Leaks and Water Use Representation in Water Distribution System Models: Finding a Working Equivalence

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Abstract: The challenge of water demand representation in water distribution systems is revisited with a brief exploration of the relationship between a pressure-dependent leak and a fixed legitimate demand. Specifically, the idea that a leak can be modeled as an increment to legitimate demand in such a way that it entails an equivalent impact on both water loss and energy consumption is explored. Conversely, the representation of demands as leaks is briefly considered. The effectiveness of pressure reduction and demand curtailment as leak management schemes are compared for a single pipe system. The influence of pipe resistance on this relationship is assessed, suggesting that such schemes are more important in rougher pipes. In general, the notion that leakage and demand analysis/management are two sides of the same coin, and that pressure/demand management is essentially conservation, is put forth.

DOI: 10.1061/(ASCE)0733-9429(2009)135:3(234)

CE Database subject headings: Water demand; Water distribution systems; Leakage.

Introduction

Since the origins of planned water supply, designers, operators, and those responsible for the provision of this vital resource have wrestled with the twin issue of demand evaluation and representation. Conventional wisdom has treated, forecasted, and approximated demands as fixed quantities which are then employed as certain inputs to sophisticated models dealing with a broad range of issues from leak detection and pipe size optimization to the temporal/spatial distribution of water quality.

Leaks are often thought of and modeled separately from "legitimate" demands for understandable reasons. One obvious inducement for this segregated treatment is that the motivation for building and maintaining a distribution system is to satisfy demand and not to supply leaks. Further, leaks possess an enigmatic quality in that their location and discharge usually remain unknown, reinforced by the additional mystery of how they evolve with time. It is now commonly accepted to model leaks with an orifice function, for which appropriate coefficient and exponent values are still the subjects of research and conjecture (Lambert 2000). However, representing leaks by applying a "safety" factor to assumed demand values at known extraction points has also been a common modeling expedient. What is the best way to represent leaks in a system? Is it appropriate to treat them as additional demand and, if so, what might be the limitations of doing so?

The answers to these questions depend on what exactly is being modeled. Physically, leaks can be represented as demands and vice versa; they are both merely outflows from the system. Thus, in some circumstances a leak can be modeled as a fixed draw on the system and in others a demand can be represented by an orifice function. The choice depends on the temporal/spatial scale of analysis and the required accuracy of modeling results. An appreciation of how leaks affect water loss and energy use, as well as the spatial relationship among leaks, demand, system heads, and flows helps to inform this choice.

Fundamentally, a leak is a demand. Although a leak may be physically like a demand, its primary distinguishing feature is that it provides no revenue and tends to remain outside of user control. Fig. 1, which depicts a hypothetical demand profile of a water main with leakage, can help illustrate this point. Because the network is more intricate, its demand profile is a superposition of the numerous downstream demands it services. As leaks release water in accordance with system pressure, leakage tends to vary inversely with demand when there is no compensation (extra pumping) for diminished delivery pressures (left-hand side of Fig. 1). Leakage and demand at this more upstream location in the system are essentially ongoing throughout the diurnal cycle. The contrast between leakage and demand is more evident in the hypothetical bar plot representing flow through a household service connection. Demand at this connection occurs in discrete intervals according to when people use water. However, even when the faucets are shut and there is no "legitimate" demand, water still escapes from the pipe. If the service connection is supplied by a constant head source, leakage will actually increase when the faucets are closed. This continuous discharge is what nocturnal water audits seek to reveal and, in a more sophisticated way, inverse transient techniques similarly employ demand profile comparisons in order to expose leakage (Kapelan et al. 2003).

Impact Equivalence

When modeling leakage as extra demand at the legitimate, or known, demand location it is valuable to recognize certain limitations or peculiarities, especially with regard to the impact on system performance. As leaks entail both water loss and extra

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Note. Discussion open until August 1, 2009. Separate discussions must be submitted for individual papers. The manuscript for this technical note was submitted for review and possible publication on May 4, 2006; approved on August 14, 2008. This technical note is part of the ***Journal of Hydraulic Engineering***, Vol. 135, No. 3, March 1, 2009. ©ASCE, ISSN 0733-9429/2009/3-234–239/$25.00.

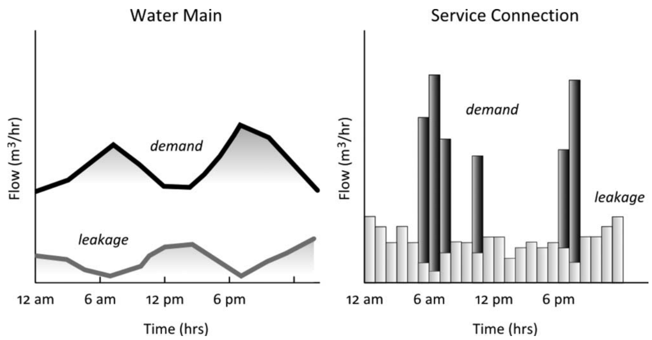

Fig. 1. Hypothetical diurnal flow profiles for conduits with leakage. Proportions are not necessarily to typical scale and, for simplicity, residential demand is lumped into a few single hourly impulses on the right-hand side.

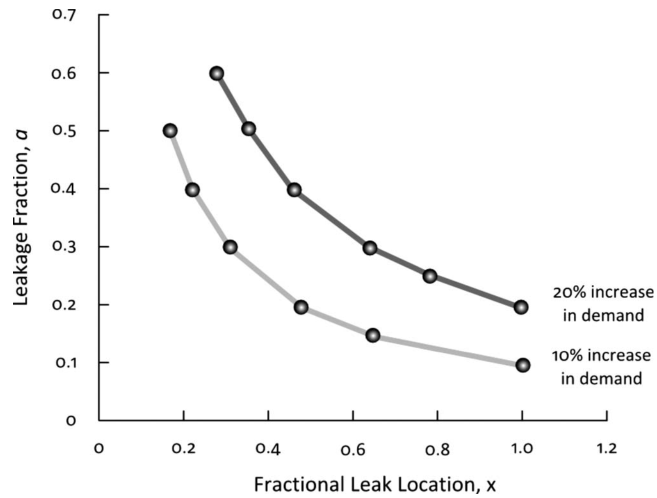

Fig. 2. Equivalency of water loss between upstream leakage and demand increase at the delivery point

energy use, their influence on both should be acknowledged by any surrogate demand representation scheme (Colombo and Karney 2002). There is an important spatial relationship between a leak and any demand increment that represents it.

Water Loss Equivalence

To capture its pressure dependence, the commonplace representation of leaks is with an orifice function of the form

$$Q_l = CA(2g)^{\alpha}H_l^{\alpha} = C_E H_l^{\alpha} \tag{1}$$

where Q_l=flow through the leak (m^3/s); A=leak area (m^2); C=dimensionless discharge coefficient; C_E=emitter coefficient ($m^{3-\alpha}/s$); H_l=pressure (m) at the leak; and α=emitter exponent. If a leak is situated at a fractional distance x along the pipe, the pressure head at the leak is given by $H_l = H_d + (l - x)H_f$ where H_d=delivery head in meters at the downstream, or delivery, end of the pipe and H_f=friction slope associated with the delivery flow Q_d downstream of the leak. This expression results from the hydraulic grade line's (HGL's) assumption of a piecewise structure when different flows up- and downstream of the leak occur as a result of the leak. The friction slope can be determined using either the Darcy–Weisbach or Hazen–Williams equations.

Because leakage at the demand location is akin to an increase in legitimate demand when leaks are absent, it is a simple matter to represent a leak anywhere along the pipe as an extra demand, or another leak, at the downstream extraction point. As the pressure is lowest at its downstream terminus, an equivalent leak situated here must be physically larger in order to ensure equal water loss. The ratio of emitter coefficients for a leak somewhere upstream of the demand point C_{El} and a leak at the demand point C_{Ed} is expressed as

$$C_{\mathrm{El}}/C_{\mathrm{Ed}} = H_d^{\alpha}/[(1-x)H_f + H_d]^{\alpha} \tag{2}$$

If the delivery head remains fixed, the corresponding demand point leak can be determined for a given upstream leak at any location. For example, a leak situated halfway along a pipe (L=1 km, D=0.3 m, C=100) which supplies 0.1 m^3/s at an H_d of 40 m has an emitter coefficient that is 94.3% that of a leak downstream at x=1. Therefore, the rate of water loss from this leak could be represented by a surrogate demand increase equal to the discharge of a leak at the downstream terminus possessing a 6% larger C_{Ed}.

Head Loss Equivalence

Because leaks also represent a loss of energy, this effect should also be preserved when a leak is represented by a surrogate adjustment in demand. It is possible in the single pipe case to represent a leak with an increment in demand (or leak in another location) such that the head drop across the segment is identical.

The frictional head loss for a pipe segment with a single leak located at x_iL is given as (Colombo and Karney 2002)

$$H'_{fi} = H_s - H_d = [1 + a_i x_i (a_i + 2)]H_f \tag{3}$$

where H_s and H_d=pressure heads at the source and delivery (demand) points, respectively, and a_i=leakage fraction associated with x_i; that is, the percentage of delivery flow Q_d that passes through the leak. Moving a leak with constant geometry (i.e., fixed C_E) to different points along the segment involves a different leakage fraction corresponding to each location.

If two leak positions are selected, and the leakage fraction for one of the two locations is fixed, the other leakage fraction is determined by equating head drop across the segment ($H'_{f1}=H'_{f2}$). In the case where one leak is assigned to the delivery/demand point (x_1=1) a quadratic expression in terms of the unknown leakage fraction a_2 results

$$a_2^2 + 2a_2 - (a_1^2 + 2a_1)/x = 0 \tag{4}$$

where a_1 could be considered as either a leak at the downstream terminus or an increase in demand Q_d. Establishing the reference leak or demand increment at the delivery end, Fig. 2 plots the required leak size a_2 against position that would be necessary in order to maintain a consistent head drop across the entire segment. The two curves correspond to 10 and 20% leakage or demand increase (a_1=0.1 and 0.2, respectively) at x=1. Because the HGL governs the magnitude of leakage, leaks situated upstream of the delivery end have the same energy loss effect as correspondingly smaller increases in demand at the delivery end. Thus, if one wanted to determine the delivery pressure effect of 30% leakage at 30% distance from the source, adding 10% to demand for a leak-free pipe would entail similar head loss. Of course, knowledge of leak location cannot usually be presumed and the 30% estimated leakage could be distributed among different points along the pipe. Eqs. (2) and (3) could be modified to account for several leaks but, in practice, accurate knowledge of leak sizes and locations is highly unlikely. Consequently, incrementing demand by some factor in order to test performance under a given loading condition is both common and understandable even if it simplifies key spatial relationships. It should also be mentioned that the formulations here are based on a horizontally oriented pipe, avoiding direct consideration of elevation differences which combine with head loss to determine pressure at points of leakage. Thus, more generally, pressure differences, and not strictly head loss, play the governing role.

Simultaneously capturing both the water loss and energy consumption effects of leakage with one increment in demand is not possible without an alteration of pipe resistance characteristics. Therefore, two different changes in delivery flow are necessary to consider for the purpose of determining equivalent water and head loss impacts for a leak of fixed size and location anywhere upstream of the delivery end. In general, systems with lower resistance exhibit a weaker spatial relationship, perhaps rendering it less problematic to account for the performance impact of leaks by simply applying "safety" factors to estimated demand. Naturally, sensitivity analyses for water loss and pressure effects can be conducted by simulating network performance for different leak sizes, locations, and friction characteristics.

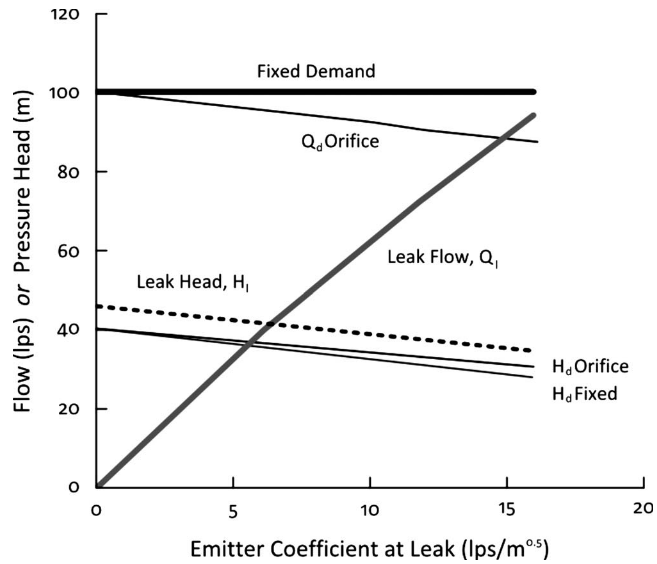

Fig. 3. Pressure and flow interactions for a pipe segment in which the downstream demand and a midpoint leak are modeled as orifices. The downstream delivery conditions are indicated as Q_d, orifice and H_d, orifice. As a reference, the delivery head for a fixed downstream demand (i.e., not an orifice) as a function of leak size, H_d, is indicated.

Modeling Demand as Leakage

Traditionally, network nodes are assigned fixed flow demands for a given time period in the diurnal cycle. As shown in Fig. 1, this is reasonable for water mains and upstream pipes that undergo fluctuating and continuous, but less erratic, demand profiles than service connections or pipes situated near extraction points. Essentially, many system demands are technically orifice discharges; sink faucets, shower heads, sprinklers, and appliance valves can all be modeled as emitters after Eq. (1).

In practice, however, end-user or device behavior tends to render such withdrawals more like fixed demands as consumers typically adjust their plumbing fixtures in such a way to achieve a desired volume in a specific time or to modify, to the extent possible, flow rate. This is especially the case with many domestic appliances which require a certain volume of water, and obtain it more or less quickly depending on available pressure (known as volumetric demand). Use of fixed demands stems from system design as the choice of appropriate pipe diameters is governed by estimates of steady state flow, whereas incorporating pressure dependence in a model may be more pertinent to certain operational concerns such as fire hydrants and sprinklers. In general, this depends on what the analyst seeks and what is the relevant spatial–temporal scale. For most extended period simulations, fixed demand representation is probably appropriate.

Fig. 3 depicts several pressure head/flow curves for the above-described single pipe system when downstream demand is modeled with an orifice of fixed size (C_E=15.8) and a 50.5 m constant head source drives the system. When there is no leak (or extra demand) in the system, Q_d=0.1 m^3/s and H_d=40 m. Situating a leak at x=0.5 and gradually increasing its emitter coefficient from 0 to 16 L ps/m$^{1/2}$ illustrates how leaks and demand behave similarly. As C_E for the leak is increased, flow through the leak Q_l increases. It increases at a decreasing rate, however, because the larger discharge through a leak of greater C_E relieves increasingly more internal pressure; that is, H_l decreases. As the leakage escalates, Q_d declines because H_d is smaller. Thus, the leak robs pressure from the downstream stretch of pipe. The total flow through the system (Q_l+Q_d) increases because Q_l grows faster than Q_d shrinks and the system is now a better conductor of water even if less of this water is reaching the desired point at x=1.

If the downstream demand is modeled as a fixed 0.1 m^3/s flow, the realistic interaction between the leak and legitimate demand is not accurately portrayed. First, the flow drops with increasing leak size and does not remain constant. Second, H_d is lower than for the orifice case because of the greater frictional head loss associated with the full passage of 0.1 m^3/s downstream of the leak. Consequently, modeling the downstream demand as a fixed flow overestimates flow and underestimates head.

Compensation for Leaks and Conservation

Although leaks involve an inescapable loss of water and energy, it is possible to compensate for some of this burden if the requirement of equivalent service (Colombo and Karney 2002, 2005) is relaxed. By reducing either Q_d, H_d, or both, it may be possible to tolerate leakage and delay capacity expansion. Fundamentally, compensating for leakage with pressure or demand management is just a way of treating a symptom rather than the problem. If reduced service is tolerable when leakage is present, why not accept this reduced service when leakage is absent and redirect the extra resources elsewhere? If leakage is viewed as a surrogate for higher demand (as in a climate change scenario, for example), analysis of pressure/demand management for coping with leakage is analogous to testing conservation measures for dealing with greater demand.

Pressure Management

Minimizing excess system pressures to curtail water loss is a common strategy of leakage control (Germanopoulos and Jowitt 1989; Savic and Walters 1996). If the demand or delivery head H_d is lowered, pressures at leak locations follow suit and water loss is diminished. Fig. 4 shows how the single pipe system responds

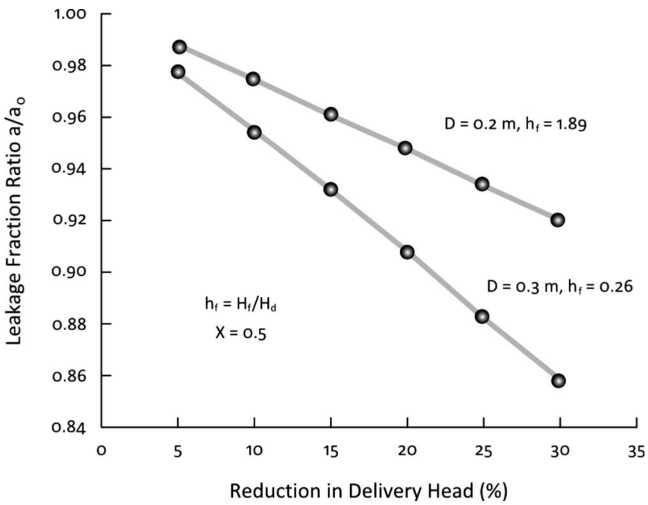

Fig. 4. Reducing delivery pressure to curtail leakage (assuming a fixed demand at the outlet)

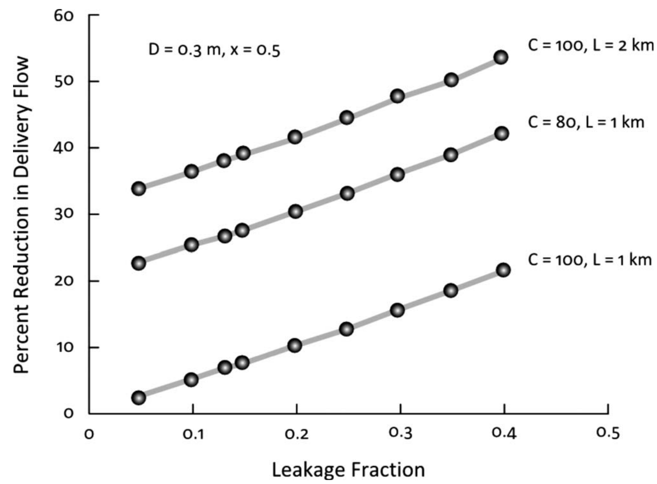

Fig. 6. Reducing delivery flow to restore delivery pressure

to this strategy. A leak bisecting the pipe ($x=0.5$) establishes the reference leakage a_0 when H_d is maintained at 40 m. As H_d is curtailed, water loss abates, evident by the declining leakage ratio a/a_0 in which a is the leakage fraction for the pressure reduced ($<H_d$) case. Two friction scenarios are established by assigning pipe diameters of 0.2 and 0.3 m (relative head losses of 1.89 and 0.26, respectively). Clearly a reduction in delivery head constricts leakage for both friction scenarios; however, it is the smoother pipe that responds more noticeably due to its flatter HGL (the smoother pipe is a better conductor of reservoir head). As x increases, the lines converge (not shown in Fig. 4) with that for $h_f=1.89$ becoming steeper. The increased frictional loss associated with a downstream leak makes pressure reduction a more effective leak management tool in this case.

Demand Management

Fig. 5 depicts the single pipe/reservoir system with several HGLs representing no leak and uncompensated/compensated leak scenarios. In the case of no leak, and a delivery flow of Q_d, the HGL follows the line AB and delivers flow at a head H_{d1}. If a flow greater than Q_d is delivered, the HGL follows the steeper line ADF resulting in a delivery head H_{d3} which is less than H_{d1}. When a leak appears at xL, and Q_d remains constant, the HGL follows ADE, where the segment DE is parallel to AB and the upstream segment that carries a flow larger than Q_d is steeper. This results in the intermediate delivery head H_{d2}. When the delivery flow, or demand, is reduced so that H_{d1} is restored, the HGL follows ACB.

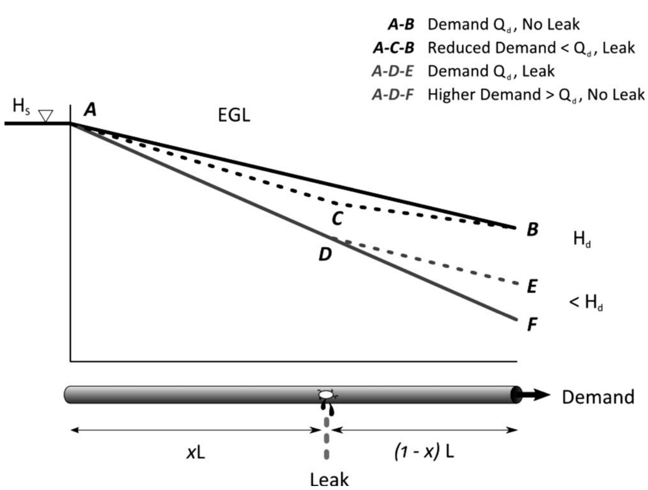

Fig. 5. Restoring delivery pressure lost due to leakage via demand reduction

Fig. 6 provides a numerical example of this manner of pressure compensation for the example pipe system ($D=0.3$ m, $H_s=50.5$ m, $x=0.5$). Three friction scenarios are considered by changing either the Hazen–Williams C coefficient or pipe length. Not surprisingly, as leakage increases so does the required reduction in delivery flow to achieve pressure compensation (H_d restored to 40 m). Moreover, the higher friction scenarios necessitate greater reductions in demand in order to realize such compensation. This illustrates how more severe conservation measures would be required in older systems to maintain pressures, or how leak repair and system rehabilitation can act as hedges against increased future demand. In this way, leakage impact analysis is a quasisurrogate for increased demand analysis.

Controlling Energy Use and Leakage

From the perspective of energy consumption it is interesting to compare the relative impact of pressure reduction and demand curtailment measures. Using the single pipe system ($L=1$ km, $C=100$) with a fixed leak size and location ($x=0.5$, $C_E=0.5\ \mathrm{m}^{2.5}/\mathrm{s}$) a sensitivity analysis of supply power P_S is determined for independent changes in H_d and Q_d about 40 m (Q_d held at 100 L/s) and 100 L/s (H_d held at 40 m), respectively. Supply power is determined as the product of total flow Q_T (Q_l+Q_d) through the system, supply head H_S, and the specific weight of water 9.81 kN/m^3. It represents the energy input for the system that must be supplied either by gravity or pumping. Two friction scenarios governed by the pipe diameters 0.25 and 0.3 m are considered (the curves denoting the smaller diameter conduit resting above those of the larger).

Fig. 7 presents the results of this analysis in which demand management impacts supply power more profoundly than pressure control [comparing Figs. 7(a and c)]. The obvious role of friction is once again evident in these plots, especially the rela-

Fig. 7. Impact on mechanical supply power and leakage of changes in delivery conditions for two friction scenarios (pipe diameters 0.25 and 0.3 m). (a), (c) Energy input as a function of delivery head and flow, respectively. (b), (d) The impact on leakage of these conditions. The upper curves in all four parts represent the higher friction scenario (smaller diameter of 0.25 m).

tionship between Q_d and P_S for the higher friction case ($D = 0.25$ m) in Fig. 7(c). Although changing Q_d does not alter Q_l as strongly as changing H_d [Figs. 7(b and c)], leakage is constrained over a narrower range. Thus, for this rudimentary system, demand management is a more effective strategy for countering water loss and energy use.

Conclusions

Modeling demands and leaks as either fixed flows or orifices may be appropriate depending on analysis scale and required accuracy. Fundamentally, leaks are demands, distinguished by the fact that they do not generate revenue. In essence, most system demands exhibit pressure sensitivity and thus can (and in some cases, should) be modeled as orifices. Treating demand as invariable flow is appropriate for system design when considering upstream pipes such as water mains. For operational modeling, however, fixed flow representation may not always be inappropriate and the choice of fixed-volume demand or orifice demand representation will depend on the role of consumer behavior in drawing water and on the spatial–temporal scale considered (at smaller scales especially, orifice flow representation may be most appropriate). In considering the appropriate flow representation, the following points should be kept in mind:

- It is not possible to use the same increment in nodal demand to capture both the water and head loss affects of a leak occurring somewhere along the pipe.
- When consumer demands are pressure dependent, the presence of leaks reduces actual delivery.
- When demands are modeled as fixed flows, the presence of leaks causes them to be overestimated, whereas heads are underestimated.
- Pressure management aimed at leakage reduction is more effective in newer systems with smoother pipes.
- In reducing system energy consumption, demand reduction seems to play a bigger role than pressure reduction, potentially influencing conservation strategies and resource management.
- As always, the choice of the model should reflect the purpose it is to serve, and the most powerful tool is always the understanding of the modeler.

Analyzing a system for leakage effects can be a surrogate for increased demand analysis and is thus relevant for illustrating the role of conservation. The interchangeability of models having either leaks or demands as a primary focus depends on the particulars of a system, the model used to represent it, and what results and accuracy are sought. Perhaps the most obvious recommendation would be to perform a dynamic sensitivity analysis on the system/model in order to assess how sensitive withdrawals might be to pressure and consider this in the context of analysis scale and accuracy requirements. Because leaks are system withdrawals, it seems reasonable to view leak-related modeling as being closely allied with demand modeling/management. In fact, depending on analysis goals, they may be one and the same.

Acknowledgments

The writers wish to thank NSERC and the Canadian Climate Change Action Fund (CCAF) for funding this research.

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