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Optimal design and operation of irrigation pumping stations using mathematical programming and Genetic Algorithm (GA)

Conception et opération optimales des stations de pompage d'irrigation utilisant la programmation mathématique et l'algorithme génétique (GA)

MAHDI MORADI-JALAL, Associate Researcher, *Department of Civil Engineering, University of Toronto, Toronto, Ontario, M5S 1A4. E-mail: mahdi.moradijalal@utoronto.ca*

BRYAN W. KARNEY, Professor, *Department of Civil Engineering, University of Toronto, Toronto, Ontario, M5S 1A4. E-mail: karney@ecf.utoronto.ca*

ABSTRACT
For many water authorities worldwide, one of the greatest potential areas for energy savings is in pump selection and in the related effective scheduling of daily pump operations. The optimal control and operation of an irrigation pumping station is achieved here by first solving the nonlinear governing model using Lagrange Multipliers (LM) and then through Genetic Algorithm (GA) approach. Computation in both methods is driven by an objective function that includes operating and capital costs subject to various performance and hydraulic constraints. The LM approach first specifies the annual energy costs and minimizes the total cost for all sets of pumping stations; the method then selects the least-cost pumps from among the feasible sets. The GA model simultaneously determines the least total annual cost of the pump station and its operation. The solution includes the selection of pump type, capacity, and the number of units, as well as scheduling the operation of irrigation pumps that results in minimum design and operating cost for a set of water demand curves. Application of the two models to a real-world project shows not only considerable savings in cost and energy but also highlights the efficiency and ease of the GA approach for solving complex problems of this type.

RÉSUMÉ
Pour beaucoup d'administrations des eaux dans le monde entier, un des plus grands secteurs potentiels pour des économies d'énergie est dans le choix des pompes et l'établissement d'un programme efficace des opérations quotidiennes de pompage. La commande optimale et le fonctionnement d'une station de pompage d'irrigation sont réalisés ici en résolvant d'abord le modèle non-linéaire du problème et en utilisant les multiplicateurs de Lagrange (LM) et puis par l'approche de l'algorithme génétique (GA). Le calcul dans les deux méthodes est conduit par une fonction objective qui inclut le fonctionnement et les frais financiers dus aux diverses opérations et contraintes hydrauliques. L'approche LM fournit d'abord les coûts énergétiques annuels et minimise le coût des ensembles de stations de pompage ; la méthode choisit alors les pompes à moindre coût parmi les ensembles possibles. Le modèle GA détermine simultanément le moindre coût annuel de la station de pompage et de son fonctionnement. La solution inclut le choix du type pompe, de la capacité, et du nombre d'unités, aussi bien que le programme de fonctionnement des pompes d'irrigation pour un coût minimum de conception et de frais d'exploitation correspondant à un ensemble de courbes de demande de l'eau. L'application des deux modèles à un projet réel montre non seulement l'économie considérable faite sur le coût et l'énergie mais met également l'accent sur l'efficacité et la facilité de l'approche GA pour résoudre des problèmes complexes de ce type.

Keywords: Energy costs, genetic algorithms, irrigation, lagrange multipliers, operation, optimal design, pumping stations

1 Introduction

The energy required for operating pumping stations to supply water for irrigation is often significant. The large costs of establishing, maintaining and operating pumping stations, particularly at a time of increasing energy costs, have motivated a search for improved design approaches and better operation of pumping stations (e.g. Ashofteh, 1999; Boulos *et al.*, 2001; Moradi-Jalal *et al.*, 2003). Concerted attempts are being made to increase the efficiency of existing or newly developed pumping stations through both pump selection and by efficient pump scheduling and operation.

Mathematically, the optimal design and operation of pumping stations is a large-scale nonlinear programming (NLP) problem. The objective is to minimize annual design and operational costs over a planning horizon subject to a set of hydraulic constraints, bounding values on the decision variables, and constraints reflecting operator preferences and system limitations.

There have been several recent attempts to develop optimal design and control algorithms to assist in the operation

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of complex water distribution systems (Alperovits and Shamir, 1977; Eiger *et al.*, 1994; Su *et al.*, 1987; Lansey and Mays, 1989; Savic and Walters, 1997; Prasad and Park, 2004). The various algorithms were oriented towards determining least-cost pump scheduling policies (typically proper on-off pump operation) based on the optimization tools such as linear, nonlinear and dynamic programming, enumeration techniques, general heuristics, and genetic algorithms. The success of these procedures has been limited and few have been applied to real water distribution systems (Cohen, 1982; Ormsbee *et al.*, 1989; Zessler and Shamir, 1989). Limited acceptance of optimal control models in engineering practice may stem from several factors:

(1) such techniques are generally quite complex involving a considerable amount of mathematical sophistication (e.g. requiring extensive expertise in systems analysis and careful selection and fine-tuning of parameters);
(2) they are generally highly dependent upon the number of pumps being considered along with the duration of the operating period;
(3) they are generally subject to oversimplification of model components along with several restrictive assumptions to accommodate the nonlinear hydraulic constraints that require, for example, demands to be known with certainty;
(4) they tend to be computationally demanding, leading to added costs and delays; and
(5) they may be easily trapped at a local optimum.

A contributing factor might also be the unavailability of suitable and user-friendly pump optimization packages. As a result, most optimal control models have only been used to support research, and not for real system decision-making.

Genetic Algorithms (GAs) have rapidly increased in popularity as a way of identifying superior, low-cost system expansion and operational alternatives. GAs have been applied to all civil engineering sub-disciplines within as including the design of water distribution systems (Simpson *et al.*, 1994; Reis *et al.*, 1997; Savic and Walters, 1997; Boulos *et al.*, 2000; Moradi-Jalal *et al.*, 2004). This paper illustrates the determination of an optimum set of pumps and associated annual operational rules for an irrigation pumping station. The GA solution is compared and contrasted to an alternate mathematical solution approach using Lagrange Multipliers.

2 Mathematical model development

A pumping station typically includes a number of pumps to meet specified demand characteristics. The overall goal is to minimize total annual cost which includes both annual energy consumption of each candidate pumping system, based on the increment discharge time of duration curves and the annual depreciation cost of associated capital investments. Thus, the objective function may be expressed as:

$$\min(\text{ATC}) = \sum_{i=1}^{n} C_{\text{RF}} \cdot C'_{\text{i}} + C_{\text{E}} \cdot E_{\text{T}} \tag{1}$$

$$C'_{\text{i}} = \left(1 + \frac{r \times TC}{2}\right) \cdot C_{\text{i}} \tag{2}$$

where C_{E} is the unit energy cost, C_i and C'_i one cost of the ith pump and equivalent cost of ith pump after construction time respectively; C_{RF} is the capital recovery factor; TC is the length of construction and r is the rate of interest. Issues such as the project's useful life, rates of interest and depreciation, capital cost, and length of construction, all enter into this determination. The annual consumed energy E_{T} is determined as:

$$E_{\text{T}} = \rho \cdot g \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{Q_{i,j} \cdot H_{i,j}(Q_{i,j})}{e_{i,j}(Q_{i,j})} \times \Delta t_{i,j} \tag{3}$$

in which E_{T} is the total annual consumed energy; $Q_{i,j}$ is the discharge from ith pump at jth time step; $e_{i,j}$ is the efficiency of ith pump at jth time step; $\Delta t_{i,j}$ is the associated time step of pump operation; ρ is the density of water; and g is the gravitational acceleration. Note that pump efficiency is a function of pump discharge, which in turn is related to the total discharge at the jth time step.

The objective functions (1) and Eq. (3) are constrained by

$$0 \leq Q_{i,j} \leq Q\max_i \tag{4}$$

$$\sum_{i=1}^{n} Q_{i,j} = (Q_{\text{N}})_j \tag{5}$$

$$H_{\text{min}_i} \leq H_{i,j} \leq H_{\text{max}_i} \tag{6}$$

where $Q\max_i$ is the Maximum allowable discharge of ith pump, $(Q_{\text{N}})_j$ is the total demand discharge required to be supplied at jth time step; these constraints are valid for all pumps at all times ($i = 1, \ldots, n$ and $j = 1, \ldots, m$).

The net pumping head $H_{i,j}(Q_{i,j})$ is also related to the static head and the total head losses. Head loss can be obtained, for example, by the Darcy–Weisbach equation applied in this paper.

3 Model simplification

The number of equations in the model is directly dependent on the type, size, and number of pump units, as well as on both the demand curves and their temporal discretization. Using a time increment of one month simplifies the solution and the discretization of the demand duration curves. By introducing these assumptions into Eq. (3), the formulation of consumed energy of pumping station E_{T} is reduced to

$$E_{\text{T}} = \rho \cdot g \sum_{i=1}^{n} \sum_{j=1}^{m} H_{i,j}(Q_{i,j}) \cdot \frac{Q_{i,j}}{e_{i,j}(Q_{i,j})} \cdot \Delta t_{i,j}$$

$$i = 1, \ldots, n \quad \text{and} \quad j = 1, \ldots, m \tag{7}$$

where E_{T} is the consumed energy of pumping station, subjected to constraints (4)–(6);

For further simplification in the calculation process, it is assumed that the pump efficiency curve is a function of discharge as follows:

$$e_{i,j}(Q_{i,j}) = a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i \quad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,m \tag{8}$$

where a_i, b_i, and c_i are performance coefficients found for the ith pump. By substituting Eq. (8) into Eq. (7), the annual consumed energy reduces to

$$E_T = \rho \cdot g \sum_{i=1}^{n}\sum_{j=1}^{m} H_{i,j}(Q_{i,j}) \cdot \frac{Q_{i,j}}{(a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i)} \cdot \Delta t_{i,j} \quad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,m \tag{9}$$

The final step in the optimal design is to select an appropriate pumping station system based on the minimum cost, number and type of pumps, demand curve characteristics, feasibility and personal preferences based on experience by considering availability for maintenance operation of pump type and similar background of pumping operation in other previous pumping stations.

It should be noted that as the values of pumping head, $H_{i,j}(Q_{i,j})$, have less variation comparing to values of efficiencies $e_{i,j}(Q_{i,j})$ so values of pumping head are nearly considered constant in order to do more simplification for the rest of calculations.

4 Solution of the mathematical model by Lagrange Multiplier (LM) method (ODIPS program)

For the optimal design of pumping station systems (with N variables, N_1 equality constraints, and N_2 inequality constraints), the LM method is used to solve the aforementioned mathematical model. The first step in this method is adding slack variable to create an equality constraint from Eq. (4) as:

$$Q_{i,j} + x_i^2 = Q\max_i \quad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,m \tag{10}$$

By combining the right-hand side of Eq. (9) and constraints (5) and (8) and extracting partially constant values for pumping head, the Lagrange function is expressed as:

$$\psi = \sum_{i=1}^{n}\left[\left(\frac{Q_{i,j}}{a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i}\right) + \lambda_i(Q_{i,j} + x_i^2 - Q\max_i)\right] + \lambda\left(\sum_{i=1}^{n} Q_{i,j} - (Q_N)_j\right) \tag{11}$$

where λ and λ_i are the introduced Lagrange multipliers.

To minimize Eq. (11), subject to constraints (4)–(6), the following conditions are applied:

$$\frac{\partial\psi}{\partial Q_{i,j}} = 0 \Rightarrow \frac{(a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i) - (2a_i Q_{i,j} + b_i)Q_{i,j}}{(a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i)^2} + \lambda_i + \lambda = 0 \tag{12}$$

$$\frac{\partial\psi}{\partial x_i} = 0 \Rightarrow 2X_i\lambda_i = 0 \tag{13}$$

$$\frac{\partial\psi}{\partial\lambda_i} = 0 \Rightarrow Q_{i,j} + x_i^2 - Q\max_i = 0 \tag{14}$$

$$\frac{\partial\psi}{\partial\lambda} = 0 \Rightarrow \sum_{j=1}^{m}\left(\sum_{i=1}^{n} Q_{i,j} - (Q_N)_j\right) = 0 \tag{15}$$

Equations (12)–(15) are valid for $i = 1,\ldots,n$ and $j = 1,\ldots,m$.

The solution of the nonlinear equations yields the optimal monthly discharge, $Q_{i,j}$, for a proposed set of pumps. Finally, the minimum total annual cost for each pumping system is found from Eq. (1).

Equation (13) is satisfied if $X_i = 0$ or $\lambda_i = 0$. For $X_i = 0$, Eq. (14) yields $Q_{i,j} = Q\max_i$. If $\lambda_i = 0$, a new discharge distribution is needed for the pump set. In this situation, Eqs. (12) and (15) are combined to give:

$$\lambda_i = 0 \Rightarrow \lambda = \frac{a_i Q_{i,j}^2 - c_i}{(a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i)^2} \quad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,m \tag{16}$$

$$(a_i^2\lambda)Q_{i,j}^4 + (2a_ib_i\lambda)Q_{i,j}^3 + (b_i^2\lambda + 2a_ic_i\lambda - a_i)Q_{i,j}^2 + (2b_ic_i\lambda)Q_{i,j} + c_i^2\lambda + c_i = 0 \tag{17}$$

Now, for a given value of λ in Eq. (17), only those values for $Q_{i,j}$ which satisfy the constrained Eq. (18) are acceptable.

$$0 \le Q_{i,j} \le Q\max_i \quad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,m \tag{18}$$

The solution of this part of the mathematical model is complete if the $Q_{i,j}$ of Eq. (17) satisfies Eq. (15); otherwise, a new value of λ is selected and computation is repeated to find a solution. In this case, the number of iterations is related to the number of time steps the discharge demand curve and the number of proposed sets of pumps.

In practice, it is important to determine a suitable domain for λ and then obtain the value of λ by iteration (e.g. by using the bisection method). As stated earlier, the Lagrange parameter λ is a function of $Q_{i,j}$ [refer to Eq. (16)], and considering Eq. (18), the feasible domain of x_{i2} for each i is 0 and $Q\max_i$. Thus, the values at the boundaries of the $Q_{i,j}$ domain are:

$$Q_{i,j} = 0 \quad \forall i \Rightarrow \lambda_{1i} = \frac{-c_i}{c_i^2} = \frac{-1}{c_i} \tag{19}$$

$$Q_{i,j} = Q\max_i \quad \forall i \Rightarrow \lambda_{2i} = \frac{a_i(Q\max_i)^2 - c_i}{[a(Q\max_i)^2 + b_iQ\max_i + c_i]^2} \tag{20}$$

And the extreme (min. or max.) value of λ, here labeled λ_{3i}, is obtained from Eq. (16):

$$\frac{\partial\lambda}{\partial Q_{i,j}} = 0 \Rightarrow a_i^2 \cdot Q_{i,j}^3 - 3a_i \cdot c_i \cdot Q_{i,j} - b_i \cdot c_i = 0 \quad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,m \tag{21}$$

By solving Eq. (21), a new $Q_{i,j}$ can be obtained; by substituting this into Eq. (16), λ_{3i} will be obtained. It must be noted that only the values of $Q_{i,j}$ that satisfy Eq. (18) can be accepted. λ_{1i}, λ_{2i} and λ_{3i} are Lagrangian parameters which are obtained through Eqs. (19)–(21), so boundary variations of λ are:

$$\mathrm{Min}(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}) \leq \lambda \leq \mathrm{Max}(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}) \quad (22)$$

There is a special situation in discharge distribution through pumps of a set, If the demand is small compared with the capacity of the individual pumps in the set, then the discharge should be applied to the pump with the lowest energy consumption with the best pump efficiency.

Once the optimization process and the method of solution are identified, a computer program is used to find the Optimal Design of the Irrigation Pumping Station (ODIPS) using this LM method. The main program consists several subroutines which assigned for several purposes; subroutines are used to generate different feasible combination of pump types and the optimal distribution of demand discharge to each time step. Subroutines are also needed for several tasks:

(1) to convert the efficiency curves into polynomial functions of Degree 2 by a least-squares method by inputting several points in efficiency-discharge curve;
(2) to compute the Lagrangian parameters λ_{1i}, λ_{2i}, λ_{3i}; and
(3) to solve the set of nonlinear equations and optimizing the operation based on the Lagrange method.

In LM method, at first ODIPS program generate all feasible pump combinations from selected pump types, (more than thousands sets, in this problem) and then ODIPS starts to solve mathematical equations for all generated sets to find optimal monthly operation schedule individually, after computing annual operation cost for all sets and adding them with corresponded annual depreciation cost of generated pump sets, values of total annual cost for all pump sets are obtained and by ranking them, the best pump sets are extracted. An example application is presented later.

5 Solution of the mathematical model by Genetic Algorithm (WAPIRRA program)

The GA approach is a probabilistic global optimization technique based on the mechanics of natural selection and genetics (Holland, 1975). Numerically the process uses reproduction, crossover, and mutation to evolve encoded variables. The algorithm is designed to produce "populations" of solutions whose "offspring" display increasing levels of optimality (Goldberg, 1989). The optimization model employed here uses an efficient GA technique to obtain optimal solutions to the pump-scheduling problem. The GA is designed to perform search procedures of an artificial system by emulating evolution. The principal advantage of GAs is their inherent ability to intelligently explore the solution space from many different points simultaneously, enabling a higher probability for locating a global optimum without having to analyze all possible solutions available and without requiring derivatives (or numerical approximations) or other auxiliary knowledge.

Using a GA approach to optimize the design and operation of a pumping station involves the following steps:

(i) Randomly generating an initial set of pump combinations to for a given demand values;
(ii) Minimizing the total annual cost, which includes operation, maintenance and depreciation costs, by changing the set and discharge of the pumps based on the performance evaluated by the GA process; and
(iii) Achieving the final criterion to stop the optimization process and reporting the number of pumps and pump types, value for output discharge on every time step for the optimum set of pumps, the initial investment and the annual costs of depreciation and operation, and the total costs for the optimum set.

Specifically, the WAPIRRA program uses the GA approach to optimize the aforementioned model. The program is applicable to any number of pumps, pump types, time steps, and different unit costs of energy at every time step, but the program limits the maximum number of pumps in a station.

There are two ways to proceed with the optimization: (1) to optimize a given solution as an initial set; and (2) to optimize a randomly generated initial set. These two ways of setting a pump set are appropriate for the optimization of different pumping stations. For example, with a given set of pumps, an optimal solution for the operating schedule of existing pumping stations can be determined without changing the pump types in the stations by only adjusting their schedule to minimize the consumed energy. Similarly, for any number of pumps, an optimal solution for both the design and operation of the pump set and its schedule can be found.

6 Application

Iran, with an area of 165 million hectares, is located in a semiarid region of the Middle East. Distribution of precipitation is uneven with an average precipitation of less than one-third of the world average (Alizadeh and Keshavarz, 2005). In the year 2000, about 43 billions cubic meters of surface water resources, including regulated flows, were used by reservoir dams, pumping stations, small-scale water supply projects, or traditional stream systems (Jamab Consulting Engineers, 1999).

As a case study, the main pumping station of the Farabi Agricultural and Industrial Complex in Iran, is considered. It consists of a 20,000-hectare agricultural land, which is located in the Khoozestan province in southwestern Iran (Fig. 1). Irrigated water in this project is used for sugar cane and other crops. In this station, demanded water is supplied for agricultural use from the Karoon River to the main lot. Karoon River is 890 km in length with a catchment area of 66,930 km^2, is the longest river in the country which flows along many industrial and agricultural areas. Karoon water is also used for water supply of Ahvaz city, the capital city of Khoozestan province.

Figure 2 shows the demand duration curve, discretized in monthly segments that must be supplied by the main pumping

Table 6 Typical output of discharge for ODIPS/WAPIRRA optimum set of pumps

	Month											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
ODIPS Results												
Disch. P #1 (P.T = 1)	0	0	0	5.33	5.7	6.29	7.35	7.14	5.89	5.88	0	0
Disch. P #2 (P.T = 1)	0	0	5.44	5.33	5.7	6.29	7.35	7.14	5.89	5.88	5.55	0
Disch. P #3 (P.T = 1)	0	0	5.44	5.33	5.7	6.29	7.35	7.14	5.89	5.88	5.55	5.34
Disch. P #4 (P.T = 2)	2.32	2.26	0	0	2.32	2.5	2.92	2.83	2.34	2.34	0	0
Disch. P #5 (P.T = 2)	2.32	2.26	0	2.1	2.32	2.5	2.92	2.83	2.34	2.34	2.2	0
Disch. P #6 (P.T = 2)	2.32	2.26	2.16	2.1	2.32	2.5	2.92	2.83	2.34	2.34	2.2	2.13
Disch. P #7 (P.T = 2)	2.32	2.26	2.16	2.1	2.32	2.5	2.92	2.83	2.34	2.34	2.2	2.13
Disch. P #8 (P.T = 4)	0	0	0	0	0	1.65	1.93	1.89	1.55	1.54	0	0
Disch. P #9 (P.T = 4)	0	0	0	0	0	1.65	1.93	1.89	1.55	1.54	0	0
Disch. P #10 (P.T = 4)	0	1.51	0	0	0	1.65	1.93	1.89	1.55	1.54	0	1.4
WAPIRRA Results												
Disch. P #1 (P.T = 1)	0	0	0	5.42	5.39	6.17	7.03	6.84	5.63	5.65	0	0
Disch. P #2 (P.T = 1)	0	0	0	5.7	5.35	6.08	7.05	6.85	5.66	5.59	5.88	0
Disch. P #3 (P.T = 1)	0	5.31	5.57	5.58	5.42	6.05	7.05	6.87	5.62	5.73	5.94	5.5
Disch. P #4 (P.T = 1)	5.2	5.19	5.33	5.6	5.25	6.02	7.08	6.94	5.77	5.73	5.88	5.5
Disch. P #5 (P.T = 2)	2.05	0	2.1	0	0	2.39	2.8	2.75	2.24	2.22	0	0
Disch. P #6 (P.T = 2)	2.05	0	2.2	0	2.14	2.39	2.89	2.72	2.25	2.22	0	0
Disch. P #7 (P.T = 4)	0	0	0	0	0	1.56	1.9	1.8	1.53	1.48	0	0
Disch. P #8 (P.T = 4)	0	0	0	0	1.42	1.59	1.85	1.83	1.5	1.47	0	0
Disch. P #9 (P.T = 4)	0	0	0	0	1.43	1.55	1.85	1.8	1.5	1.51	0	0
Total discharge	9.3	10.5	15.2	22.3	26.4	33.8	39.5	38.4	31.7	31.6	17.7	11

Table 7 Specification of ten best sets of pumps in ODIPS model

Ranking sets	Annual cost	# of pumps	# P.T-1	# P.T-2	# P.T-3	# P.T-4
1	182.40	10	3	4	0	3
2	182.52	8	4	3	0	1
3	182.83	10	3	5	0	2
4	182.83	7	3	3	0	1
5	183.12	10	3	3	1	3
6	183.13	7	4	2	1	1
7	183.14	10	3	4	1	2
8	183.36	8	4	4	0	0
9	183.45	8	4	3	1	0
10	183.59	8	4	1	2	1

active pumps is divided equally among identical pumps. Thus, pumps are either off or have the same discharge as other identical pumps in each irrigation period. Table 8 shows relative discharge results of ODIPS/WAPIRRA programs, which are ratios of optimum discharges of pumps to maximum allowable discharges of pumps (all are greater than 0.5).

A typical view of the WAPIRRA program is shown in Fig. 4. Input and output worksheets of the program, which are in MS Excel 2000 format and are used in the optimization model, are provided in Fig. 5. The results of this case study are achieved through the use of the aforementioned inputs and optimization equations and with the aid of the computer programs previously described.

7 Conclusions

The energy required for operating pumping stations in an irrigation area is often significant, as is the capital/annual costs of the required pumps. Thus, improvements in the design and operation efficiency of existing or newly developed pumping stations can be important.

Mathematically, the optimal design and operation of pumping stations is a large-scale nonlinear programming problem. Minimizing the total annual design and operation cost over a given planning horizon based on mathematical programming and a GA approach was taken as the objective of the design. Developing a large-scale nonlinear optimization model that accounts

Notation

a_i, b_i, c_i = Efficiency curve coefficients of ith pump
C_E = Unit energy price
C_i = Cost of ith pump
C'_i = Equivalent cost of ith pump after construction time
D_i = Delivery pipe diameter of ith pump
E_T = Total annual consumed energy
e = Efficiency
$e_{i,j}$ = Efficiency of ith pump at jth month
H = Pumping head
$H_{i,j}$ = Pumping head of ith pump at jth month
$HS_{i,j}$ = Static head of ith pump at jth month
$H\max_i$ = Maximum allowable pumping head of ith pump
$H\min_i$ = Minimum allowable pumping head of ith pump
i, j = ith pump at jth month
$(Q_N)_j$ = Total demand at jth month
$Q_{i,j}$ = Discharge of ith pump i at jth month
$Q\max_i$ = Maximum allowable discharge of ith pump
$Q\min_i$ = Minimum allowable discharge of ith pump
TC = Construction length of project
X_i^2 = Variable parameter
λ, λ_i = Lagrange parameters
Δt = Time step of pumping
ψ = Lagrangian function

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