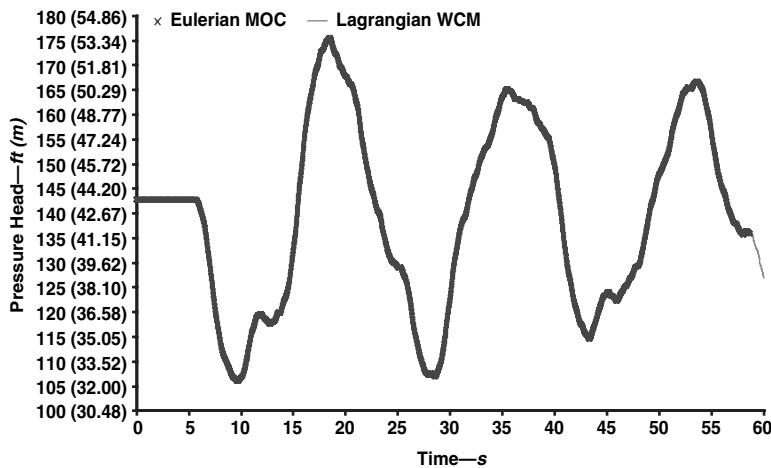
**FIGURE 5** Pressure head at node 1 for Example 3



MOC—method of characteristics, WCM—wave characteristic method

The Lagrangian WCM and the Eulerian MOC produced virtually identical results indicated by the single line.

**FIGURE 6** Pressure head at node 19 for Example 3



MOC—method of characteristics, WCM—wave characteristic method

The Lagrangian WCM and the Eulerian MOC produced virtually identical results indicated by the single line.

pressure wave travel times will be approximately a multiple of this time and this integer multiple will be calculated for each line segment. Some adjustment is normally required to obtain a time interval that is not unreasonably small. This may amount to actually analyzing a system with lengths (or wave speeds) slightly different from the true values; a tolerance should be chosen so that this is an acceptable deviation from the actual situation. Initial

flow conditions in all line segments and static pressure head (or pressure) at all junctions and components must be known. The required initial pressure head conditions may be static heads ( $P/\gamma$ ) or hydraulic grade lines (elevation +  $P/\gamma$ ).

Before transient calculations are initiated, all components must initially be in a balanced state (i.e., the initial pressure change across the component and flow through the component should be compatible with the characteristic relationship for that component). The initial flow for each component must be known, and the pressure on each side of the component must be defined. Finally, the exact nature of the disturbance must be specified. The disturbance will normally be a known change in the stem position for a valve, a change in the operational speed for a pump, or the loss of power to a pump (trip).

Results for pressure head and flow variations are calculated for each time step at all components and junctions in the pipe system. For the MOC approach, these calculations are also required at all interior locations. For the WCM approach, a single calculation is carried out for each pressure wave to determine the effect of pipe friction as the wave is transmitted through the pipeline.

**Numerical results.** Justification for the use of any transient flow algorithm rests on its ability to solve problems by means of computer implementation. This is best evaluated by comparing solutions obtained using the various approaches. This research compared solutions for a number of water distribution systems of various sizes using an equivalent time step. The number of calculations required to obtain the

solutions were also compared. This number was proportional to the execution time needed to perform a transient analysis and therefore constituted a good indicator of the computational efficiency of the numerical-solution procedures.

**Example 1.** In this example, water is flowing from a reservoir at the upstream end to the downstream end of a line of constant cross-sectional area  $A$  and of length  $L$ ,

**TABLE 4 Network characteristics for example 3**

Pipe Number	Length ft (m)	Diameter in. (mm)	Roughness	Node Number	Elevation ft (m)	Demand gpm (L/s)
1	2,400 (732)	12 (300)	100	1	50 (15)	-694.4 (-44)
2	800 (244)	12 (300)	100	2	100 (30)	8 (0.5)
3	1,300 (396)	8 (200)	100	3	60 (18)	14 (0.9)
4	1,200 (366)	8 (200)	100	4	60 (18)	8 (0.5)
5	1,000 (305)	12 (300)	100	5	100 (30)	8 (0.5)
6	1,200 (366)	12 (300)	100	6	125 (38)	5 (0.3)
7	2,700 (823)	12 (300)	100	7	160 (49)	4 (0.3)
8	1,200 (366)	12 (300)	140	8	110 (34)	9 (0.6)
9	400 (122)	12 (300)	100	9	180 (55)	14 (0.9)
10	1,000 (305)	8 (200)	140	10	130 (40)	5 (0.3)
11	700 (213)	12 (300)	100	11	185 (56)	34.78 (2.2)
12	1,900 (579)	12 (300)	100	12	210 (64)	16 (1)
13	600 (183)	12 (300)	100	13	210 (64)	2 (0.1)
14	400 (122)	12 (300)	100	14	200 (61)	2 (0.1)
15	300 (91)	12 (300)	100	15	190 (58)	2 (0.1)
16	1,500 (457)	8 (200)	100	16	150 (46)	20 (1.3)
17	1,500 (457)	8 (200)	100	17	180 (55)	20 (1.3)
18	600 (183)	8 (200)	100	18	100 (30)	20 (1.3)
19	700 (213)	12 (300)	100	19	150 (46)	5 (0.3)
20	350 (107)	12 (300)	100	20	170 (52)	19 (1.2)
21	1,400 (427)	8 (200)	100	21	150 (46)	16 (1.0)
22	1,100 (335)	12 (300)	100	22	200 (61)	10 (0.6)
23	1,300 (396)	8 (200)	100	23	230 (70)	8 (0.5)
24	1,300 (396)	8 (200)	100	24	190 (58)	11 (0.7)
25	1,300 (396)	8 (200)	100	25	230 (70)	6 (0.4)
26	600 (183)	12 (300)	100	27	130 (40)	8 (0.5)
27	250 (76)	12 (300)	100	28	110 (34)	0 (0)
28	300 (91)	12 (300)	100	29	110 (34)	7 (0.4)
29	200 (61)	12 (300)	100	30	130 (40)	3 (0.2)
30	600 (183)	12 (300)	100	31	190 (58)	17 (1.1)
31	400 (122)	8 (200)	100	32	110 (34)	17 (1.1)
32	400 (122)	8 (200)	100	33	180 (55)	1.5 (0.1)
34	700 (213)	8 (200)	100	34	190 (58)	1.5 (0.1)
35	1,000 (305)	8 (200)	100	35	110 (34)	0 (0)
36	400 (122)	8 (200)	100	36	110 (34)	1 (0.1)
37	500 (152)	8 (200)	100	26	235 (72)	Tank
38	500 (152)	8 (200)	100			
39	1,000 (305)	8 (200)	100			
40	700 (213)	8 (200)	100			
41	300 (91)	8 (200)	100			

a general pipeline profile with initial uniform velocity,  $V_0$  (or flow rate,  $Q_0$ ), and a wave speed,  $c$ . At time  $t = 0$ , a butterfly valve located at the downstream end of the line, which is completely open, begins to close.

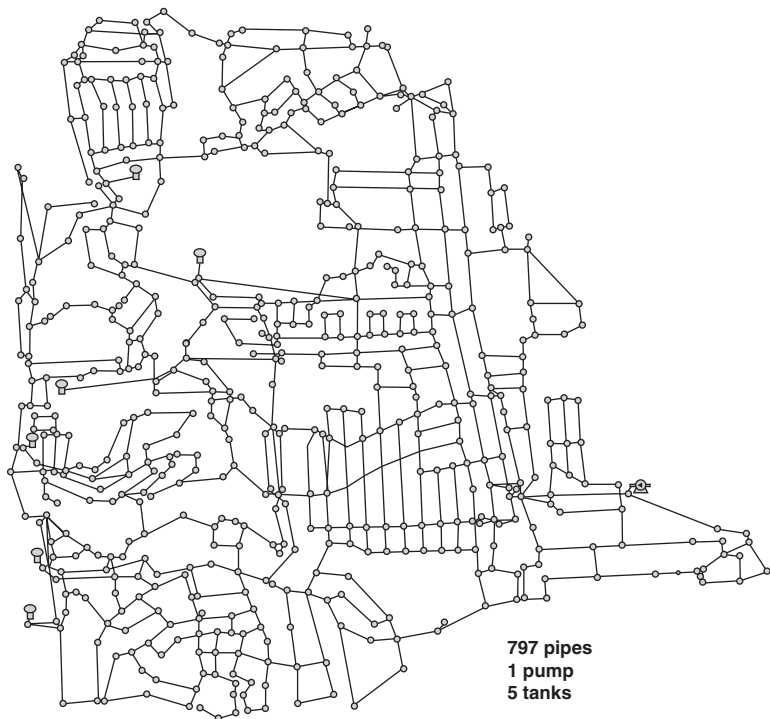
Figure 1 shows two instances of this example. In the first case, there is no orifice at the reservoir entrance, whereas in the second case an orifice is added at the

entrance. Both cases were analyzed using an exact solution of the basic partial differential equations (Eqs 1 and 2), and the actual results were compared with MOC and WCM results. Analytical and numerical solution details are provided elsewhere (Boulos et al, 2004; Boulos et al, 1990).

Calculations were carried out using the data shown in Figure 1 (cases 1 and 2) for a complete valve closure ( $c$ )

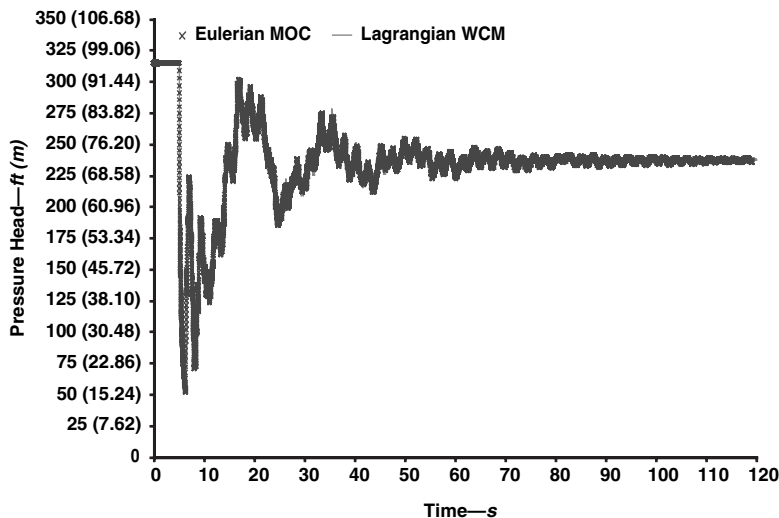


**FIGURE 7** Pipe network schematic for Example 4



Maximum length = 4,200 ft (1,280 m), minimum length = 20 ft (6 m)

**FIGURE 8** Comparison of results at pump for Example 4



The Lagrangian WCM and the Eulerian MOC produced virtually identical results, as indicated by the single line.

occurring over a time  $t_c = 10$  s. A computational time period of 1.0 s ( $L/c$ ) is necessary.  $\tau$  is the ratio of the effective flow area to the fully open flow area for the butterfly valve. For all three methods (exact, MOC, and WCM), the computations were initiated at the valve using the initial conditions and the value for the valve area ratio  $\tau$  at the end of the first time period. The computations then proceeded using results obtained from the previous calculations. Results for the three methods of analysis are shown in Table 1 (case 1), Table 2 (case 2), and Figure 2. Tables 1 and 2 compare values for the flow rate and the pressure head at the valve for time intervals of 1.0 s. For this example and for both cases analyzed, the two numerical methods produced results that were identical to the exact solution.

**Example 2.** The network used in the second example was studied by other researchers (Streeter & Wylie, 1967) and is shown in Figure 3, part A. The network comprises nine pipes, five junctions, one reservoir, three closed loops, and one valve located at the downstream end of the system. The valve is shut to create the transient. Table 3 summarizes the pertinent pipe system characteristics; the reservoir level of 626.64 ft (191 m) is shown in Figure 3, part A.

Parts B and C of Figure 3 compare the transient results obtained using the MOC and WCM solution approaches at node 3 and the valve, respectively. A 20-ft (6.7-m) length tolerance was used in the analysis, which resulted in a required time step of 0.1 s. In the figures, both solutions were plotted; the two methods produced results that are virtually indistinguishable.

**Example 3.** The methods were applied to a slightly larger, more complex system (Figure 4). This network represents an actual water system and consists of 40 pipes, 35 junctions, 1 supply pump, and 1 tank. This example was taken from the EPANET documentation (Rossman, 1993).

Table 4 summarizes the pertinent pipe system characteristics. The pump station is modeled by designating the

inflow at that location. Figures 5 and 6 compare the transient results obtained using the MOC and the WCM solution schemes at nodes 1 and 19, respectively, following a pump shutdown simulated by reducing the inflow to zero over a period of 6 s. A 20-ft (6-m) length tolerance was used in the analysis, resulting in a required time step of 0.0139 s. As the figures indicate, both methods yielded virtually identical results.

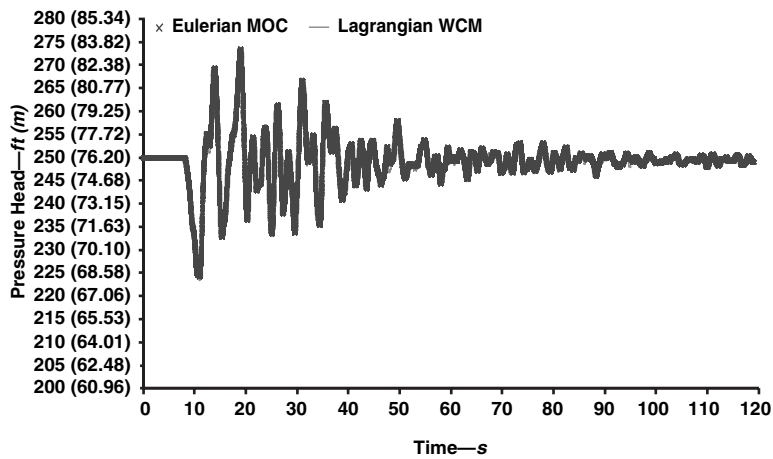
**Example 4.** To illustrate the comparable accuracy of both transient solution schemes on a larger, more complex system, the methods were applied to the network shown in Figure 7. This network represents an actual water distribution system consisting of 797 pipes, 581 junctions, 1 supply pump, and 5 tanks. (Because of the amount of data required and the fact that this model was based on an actual system, the data are not included here.)

Pipe lengths varied from 20 to 4,200 ft (6 to 1,280 m) and diameters from 4 to 24 in. (100 to 600 mm). Figures 8 and 9 compare the transient results obtained using MOC and WCM solution schemes following a pump trip. Figure 8 shows the pressure transient just downstream from the pump, whereas Figure 9 shows results at a node some distance away from the pump. The pump trip was modeled using the four quadrant pump characteristics in the form developed by Marchal and co-workers (1985). A 20-ft (6-m) length tolerance was used in the analysis, resulting in a time step of 0.0056 s. As with previous examples, the figures indicate that both methods produced virtually identical results.

## DISCUSSION

**Required calculations.** Both the MOC and the WCM require

**FIGURE 9** Comparison of results at node 3066 for Example 4



MOC—method of characteristics, WCM—wave characteristic method

The Lagrangian WCM and the Eulerian MOC produced virtually identical results, as indicated by the single line.

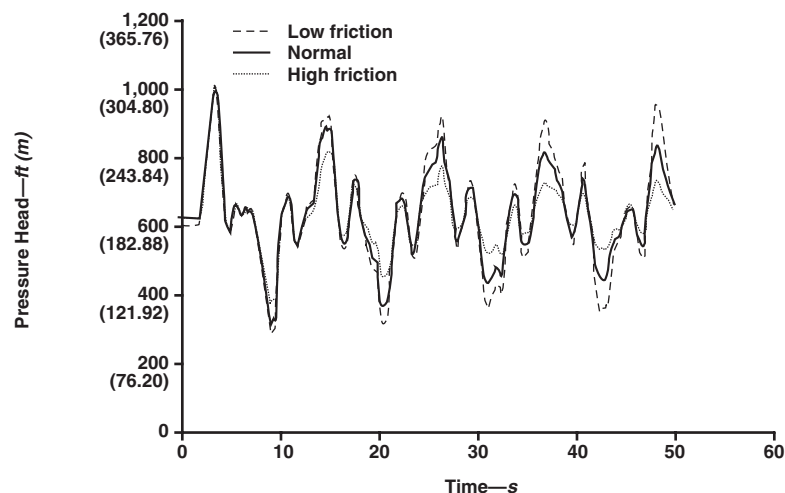
**TABLE 5** Calculation requirements for example systems

Example*	Number of Nodes	Number of Pipes	Time $\Delta t$ s	Number of Interior Points	Calculations/ $\Delta t$		
					MOC	WCM	MOC/WCM
Example 2	7	9	0.1	41	48	16	3.0
Example 3	36	40	.0139	680	716	76	9.4
Example 4	589	788	.0056	15,117	15,708	1,377	11.4
Example 5	1,170	1,676	.0067	81,508	82,678	2,846	29.0
Example 6	1,849	2,649	.0056	159,640	161,486	4,495	35.9

MOC—method of characteristics, WCM—wave characteristic method

\*Examples 5 and 6 are for large existing water distribution systems modeled but not described in the article.

**FIGURE 10** Effect of pipe friction on pressure transient



many calculations to solve the transient flow problem. These calculations involve updating the pressure and flow at required locations at increments of the time step  $\Delta t$ . In order to compare the number of calculations required, the authors defined one calculation as the operation required to update the pressure and flow at a single location.

The MOC requires a calculation at all nodes and all interior points at each time step, whereas the WCM requires a calculation at each node and one calculation for

number of calculations per time step was roughly proportional to the accuracy. For the examples given, the calculations/ $\Delta t$  required for the MOC would roughly double if an accuracy of 10 ft (3 m) is required and would be halved if an accuracy of 40 ft (12 m) is called for.

**Handling pipe friction.** The ability of the WCM to accurately model pipe friction in networks using just one calculation was substantiated by the virtually identical results obtained for the examples given here. This accuracy held true even though pipe friction has a significant effect on

## For the same modeling accuracy, the wave characteristic method will normally require fewer calculations and provide faster execution times.

each pipe at each time step. The pipe calculations are required to modify the pressure waves in that pipe to account for the effect of pipe wall and fittings friction.

The time step used in the analysis is determined by the tolerance set for the accuracy of the model pipe lengths. A time step must be chosen such that pressure waves traverse each pipe segment in a time that is a multiple of the time step. For the comparisons shown, the length tolerance was set to 20 ft (6 m). This means that the largest possible time increment was chosen so that the maximum error in the length of the pipes in the model would not exceed 20 ft (6 m).

Table 5 summarizes the calculation requirements for the three example systems (examples 2–4). In addition, the table includes data for two additional larger existing water distribution systems (examples 5 and 6) that have been modeled but are not described in this article.

The number of calculations for the WCM per time step does not change with accuracy. For the MOC, the

the solution. Figure 10 compares the transient analysis with and without calculating the effect of pipe wall friction for example 2. As the figure shows, the pressure transient was significantly modified by pipe friction.

The excellent agreement between the MOC and WCM solutions for this system (shown in Figure 3, part C) confirms that the computed effect of wall friction is similar for the two methods. Figure 10 also illustrates the growing significance of increasing the pipe resistance. Agreement for the two methods is excellent despite some long pipes and a wide range between maximum and minimum pipe lengths and diameters for the examples.

## CONCLUSIONS

Both the MOC and WCM methods are capable of accurately solving for transient pressures and flows in water distribution networks including the effects of pipe friction. The MOC requires calculations at interior

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points to handle the wave propagation and the effects of pipe friction. The WCM handles these effects using pressure waves. Therefore, for the same modeling accuracy, the WCM will usually require fewer calculations and provide faster execution times. In addition, the number of calculations per time step does not increase for the WCM when greater accuracy is required. However, because of the difference in calculation requirements and the comparable accuracy of the two techniques, use of the WCM may be more suitable for analyzing large pipe networks.

The results given here compared favorably with results obtained by other researchers (Rossman & Boulos, 1996) for solving the transient water quality transport equations. In their study, Rossman and Boulos made detailed comparisons of accuracy, computation time, and computation storage requirements for the Eulerian and Lagrangian numerical solution schemes. They found that all methods produced virtually identical results, but the Lagrangian approach was more versatile and more time- and memory-efficient than the Eulerian approach when modeling chemical constituents.

Any transient analysis is subject to inaccuracies because of incomplete information regarding the piping system, its components and degree of skeletonization, and some uncertainty with respect to initial flow distribution (Martin, 2000). However, the efficacy of transient modeling is enhanced by ensuring proper network model construction and calibration. Properly developed and calibrated models for transient analysis greatly improve the ability of water utilities to determine adequate surge protection, strengthen the integrity of their systems, and forge closer ties with their customers as well as the surrounding community. Water utility engineers can effectively use these models to predict unacceptable operating conditions developing in their distribution systems, iden-

tify risks, formulate and evaluate sound protective measures, and implement improved operational plans and security upgrades.

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