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An exploratory approach to teaching gradually varied flow

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Abstract

An innovative approach to teaching the principles of gradually varied flow is presented in the larger context of water resource engineering. The paper illustrates an approach to gradually varied flow (GVF) by directly tracking the "deposits" and "withdrawals" of mechanical energy to a channel and then using this balance to show that certain evolutions on the specific energy diagram are precluded, and others permitted, under steady flow. Based on this well known approach, but now applying second law of thermodynamics, students are encouraged to first explore, and then critique, an innovative approach for GVF calculation. This "trial" approach minimizes the total input of mechanical energy to the channel and, perhaps surprisingly, succeeds in calculating water surface profile under both in sub- and supercritical regimes, and even captures the hydraulic jump in a single pass calculation. However, students are challenged to understand both the power and the limitations of this formulation. The overall educational thrust is not to present yet another method of computing flow profiles; rather, the goal is to help students establish a more complete understanding of the computational constraints and options, and how these manipulations must always reflect the underlying physics.

Keywords: Education; Energy methods; Minimization; Gradually varied flow; Shallow flows; Thermodynamics

1. Introduction

Both educationally and intrinsically, hydraulic engineering is a broad and dynamic field with important historical, cultural and environmental implications. Hydraulic engineering, perhaps particularly over the past century, has contributed to society in many positive ways (Liggett, 2002). Yet, hydraulic projects are inevitably complex, not only achieving their primary and intended benefits but, sometimes (particularly if poorly implemented), detracting from, other ecological, human and economic functions. This larger reality excludes the possibility of a comprehensive hydraulic "cookbook," since not even a well conceived set of recipes could hope to cover the variegated scope of problems that might arise over a student's professional career.

In fact, hydraulic engineering applications invariably require not only a thorough comprehension of the principles of hydraulics, but a deep appreciation for issues of local context. Such contextual issues relate to many matters, including local culture, which influences both a system's operation and maintenance, but also involve the functioning of the natural system with both conditions and is modified by the design, and of historical influences that impinge on issues of evolution, ownership, governance and legal responsibility. Only a scant few applications, well rehearsed and long-understood, can be handled straightforwardly.

The educational conclusion is as obvious as it is difficult to negotiate. Preparing students for dealing with dynamic and challenging problems requires instructors to balance many considerations, ranging from the adopted teaching style to specific questions of curriculum and course coverage, while always emphasizing an integrated and broad education (Liggett and Ettema, 2001). The importance of the topic and the realization that some facets of a traditional engineering

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education must be modified to fill the gap between teaching and practice has been recognized by many organizations and instructors. One significant effort to address the educational issues was undertaken by the Task Committee on Teaching Hydraulic Design through organizing a conference session. The conference was well received and the expanded version of the selected papers appeared in a special issue of the *Journal of Hydraulic Engineering* (JHE) in 2001. The published articles contain excellent material that can both aid and inspire instructors. One of the included papers specifically discusses the future of civil engineering education for hydraulic engineers (Liggett and Ettema, 2001); others focus on examples of design projects for use in the classroom (Tullis and Tullis, 2001; Finnie, 2001; Jewell, 2001), a few present local teaching with examples from Australia, England, and the United States (Hotchkiss et al., 2001; Novak and Valentine, 2001; Heyder, 2001; Chanson, 2001), and one reviews documents from various sources for use in teaching hydraulic design (ASCE Task Committee, 2001). All this work appreciably contributes to the subject and sheds light on difficult and, often at least partly, intractable issues.

A truism that the current paper accepts is the teaching of a dynamic subject should be experienced dynamically. While the elements are invariably individual, depending on the resonance between the subject and both the teacher and the students, it is maintained here that a dynamic style needs to focus on the physics of the problem rather than to be carried away by the mathematical formalism. The current paper aims neither to dissect different components of such a teaching style nor to quantitatively evaluate the performance of this method over more traditional ones. Instead its aim is to share an innovative approach to teaching the principles of gradually varied flow. This approach has been suggested through many years of experience with graduate classes on water resources systems modelling at the University of Toronto. The application of these principles provides an insightful way of teaching gradually varied flow that has been warmly welcomed by students in the class and leads to a novel exploration of GVF.

2. Course background and context

The graduate course “water resources systems modelling” covers topics of hydrology, reservoir routing, open channel, ground water and pressurized flow, and seeks to present aspects of the formulation and modelling that are common to this diverse range of applications. The approach begins with two well-known premises that are reinforced throughout all course topics. The first of these principles is that all conserved substances are stored in physical systems in direct relation to the imbalance in the net flux (inflow and outflow) of the conserved substance. The formal statement of this law is of course summarized in the well-known Reynolds Transport Theorem. The second modelling challenge is to consider, from elementary system considerations alone, what physical mechanisms are available in each system that permit storage of any resulting imbalance. These considerations lead formally to an equation of state, but students are encouraged to ask a more essential or intuitive question: what physical changes actually come about (could, at least in theory, be measured) in the system as a result of an imbalance in any conserved quantity? It should be emphasized that the pedagogy goal is to ground these considerations both within the system being modeled (its internal characteristics) and within the human context of the decision to be made (the external characteristics). Indeed, it is often the fact that a *decision* is insensitive to the modelling assumptions (e.g., isothermal vs. temperature-variable flow, or the assumption of incompressibility) rather than the flow itself that justifies a particular set of simplifications.

These general statements can be made clearer to students by grounding them in examples and applications, and many are better than a few. Thus, this point in the discussion leads naturally to an opportunity to consider, for example, how open channels models essentially differ from pressurized ones, or on what justifies the selection of a 1-D, 2-D and 3-D flow representation. However, this is also a time to provide a more mature perspective on how equilibrium is actually achieved in systems, thus allowing transient states to be taken as natural, reasonable and indeed inevitable outcomes of any analysis. If this approach is adopted, the overall approach to teaching hydraulics becomes inherently transient; steady state flows are thus implicitly treated as a special case rather than the norm. The convergence to equilibrium is seen from this perspective as a set of physical and incremental adjustments that progressively resolve the imbalance present as the storage equations are continuously applied and worked out for all conserved quantities. More profoundly, the various approximations that are often imposed in the modelling approach—whether steady, or isothermal, or incompressible, or indeed pressure vs. free surface flow—arise as natural approximations in the governing laws (i.e., internal considerations) in the context of specific decisions (i.e., the external context). Thus, modelling simplifications can be tested and confirmed through the expedient of the modeler’s purposes and flow properties, rather than imposed on the solutions *a priori*. The key question here is invariably pragmatic: what does it matter? What difference would it make? What if it were not so?

To make this somewhat abstract discussion more educationally tangible, this paper applies the approach to gradually varied flow. In particular, by considering the “deposits” and “withdrawals” of mechanic energy that occur in steady flow in a sub- and supercritical flows in steep and mild channels, it is easily shown that certain evolutions on the specific energy diagram are precluded under steady flow, and that only certain transitions are permitted. The students then see that the precluded and permitted transformations arise naturally and produce the normal rules of upstream and downstream control in super- and sub-critical flows. Such insights should not be imposed from above through a traditional lecture, but are best to evolve from an interactive “discovery” session, so that these insights become logical outcomes of the thought processes applied nature of steady flow and normal depth, coupled to the basic shape of the specific energy diagram. With these aides the insight become visual and direct, and a logical implication of various flow states. The by-product is that the discussion nicely demonstrates why such transformations naturally give rise to a degree of unsteadiness and, overall, more tightly and physically couple the steady flow equations to those for unsteady flow. More specifically, the implications of the evolution of flow within the channel are extended to consider the nature of the storage and accumulation along a channel length, or in other words, the traditional topic of gradually varied flow (GVF).

3. Gradually varied flow

Water-surface calculations along the length of the channel for a specified discharge provide useful information for many hydraulic engineering problems. These computations are essential in the analysis of various problems, including three specific applications: 1) the determination of the effect of hydraulic structures on the upstream and downstream channels; 2) the estimation of flood plane; and 3) the determination of the safe and optimum operation of hydraulic structures.

Several numerical procedures, such as the Standard Step method and the Direct Step method, have been proposed to compute GVF, as described in excellent standard texts like Chow (1973) and Henderson (1966). The computational procedure usually starts from a point at which the hydraulic characteristics are clearly defined and proceeds in the upstream direction in sub-critical flow and downstream in supercritical flow. Although the procedure is easy to understand, many students wonder why the direction of calculation differs, and clear answers are often short, missing, or imprecise. For example, Chaudhry (1993) discusses the issue briefly in a footnote, stating that from numerical point of view, the water surface profile in either sub- and supercritical flow can be calculated in both direction, but begins from the first hydraulically-defined point. Since in sub-critical flow disturbances are initiated from downstream side of the channel, the calculation is begun there and proceeds to the upstream side. Conversely, supercritical flows have known conditions upstream.

Although the above reasoning is acceptable, it is perhaps not completely satisfying and begs the student to ask why conditions are known where they are. Indeed one may argue that in a hydraulically long sub-critical channel which is disturbed at the downstream end, the upstream flow condition is known beforehand (uniform flow), but it is impossible to start the GVF calculation from that point and to converge to the already known downstream condition. But the whole approach seems ad hoc and arbitrary to the novice, an uneasy feeling that likely prolongs not just the backwater calculations but perhaps the learning process.

4. Formation of GVF surface profiles

Rather than arm waving, it is perhaps helpful to return to more basic considerations, and this approach, though initially more demanding, has the reward of visualizing the system's evolution. The kind of reasoning process that students are encouraged to engage in can be summarized as follows.

If no external forces other than gravity and bed shear forces are applied to an open channel flow, uniform flow can be theoretically established in a channel as the balance of these key forces. In this kind of flow, the bed shear force and gravity force become equal and the constant energy in the channel now means the water surface profile must be parallel to the channel bed (the "no accumulation" condition). Now we consider a perturbation to this condition. As soon as the flow at a point is disturbed by any external force whatever, an instantaneous imbalance of mechanical energy is set up which tends to cause a disturbance to move away from its point of creation. Thus, at this moment, according to the Reynolds Transport Theorem, instantaneous transient flow is locally formed and the energy starts accumulating in, or depleting from, the channel. In open channel flows, an instantaneous imbalanced mechanical condition also results in an imbalanced condition in mass flow rate, so that mass begins to accumulate or deplete which is manifested as a change in the flow depth. As soon as this disturbance 'information' reaches the immediate upstream or downstream points, a similar imbalanced condition is set up and propagates further. This process is continued until the imbalanced condition reaches a particular length of the channel and until the flow eventually re-establishes a steady state condition. This point is obviously reached when we will again observe an equilibrium condition as a uniform flow.

Even more specifically, consider the uniform flow in a mild channel having the uniform cross section along its length and fed by a lake in the upstream end. The uniform flow is then partially obstructed by a sluice gate at the downstream end. At this moment energy level in channel at gate location is not sufficient to move the initial full discharge through the gate. Consequently the discharge at the gate place decreases temporarily and a mass imbalance occurs in the vicinity of the gate. This causes the flow depth at the gate location to increase and affects upstream side of the channel. This process is continued until the energy at the gate is large enough to push the original discharge through the gate; thus, by the process of routing or accumulation, the discrepancy in the mechanical energy is progressively resolved over the length of the channel. Needless to say the channel gradually experiences its new steady state in which part of channel now carries a backwater curve whereas other parts remain in their original (undisturbed) form. Thus, to provide required energy at the gate, mechanical energy is accumulated in part of the system through a mechanism in which part of available gravity energy is preserved in the channel by decreasing head loss in the backwater zone; this is naturally achieved by an increase in water depth that decreases the rate of energy dissipation since the original slope is unlikely to be adjusted quickly (if at all).

An exactly analogous set of arguments can be used to explain energy withdrawal from a mild channel creates a drawdown curve, or to explain how energy accumulation or withdrawal in a steep channel carries different shape of profiles, owing to how energy is partitioned between gravity and kinetic terms in the different kinds of channels. These additional arguments are naturally left as exercises for the aspiring student, or indeed to the intrepid reader.

5. Permitted calculations directions

The above discussion leads student to quite naturally reconsider the feasible flow directions associated with GVF calculations. Consider first a mild channel carrying a given discharge. As shown in Fig. 1, the normal flow condition is disturbed at the downstream end in one of two different ways, leading to backwater and drawdown curves.

It can be easily shown analytically that while flow moves from the upstream to the downstream ends energy is accumulated in the channel in case of backwater and lost from the channel in the other case. To show this, consider the following equation which is the general governing equation of gradually varied flow (Chow, 1973).

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \quad (1)$$

As can be seen, the sign in the numerator is always positive for sub-critical flow, but the sign in denominator is positive in the backwater case ($S_0 > S_f$) and is negative in the drawdown case ($S_0 < S_f$). So dy/dx in the backwater and drawdown cases are positive and negative respectively. Applying this information to the specific energy curve (Fig. 2) shows that in case of backwater case the energy is accumulated in the channel as moving in direction of flow and in the other case the energy is lost from the system as moving in the same direction. According to our previous discussion, the only physically sensible way for GVF calculation to proceed is to move toward equilibrium status.

This shows we cannot generally start computation from point A and to reach to points B or C. This is because as we move downstream the energy is continually accumulating (in the case of backwater) or leaking from the system (in the case of drawdown) in or from the channel and an equilibrium status can be never reached. However starting either from point B or C or even any other depth in the downstream, the computations are automatically led to the equilibrium status that is uniform flow.

In the case of a backwater calculation, the energy at point B is greater then uniform flow, so to reach equilibrium status the calculation should be done in the way that the system loses energy. Considering Eq. (1) and Fig. 2 together indicate that the only possible way is to move upstream. In the case of drawdown, by contrast, the energy at point C is less than that at uniform flow, so the direction of calculation could be selected by the student in such a way that the system gains energy. Again considering Eq. (1) and Fig. 2 together indicate that the only possible way is to move upstream. The obvious conclusion is that the only feasible direction for GVF calculation in sub-critical flow is from downstream to upstream. The similar approach can be taken to show that the only possible direction of GFV calculation in supercritical flow is from the upstream to downstream end.

6. GVF calculation formulation

As shown earlier, the water surface profile in gradually varied flow is formed as a result of the deposits or withdrawals of mechanical energy from the channel. The amount of accumulation or withdrawal of energy along the channel now obviously depends on many factors including channel bed slope, channel roughness, discharge, and the degree of energy level by which flow is disturbed. Whatever the amount of mechanical energy deposition or withdrawal is along the channel, this value is also limited by the second law of thermodynamics.

Although students initially struggle with this concept, it is important for them to see that the second law of thermodynamics governs all real process in nature, an insight that can be presented in many forms. One popular presentation of this law, as stated by Abbott & Minns (1998) is that "there is a tendency on part of the nature to proceed toward a status of greater disorder." If the students are encouraged to at least provisionally assume that the greater disorder is equivalent to greater energy dissipation (that is, conversion of energy from mechanical to thermal forms), then in the context of open channel flow this law can be restated as 'there is a tendency in open channel flow to proceed toward greater energy dissipation'. This law then shows that the water surface profile in the open channel tends to be formed in such a way that the least energy is "deposited" or, in other words, that the dissipative withdrawal is maximized along the channel.

By applying this principle it is possible for the students to formulate an alternate formulation of GVF. In this approach the water depths in the specified points of the channel can be calculated by an optimization procedure in which the total amount of energy dissipation is maximized. To facilitate formulation consider Fig. 3. As shown, the channel is divided to N equal distance sections and the flow depths in intermediates computational points (Y_2 through Y_N) are taken as the unknowns. The unknowns then are implicitly calculated using the following optimization formulation through which the total head loss along the channel is maximized while the momentum residual in each two subsequent sections is forced to a predefined tiny value.

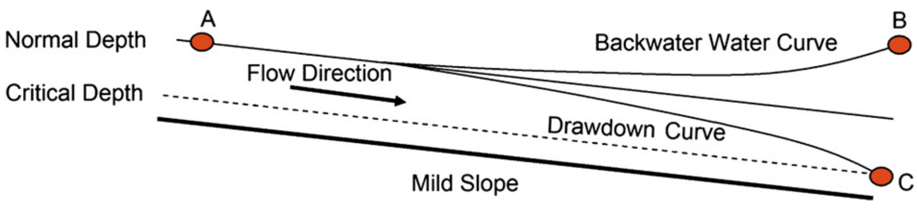

Fig. 1. Disturbed flow in a sub-critical open channel flow.

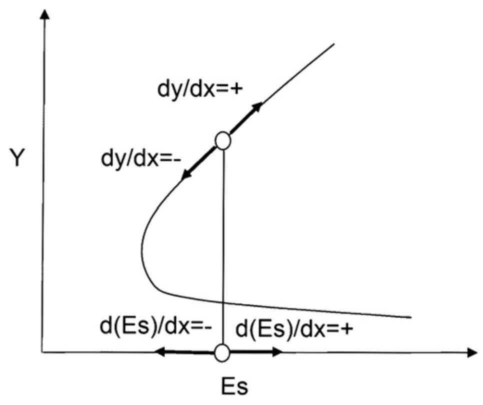

Fig. 2. Energy level changes in sub-critical flow.

$$\text{Maximize} \sum_{i=1}^{N} \left(\overline{S_f}\right)_i \times \Delta x \tag{2}$$

Subject to:

$$\left|\left(\Delta F_P + \overline{W}\sin\theta - F_f\right)_i - \left(\omega Q(V_{i+1} - V_i)/g\right)\right| \le \varepsilon \quad (i = 1, 2, \ldots N) \tag{3}$$

$$Y_1 = C_1 \tag{4}$$

$$Y_{N+1} = C_2 \tag{5}$$

$$Y_2\ , Y_3, \ldots, Y_N \ge 0 \tag{6}$$

Where,

$$\overline{S_{f_i}} = \frac{S_{f_i} + S_{f_{i+1}}}{2}, \ S_f = \frac{Q^2 n^2}{A^2 R^{1.33}} \tag{7}$$

$$\overline{W} = \omega \Delta x \left(\frac{A_i + A_{i+1}}{2}\right) \tag{8}$$

$$\Delta F_P = \omega A_i (Y_{i+1} - Y_i) \tag{9}$$

$$F_f = \omega \Delta x \left(\frac{A_i + A_{i+1}}{2}\right) \overline{S_{f_i}} \tag{10}$$

In the above equations Q = flow discharge, V = velocity, n = Manning roughness coefficient, A = cross sectional area, R = hydraulic radius, Y = channel depth, θ = channel bottom angle, g = gravitational acceleration, i = cross sectional index, ω = unit weight of water, C_1, C_2 = upstream and downstream depth values, Δx = distance interval, Sf = friction slope, $\overline{W}$ = averaged water weight, F_f = friction force, ε = a constant-tiny value.

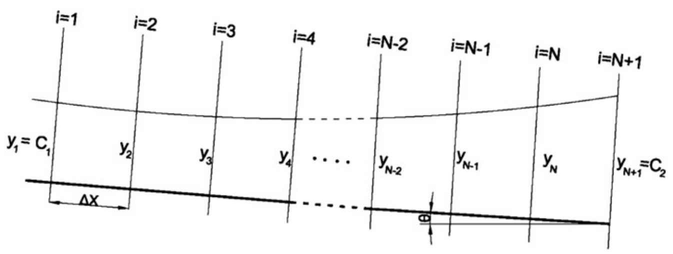

Fig. 3. A schematic discritization of the channel.

It should be noted that unlike the traditional GVF calculation methods in which just one point boundary is needed for water surface calculations, the proposed method could be served by either one or two boundary points, one at the upstream end and one at the downstream end. This means the method can consider the effects of both boundaries on the flow simultaneously. However, one may argue that in trans-critical flow the energy loss in the system is not just limited to the bed friction loss and the system will receive additional energy loss through the formation of the hydraulic jump. That is true, but as we carefully look at the formulation we will find that we will find that the energy losses associated with the formation of any discontinuities flow is implicitly considered through forcing the momentum balance between two subsequent computational sections. This method indeed is a physically sensible way to solve a set of ill-posed nonlinear equation having one equation more than the unknowns. Of course, discovering a variety formulation is challenging but learning its limitations is a great discovery, one that lays down a memorable lesson for the student to bear in mind in their own work and designs.

As can be seen the obtained optimization set of equations are nonlinear in the nature and should be solved using a standard iterative method. To solve the equations, the Solver toolkit of the Microsoft Excel program is employed herein; this versatile set of tools is an excellent one to expose (or remind) the students of.

7. Numerical results

To illustrate the accuracy of the proposed approach two different examples are presented. In the first example the backwater curve is calculated in a rectangular channel with the following characteristics: width = 100 m, discharge = 230 m^3/s, bed slope = 0.002, normal depth = 0.93 m, and Manning roughness coefficient = 0.017.

The backwater curve is generated in the channel by a sluice gate producing a depth of 3 m at the downstream end of the channel. Fig. 4 shows the water surface profiles calculated

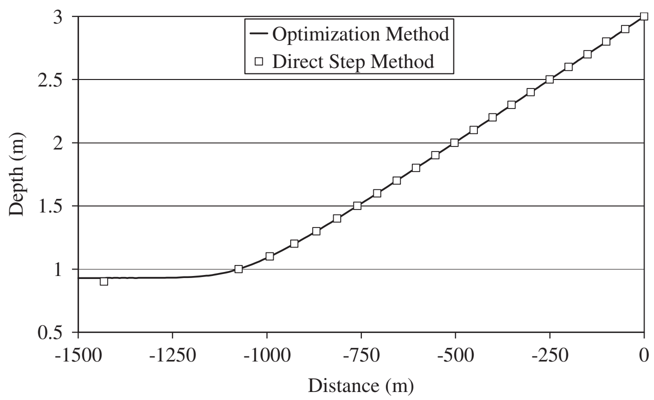

Fig. 4. Backwater calculation in a mild channel.

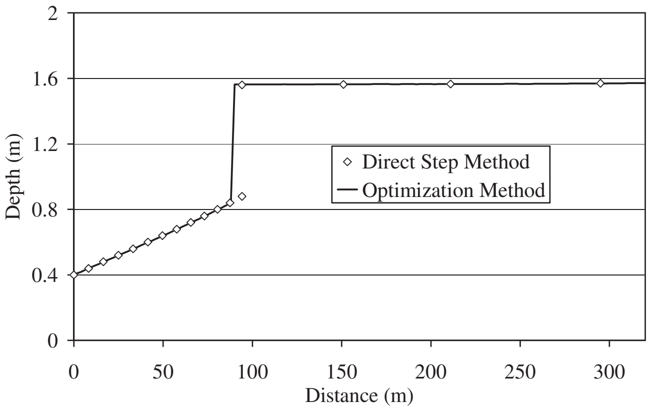

Fig. 5. Hydraulic jump calculation in a mild channel.

both with the Direct Step method and the proposed method. As can be seen the results obtained by proposed method precisely resemble those of the Direct Step method. It can be also seen that the proposed method succeeds in capturing water surface profile in both GVF and normal flow zone; exactly the similar results were obtained when just downstream boundary point was considered in the formulation.

To confirm whether the proposed method is able to capture a hydraulic jump, a second example is presented. In this example a sluice gate at the upstream end of a mild channel releases supercritical flow with the depth of 0.4 m into the channel of width 5 m, discharge 20 m^3/s, bed slope 0.002, normal depth 1.56 m, and Manning roughness coefficient 0.017. Fig. 5 shows the results obtained both from Direct Step method and the optimization method. To increase the resolution of the results in hydraulic jump zone, water surface profile is shown just in the first 300 m of the channel. As can be seen the results obtained from the optimization method are in close agreement with those obtained from the Direct Step method.

It should be finally noted that in Direct Step method the location of hydraulic jump is determined using a trial and error procedure whereas in the optimization method the location of the jump is captured in only a single pass. However, the lesson is not complete until the students can not only discover the merits of this optimized formulation, but also its limitations.

8. Conclusion

An innovative approach to teaching the principles of gradually varied flow is presented in this paper. The paper encourages students to explore a systematic approach to gradually varied flow by considering from first principles the essential “deposits” and “withdrawals” of mechanical energy. This approach naturally reveals to students that certain evolutions on the specific energy diagram are precluded under steady flow, and only certain transitions are permitted. The goal is to visualize the nature of the process before they are calculated, but to always confirm intuition through careful and painstaking confirmation.

Based on this approach, and applying second law of thermodynamics, an innovative approach is formulated for GVF calculation. To confirm the accuracy of the proposed method, two examples are presented. The results show that the method not only succeeds in calculating water surface profile but it is also able to capture the hydraulic jump in a single pass calculation.

Finally it should be emphasized that although the proposed method is more efficient than the traditional GVF calculation methods in capturing hydraulic jumps, it is not presented as an alternative method for this purpose, because it is much expensive in terms of computational time. Instead the proposed approach can give much insight about the formation and calculation of the GVF in the open channels.

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