EFFICIENT VALVE REPRESENTATION IN FIXED-GRID CHARACTERISTICS METHOD

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ABSTRACT: This work formulates a unified set of boundary conditions to efficiently represent the majority of valve and orifice devices found in water supply, transmission, and distribution systems. A particularly useful combination of mathematical components results when a lumped inertia model is linked with a throttling device. This combination of elements, termed a pipe replacement element/valve-in-line (PREVIL), has been constructed to permit a wide range of control-valve/short-pipe combinations to be conveniently modeled with the method of characteristics. The solution is quadratic in form and explicit, regardless of the number of pipes that are connected to the boundary condition. A variety of on-off and modulating valves can be accurately handled within this framework. An additional feature developed in this treatment of pressure-reducing and pressure-sustaining valves, flow controllers, and other similar devices permits a more realistic representation of these important control devices. The response time of the regulating valve on opening or closing can be readily approximated so that a greater range of control behavior can be simulated. Application of the combined boundary condition is illustrated in a pipe network.

INTRODUCTION

A glance at any design text or valve supplier's catalog reveals a plethora of valve types for use in water distribution and industrial applications. A valve can be custom manufactured to suit almost any need. Despite the seemingly endless variety of valve types and applications, most of these complex behaviors are built up by combining several valve functions into one unit. Although many classification schemes have been proposed to aid in selecting appropriate valves for a particular use, none of the taxonomies is ideal for computer modeling. The classification schematized in Fig. 1 is based on valve characteristics that are important in the current context (dashed lines indicate that items are not mutually exclusive). Some useful definitions are

- On-off control: event-actuated valves that have a predefined motion
- Modulating control: valves that can continually adjust their setting in response to sensed pressure differences

The vast majority of modulating valves sense and react to pressure differences by means of a pilot system. Air valves, vacuum-breaking valves, and combination air/vacuum valves are not considered in this paper.

Modulating valves that maintain a preset upstream level of head are often called pressure-sustaining or backpressure valves. Those valves that maintain a preset downstream pressure head are referred to as pressure-reducing valves. Valves that maintain a constant head loss across the orifice are called rate-of-flow controllers. Some valves may combine reducing or sustaining behavior with other flow-control functions; others may act with both modulating and on-off control, thus acting as dual- or multifunction valves. All types may have check action. In addition on-off control valves may be of the resetting type. Hydraulically activated valves are controlled by local parameters while electronically actuated valves may respond to local or remote parameters. As "intelligent" control systems based on one or more sensing devices in combination with electrical relays or programable controllers become more common, valve control and behavior will increase in complexity.

In this paper the traditional mathematical representation of orifices and valves is presented, along with an original development that highlights the relationship between the formal mathematical expressions used to describe an orifice element, the physical device itself, and the modeling representation. A new and useful modeling element that combines all the features of valves and short-pipe elements, connected at either end to any number of pipes, is introduced. This versatile and practical mathematical element includes connector inertia and frictional effects as well as the orifice characteristics. It can be used to represent virtually any valve and pipe combination and can be efficiently combined with many other important boundary conditions to provide an improved system representation with minimal memory use and execution time. The acronym PREVIL (pipe replacement element/valve-in-line) is coined here to refer to this particular combination of mathematical elements.

PIPE REPLACEMENT ELEMENT/VALVE-IN-LINE

To apply the method of characteristics (MOC) to a network, Karney and McInnis (1992) considered a frictionless junction of any number of pipes meeting at a node. The set N_1 indicates those pipes whose assumed direction is toward the node; N_2 indicates those pipes whose assumed direction is away from the junction; while H_i^{n+1} and Q_e^{n+1} represent the nodal hydraulic gradeline elevation and external discharge at the $n + 1$ time step. The external discharge is governed by an auxiliary or boundary relation, with positive external flows assumed to be from the junction. The resulting linear equation is

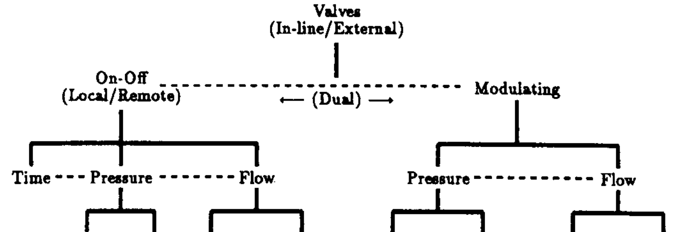

FIG. 1. Operational Valve Modeling Scheme

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Note. Discussion open until January 1, 1998. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on May 15, 1995. This paper is part of the ***Journal of Hydraulic Engineering***, Vol. 123, No. 8, August, 1997. ©ASCE, ISSN 0733-9429/97/0008-0709–0718/$4.00 + $.50 per page. Paper No. 10737.

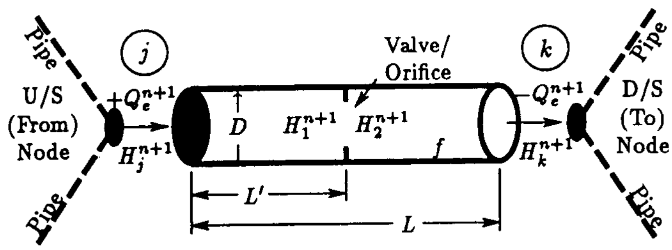

FIG. 2. Simplified PREVIL Element

$$H_i^{n+1} = C' - B'Q_e^{n+1} \quad (1)$$

in which

$$B' = \left(\sum_{i \in N_1} \frac{1}{B_{P_i}} + \sum_{i \in N_2} \frac{1}{B_{M_i}} \right)^{-1}; \quad C' = B' \left(\sum_{i \in N_1} \frac{C_{P_i}}{B_{P_i}} + \sum_{i \in N_2} \frac{C_{M_i}}{B_{M_i}} \right) \quad (2, 3)$$

and B_P, B_M, C_P, C_M are pipe characteristic constants (Karney and McInnis 1992). This basic MOC equation is now applied to the special case of two nodes connected by a PREVIL element.

By appropriately specifying the characteristics of the two basic elements—an inertial element and an orifice loss—the complex behavior of a wide variety of flow-control elements (check valves, isolation valves, modulating valves, or simply short pipes) can be accurately modeled. The PREVIL may occur by itself or as a subcomponent of other boundary conditions. Fig. 2 shows the device has an assumed upstream (from) and downstream (to) node. The device is characterized by four geometric variables—its length L; the distance between the "from" node and the valve L' (specified by $\theta = L'/L$); the diameter D; and wall friction f—as well as variables that specify the type of behavior of the orifice component.

Orifice-Loss Model

Since a valve produces a change in head between its upstream and downstream sides, it is natural to think of them as in-line, or two-node, devices. The equation for discharge through a valve is (Karney and McInnis 1992)

$$Q_v = s\tau E_s \sqrt{s \cdot H_v} \quad (4)$$

in which Q_v = discharge through the valve; H_v = head drop across it; τ = nondimensional effective gate opening; E_s = a valve-size parameter; and s determines the sign of the flow (+1 for positive flow and −1 for reverse flow). E_s is a function of the energy dissipation characteristics of the valve or orifice, primarily its size and geometry.

Lumped-Inertia Model

Any short element—that is, short in the sense that its wave travel time is so small relative to other connective elements (usually pipes) that communication between its ends is effectively instantaneous—may be accurately modelled by assuming flow through the device is incompressible. However, the inertia of the fluid mass and frictional effects within such a "lumped inertial" element can still be significant (Wylie and Streeter 1993). Newton's second law equates the net force acting on the fluid with the rate of change of fluid momentum within the short-pipe element as follows:

$$\rho g A \left(H_j^n - H_k^n - \frac{fL}{2gDA^2} Q_e^n |Q_e^n| \right) = \rho L \frac{dQ_e^n}{dt} \quad (5)$$

where L = length of the short-pipe element; D = internal diameter; A = internal cross-sectional area; f = friction coefficient; H_j^n = upstream piezometric head; H_k^n = downstream piezometric head; and ρ = fluid density.

Eqs. (4) and (5), when coupled with characteristics equations at the end nodes, can be solved either implicitly or explicitly. The implicit approach has an unbounded stability region while, as is discussed later, the explicit approach may lead to a more efficient code.

Implicit Approach

An accurate and stable trapezoidal approximation to (5) is

$$g\left(\frac{H_j^{n+1} + H_j^n}{2} - \frac{H_k^{n+1} + H_k^n}{2} - R_L Q_e^{n+1} |Q_e^n| \right) = \frac{L}{A} \frac{Q_e^{n+1} - Q_e^n}{\Delta_t} \quad (6)$$

where known upstream and downstream piezometric heads at the current time t are denoted by H_j^n and H_k^n, respectively; unknown heads at time step $t + \Delta t$ are H_j^{n+1} and H_k^{n+1}; the known flow passing through the element is Q_e^n and the unknown flow is Q_e^{n+1}; and $R_L = fL/2gDA^2$. Eq. (6) is rearranged to produce the following conventional form

$$H_j^{n+1} - H_k^{n+1} = G + G'Q_e^{n+1} \quad (7)$$

in which the constants $G = H_k^n - H_j^n - 2LQ_e^n/gA\Delta t$ and $G' = 2L/gA\Delta t + fL|Q_e^n|/gDA^2$. If the length of an inertial segment is zero, the constants G and G' are zero.

Thus the system depicted in Fig. 2 is completely described by five equations: two characteristic equations apply at the nodes, a quadratic head-balance expression governs the orifice, and an inertial equation applies to the upstream and downstream pipe segments

$$H_j^{n+1} = C_j' - B_j'Q_e^{n+1}; \quad H_k^{n+1} = C_k' - B_k'(-Q_e^{n+1}) \quad (8, 9)$$

$$H_j^{n+1} - H_k^{n+1} = G + G'Q_e^{n+1} + \frac{(Q_e^{n+1})^2}{s(\tau E_s)^2} \quad (10)$$

$$H_j^{n+1} - H_1^{n+1} = G_j + G_j'Q_e^{n+1}; \quad H_2^{n+1} - H_k^{n+1} = G_k + G_k'Q_e^{n+1} \quad (11, 12)$$

The constants in the characteristic equations have been defined while inertial element constants are given by

$$G_j = (H_1^n - H_j^n) - \frac{2\theta \cdot L}{gA\Delta t} Q_e^n; \quad G_j' = \frac{2\theta \cdot L}{gA\Delta t} + \frac{f\theta \cdot L}{gDA^2} |Q_e^n| \quad (13, 14)$$

$$G_k = (H_k^n - H_2^n) - \frac{2(1-\theta)L}{gA\Delta t} Q_e^n \quad (15)$$

$$G_k' = \frac{2(1-\theta)L}{gA\Delta t} + \frac{f(1-\theta)L}{gDA^2} |Q_e^n| \quad (16)$$

The composite constants G and G' in (10) are the sum of the respective constants for each individual inertial element. That is, $(G_j + G_k)$ and $(G_j' + G_k')$ are values that are independent of θ.

Explicit Approach

Eqs. (4) and (5) can be solved explicitly as well as implicitly. In order to achieve a degree of accuracy equal to that of the second-order trapezoidal method, we demonstrate the modified Euler two-step finite difference method. In symbols, the modified Euler method is

$$Q_e^{n+1} = Q_e^n + \frac{\Delta t}{2} \{ f(t, Q_e^n) + f[t + \Delta t, Q_e^n + \Delta t f(t, Q_e^n)] \} \quad (17)$$

where Q_e^n and Q_e^{n+1} are the discharges through the PREVIL at time t and $t + \Delta t$, respectively; and $f(t, Q_e^n) = dQ_e^n/dt$. To clarify, we add the head-loss relation due to the orifice to (5) and rearrange to produce

$$f(t, Q_e^n) = \frac{dQ_e^n}{dt} = \frac{H_j^n - H_k^n - R_L Q_e^n|Q_e^n| - \dfrac{Q_e^n|Q_e^n|}{(\tau E_s)^2}}{L/gA} \tag{18}$$

In addition, we write

$$f[t + \Delta t, (Q_e^{n+1})'] = \frac{(H_j^{n+1})' - (H_k^{n+1})' - R_L(Q_e^{n+1})'|(Q_e^{n+1})'| - \dfrac{(Q_e^{n+1})'|(Q_e^{n+1})'|}{(\tau E_s)^2}}{L/gA} \tag{19}$$

where primed variables represent the predicted quantities at the end of the time step. Specifically, $(Q_e^{n+1})' = Q_e^n + \Delta t f(t, Q_e^n)$ while $(H_j^{n+1})'$ and (H_k^{n+1}) are defined by (1).

It is perhaps fortuitous that the trapezoidal method is commonly used in the solution of the in-line valve boundary condition because it is unconditionally stable and second-order accurate. The same cannot always be said for the modified Euler approach. In particular under nearly steady conditions the transient dQ_e^n/dt term in (18) decays rapidly to zero while the steady terms do not change significantly. These events often create difficulties in the finite difference solution because the numerical scheme must cope with the transient term even after its contribution becomes small (Golub and Ortega 1992). This is not intuitively obvious because one typically expects the integration to improve as the solution becomes easier to approximate.

Such problems involving more than one time scale create so-called stiff systems of differential equations. Although placing significant restrictions on the time step (i.e., reducing it drastically) may ensure computational stability in stiff systems, it is inefficient in most extended period simulations. Therefore methods should be used that exhibit an unbounded region of stability (Shampine 1994) but that are not susceptible to the same time step restrictions. For example, the first-order accurate backward Euler method is globally stable and most often recommended as an alternative to multistep finite difference methods under these conditions (Press et al. 1992; Gear 1971; Miranker 1981). Higher-order backward formulations such as implicit Runge-Kutta methods are typically less satisfactory in terms of maintaining stability (Shampine 1994). In symbols, the backward Euler method is

$$Q_e^{n+1} = Q_e^n + \Delta t f(t + \Delta t, Q_e^{n+1}) \tag{20}$$

but upon substitution of (19) can be rewritten as

$$Q_e^{n+1} = -b_o + \sqrt{b_o^2 - c_o} \tag{21}$$

where

$$b_o = \frac{L/gA + (B_k' + B_j')\Delta t}{2[R_L + \dfrac{1}{(\tau E_s)^2}]\Delta t} \text{ and } c_o = \frac{\Delta t(C_k' - C_j') - LQ_e^n/gA}{\left[R_L + \dfrac{1}{(\tau E_s)^2}\right]\Delta t} \tag{22}$$

Having derived a set of equations that fully describes the operation of a PREVIL we must still define a transition criteria. During steady-state conditions the inertia of the fluid is negligible, therefore we can solve for the steady-state discharge $(Q_e^{n+1})''$ by evaluating (19) when $(dQ_e^{n+1})''/dt = 0$. Thus, it is proposed that the transition between the backward Euler and modified Euler methods be governed by the following criteria:

$$\text{If } |(Q_e^{n+1})' - (Q_e^{n+1})''| \begin{cases} \le 0.001 \text{ m}^3\text{/s} & \text{must use backward Euler [i.e., (21)]} \\ > 0.001 \text{ m}^3\text{/s} & \text{must use modified Euler [i.e., (17)]} \end{cases} \tag{23}$$

That is, (17) describes the conditions during the transient event while (21) describes the conditions outside the transient event (i.e., steady, or nearly steady, state).

Using the notation of Fig. 2, and (8)–(10), solutions for several important valve and orifice boundary conditions are presented in subsequent sections. The constants in this formulation (namely, G, G', G_j, G_j', G_k, G_k', C_j', C_k', B_j', B_k') are known from initial conditions. However, there are six potential unknowns—H_j^{n+1}, H_k^{n+1}, H_1^{n+1}, H_2^{n+1}, Q_e^{n+1}, or τ—but only five equations. The type of valve and the initial conditions determine one of these values, and the remaining five equations are combined in various ways to solve for the other unknown variables. In all cases the final expression is no higher than quadratic and can be solved explicitly. The discussion begins with on-off valves.

ON-OFF VALVES

As their name suggests, on-off valves have a predefined valve motion that they execute in response to a specified "trigger" variable, such as a particular time or a certain pressure value at some location. Whatever the activating parameter, once the triggering value (set point) has been realized the valve follows the motion prescribed by its τ-curve. Valves belonging to this category include fixed orifices, check valves, direct-acting pressure relief valves, surge-anticipating relief valves, solenoid-controlled valves, valves with external actuators (manually, hydraulically, or electrically driven), and many others. In addition, some types of modulating valve behavior may be reasonably approximated by a predefined τ motion if its gross behavior can be anticipated and the system response is not particularly sensitive to details of the valve's behavior. Considerable flexibility exists to permit complex behaviors—such as valves moving along the τ-curve, opening and closing in response to one or more setpoints—but no matter how "logically" complex the behavior may be, it is fully defined by the τ-function.

The problem for on-off valves may thus be summarized as follows: the valve setting (τ) is given from the τ-function for the valve while the unknowns include the heads (H_j^{n+1}, H_k^{n+1}, H_1^{n+1}, H_2^{n+1}) and the external flow (Q_e^{n+1}). This allows the solution to be split into two logical parts: (8)–(10) may be solved for the nodal heads and the discharge; the internal heads H_1^{n+1} and H_2^{n+1} are then obtained from (11) and (12), respectively, to update the constants for the next time step.

Substituting (8) and (9) for H_j^{n+1} and H_k^{n+1} in (10) produces a quadratic expression with a single unknown—Q_e^{n+1}. Rearranging results in the following simple relation

$$Q_e^{n+1} = -b + s\sqrt{b^2 - c} \tag{24}$$

in which $b = s(\tau E_s)^2(G' + B_j' + B_k')/2$; and $c = s(\tau E_s)^2(-C_j' + C_k' + G)$. The nodal heads can be back-calculated using (8) and (9). The sign of the flow is determined from the sign of C; i.e.,

$$s = \text{sign}\{C_j' - C_k' - G\} \tag{25}$$

In addition, the constants G and G' are calculated from the inertial equations. It is noted that if E_s is zero, instantaneous check-valve action is implied.

The constants in (10) require that the internal section heads be calculated in order to compute G and G' at the next time step. It is recommended that the constants from the characteristics equations be used rather than directly substituting the calculated values for H_j^{n+1} or H_k^{n+1}; solving in terms of Q_e^{n+1}

reduces numerical round-off errors. These errors can be significant for PREVIL elements with negligible friction and orifice losses.

Whenever an on-off valve is activated at some specified time t_{cr} in the simulation, the activation criterion is clear: if $t \geq t_{cr}$ the valve motion begins precisely at t_{cr}. Intermediate values are easily obtained by interpolation. However, if the valve is activated by exceeding a setpoint, some ambiguity exists about the precise time at which the valve motion should start. A reasonable resolution is to assign the starting time as $t_s = t - \Delta t/2$.

MODULATING VALVES

In contrast to on-off valves, the τ value of modulating valves is determined by sensed pressure differences and by a hydraulic pilot system. The pilot system permits the modulating valve to regulate the valve inlet or outlet pressure, or the head difference across the loss element. Many methods presented in the literature for dealing with modulating valves are often unsuitable. Either they are two simple to account for important behavioral factors such as response time, or they are complex, resulting in large code size, large data requirements, and inefficient solutions. More specifically, the response time of modulating valves is determined by the design and physical characteristics of both the pilot system and the valve itself. The behavior of this system is dynamic and can be described by a system of ordinary differential equations (Wylie and Streeter 1993; Chaudhry 1987), similar to a spring-mass-dashpot system with an external forcing. The rate of change of pressure in the valve cover chamber also depends on characteristics of the pilot system. Solving such a boundary condition is both computationally demanding and data intensive. The formulations presented here overcome problems associated with both unwarranted simplicity and extreme complexity, and are physically and behaviorly quite general.

By recognizing that the primary factor controlling the response time is the rate at which pressure is relieved or added to the cover chamber by the pilot system, a useful approximation to the gross opening and closing speed of the valve can be obtained. In fact, the pilot system is generally adjusted so as to permit the valve to completely open or close in a specified period of time. This can be conveniently summarized by two parameters, $\Delta\tau_o$ and $\Delta\tau_c$, which are the average rates of change in τ for opening and closing of the valve, respectively. If t_o is the time required for the valve to open fully from a closed position, then typically $\Delta\tau_o = 1/t_o$. The average rate of closure $\Delta\tau_c$ is similarly calculated. For the case of a modulating valve whose gross opening or closure schedule is a linear τ function, the approximation is exact and equals the slope of the τ-curve.

Often there is little data on the shape of the τ-curve for modulating valves. Even when it is available, it is strictly correct only when the valve closes or opens under conditions of constant inlet pressure. Nevertheless, some account can be taken of pronounced nonlinearity in the valve schedule by calculating $\Delta\tau_o$ or $\Delta\tau_c$ as the slope of the τ function at the current value of τ.

The algorithms presented here assume that the valve response time is limited by an appropriately specified value of $\Delta\tau_o$ or $\Delta\tau_c$. This simple device greatly enhances the analyst's qualitative and quantitative ability to simulate modulating valves. Moreover, the damping effect provided by explicitly defining the response time of the valve controls physical and numerical instabilities, as well as improving convergence to steady state.

PRESSURE-REDUCING VALVES

Pressure-reducing valves are common devices in distribution systems comprising several pressure zones. Whenever flow passes from an area of high pressure or elevation to an adjacent zone at lower elevation, the pressure must be reduced to maintain it within acceptable limits. This application normally requires valves that are slow to respond, as pressure changes usually take place gradually. A quite different application is required when, for example, a constant low pressure is required for some industrial process. In this case a quick-acting reducing valve is needed so that pressure fluctuations in the supply system are not transmitted to the downstream process.

The solution for pressure-reducing valves uses the known downstream head (that is, the head to be maintained by the valve) to determine the unknown external discharge; that is, (9) may be substituted into (12) to give

$$Q_e^{n+1} = \frac{H_2^{n+1} - C_k' - G_k}{G_k' - B_k'} \tag{26}$$

The sign of the flow s is simply the sign of the external discharge Q_e^{n+1}.

Once the flow through the PREVIL is known, all the required heads (H_j^{n+1}, H_k^{n+1}, and H_1^{n+1}) can be computed and an expression for the unknown valve setting derived. The upstream internal head (adjacent to the valve) can be obtained by rearranging (11) to

$$H_1^{n+1} = C_j' - G_j - (G_j' + B_j')Q_e^{n+1} \tag{27}$$

Knowing the head loss across the valve, the value of τ required to maintain the downstream head at the setpoint value is given by

$$\tau = \begin{cases} 0 & \text{if } H_1^{n+1} - H_2^{n+1} = 0 \\ \sqrt{R_\tau} = \sqrt{\dfrac{s}{E_s^2}\left(\dfrac{(Q_e^{n+1})^2}{H_1^{n+1} - H_2^{n+1}}\right)} & \text{if } H_1^{n+1} - H_2^{n+1} \neq 0 \end{cases} \tag{28}$$

The radical $\sqrt{R_\tau}$ is not defined if its denominator is identically zero. However, the physical meaning is unambiguous if the head difference across the valve is zero, and therefore there is no flow. In this case any value of τ will produce the same result. In reality the weight of the valve disk, or sometimes a weak spring pressing on the disk, will cause the valve to slowly close, hence the first line in (28).

The interpretation of (28) is clear provided the radical is positive. However, if it is negative an imaginary solution results that cannot be physically interpreted, other than to say it implies that energy must be added by the valve in order to arrive at its setpoint pressure. Since the valve can only dissipate energy, its behavior must be somehow constrained. The following logical scheme (illustrated in Fig. 3) is proposed

$$\tau = \begin{cases} \tau_{max} & \text{if } R_\tau < 0 \text{ and } Q_{ext} > 0 \\ \tau_{min} & \text{if } R_\tau < 0 \text{ and } Q_{ext} < 0 \\ \sqrt{R_\tau} & \text{otherwise} \end{cases} \tag{29}$$

Essentially, the logic reflects the fact that water flowing through the valve will induce a torque causing the valve to respond as indicated. This scheme is consistent with the known physical behavior of modulating valves described in this paper. The maximum permissible valve opening is τ_{max} and usually equals 1. The minimum permissible setting is τ_{min}, which usually equals 0.

Using any other relational criteria to establish valve behavior may unpredictably result in aberrant physical solutions. Under certain conditions the anomalous behavior is recognizable as such. In other cases the results seem strange but plausible. Such abnormal responses are not common but must be properly handled. Failure to do so impairs both the utility of the model and the confidence that may be placed in its predictive ability.

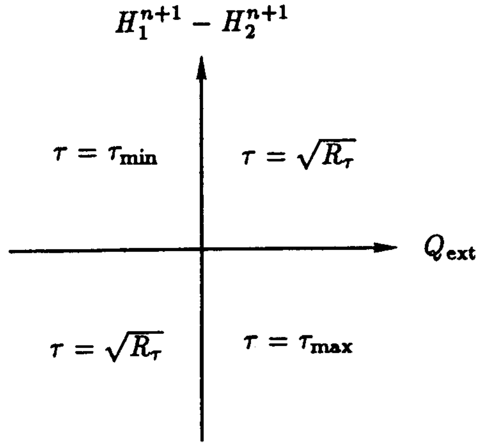

FIG. 3. Real and Complex Solutions for τ

Generally speaking the defined, positive radical case prevails and (28) is solved yielding a positive value. This does not guarantee, however, that this required valve opening is feasible with respect to either the response time of the valve or the maximum and minimum permissible values of τ. To obtain the actual valve opening, the value of τ given by (28) must be compared with the set of feasible values. Letting $\tau_{t-\Delta t}$ be the effective valve opening at the previous time step, the feasibility requirements can be tested by determining if the valve is opening or closing and setting the maximum permissible change in valve setting, $\Delta\tau\cdot\Delta t$; accordingly

$$\text{for opening valves (i.e., } \tau - \tau_{t-\Delta t} > 0\text{) set } \Delta\tau\cdot\Delta t = \Delta\tau_o\cdot\Delta t \tag{30a}$$

$$\text{for closing valves (i.e., } \tau - \tau_{t-\Delta t} < 0\text{) set } \Delta\tau\cdot\Delta t = -\Delta\tau_c\cdot\Delta t \tag{30b}$$

Next test the computed τ value against the feasible values.

$$\tau = \begin{cases} \min[\tau_{\max}, \max(\tau_{t-\Delta t} + \Delta\tau\cdot\Delta t, \tau_{\min})] & \text{if } |\tau - \tau_{t-\Delta t}| > |\Delta\tau\cdot\Delta t| \\ \sqrt{R_\tau} & \text{otherwise} \end{cases} \tag{31}$$

If the latter condition applies the solution procedure is complete, the upstream and downstream nodal heads are computed by (8) and (9).

If the final value of τ has been constrained as indicated by the former condition, the problem is transformed to that of an on-off valve with unknown heads and external discharge, but with a known τ value. In this case a second iteration is made using the solution procedure presented previously for on-off valves.

Pressure-Sustaining Valves

Pressure-sustaining, backpressure, or pressure-relief valves are all terms describing a modulating valve whose pilot system causes it to adjust its setting to maintain a present inlet pressure. The function of the valve is determined by its topological relationship to other system components. If the valve is placed in a series arrangement, it attempts to maintain a constant backpressure in the system by throttling the discharge. When the same valve is installed in a small branch downstream of a pump, it acts as an unloading or recirculating bypass. This same valve functions as a pressure-relief device if it opens rapidly and closes slowly.

The purpose of the forgoing descriptions is merely to show that all of these superficially different valves are really the same hydraulic component with particular aspects of its behavior modified. Hydraulically all of these functions are adequately simulated by varying two parameters—the inlet pressure to be maintained and the valve's response time. The solution procedure is completely analogous to that just presented for pressure-reducing valves and only those portions that are different are described below.

The solution for pressure-sustaining valves begins by using the known upstream head—that is, the inlet pressure head or hydraulic grade line elevation to be maintained by the valve—to determine the unknown external discharge. Eq. (8) may be substituted into (11), which is rearranged to solve for the external flow

$$Q_e^{n+1} = \frac{C_j' - G_j - H_1^{n+1}}{G_j' + B_j'} \tag{32}$$

The sign of the flow s is simply the sign of the external discharge Q_e^{n+1}.

Once the external flow is known, the downstream (outlet) head adjacent to the valve can be obtained from (12) as shown in the following:

$$H_2^{n+1} = C_k' + G_k + (G_k' + B_k')Q_e^{n+1} \tag{33}$$

The other internal head is the setpoint value H_{set}.

Knowing the head change across the valve, (28) is solved for the value of τ required to maintain the desired inlet pressure. The remainder of the solution is identical to that given for the pressure-reducing valve.

Rate-of-Flow Controllers

Rate-of-flow controllers differ from the two previous types of modulating valves because their pilot system adjusts the valve position to maintain a preset pressure difference across an element such as a fixed orifice. Because there is a single value of discharge associated with each unique head loss across the fixed orifice, this value limits the maximum system discharge to $Q_{\max}$.

Rate-of-flow controllers are used to prevent overpumping of wells, to maintain a constant flow for industrial processes, or to prevent a low-pressure zone from "robbing" a higher-pressure zone when the pressure differential increases between the zones. The external flow is now known: $Q_e^{n+1} = Q_{\max}$ = valve setpoint. The unknowns are the four heads (H_j^{n+1}, H_k^{n+1}, H_1^{n+1}, and H_2^{n+1}) and the valve setting τ. The external flow is assumed to equal the setpoint value $Q_{\max}$. The valve inlet and outlet heads can be computed as before, i.e.

$$H_1^{n+1} = C_j' - G_j - (G_j' + B_j')Q_e^{n+1} \tag{34}$$

$$H_2^{n+1} = C_k' + G_k + (G_k' + B_k')Q_e^{n+1} \tag{35}$$

Knowing the head change across the valve, (28) is solved for the value of τ required to maintain the desired inlet pressure. The remainder of the solution is identical to that given for the pressure-reducing and pressure-sustaining valves.

EXAMPLE 1: IMPORTANCE OF INERTIA EFFECTS

Admittedly, the inertia effects associated with many in-line valves are rarely significant. But in some special cases—such as a transition between a large-diameter pipe and a much smaller-diameter short connector where the inertia effects cannot be readily handled by conventional methods—the PREVIL approach finds great utility. The following example compares the predicted behavior of a system in which the inertia effects are included in the numerical analysis to the predicted behavior of an otherwise identical system in which the inertia effects are neglected.

System Description

Consider a simple horizontal pipeline in which water passes from a constant head supply reservoir to a constant head ter-