ASSESSING THE DEGREE OF UNSTEADINESS IN FLOW MODELING:
FROM PHYSICS TO NUMERICAL SOLUTION

Karney, B. W.¹, Jung, B.², and Alkozai, A.³

¹ Professor, Civil Eng. Dept., Univ. of Toronto, Canada, karney@ecf.utoronto.ca
² Engineer, MWH Soft, 300 North Lake Avenue, Suite 1200 Pasadena, CA 91101. BongSeog.Jung@mwhsoft.com
³ Graduate Student, Civil Eng. Dept., Univ. of Toronto, Canada, alkozai@hotmail.com

Abstract

From a long-term perspective all flows in a water system are unsteady in the sense that no single value persists throughout; all flows continuously change or adjust, either suddenly or gradually, over the system's life. From first filling to final decommission, the flow rate routinely changes from near zero to other, possibly quite high, values. Yet from either a performance or modeling perspective, a detailed or complete consideration not only of all flows, but of all possible transitions in flows, is both impossible and excessive. Thus, it is necessary to restrict attention to specific and targeted design regimes that capture in a reasonable way the key system performance issues in terms of cost effectiveness, hydraulic capacity, structural strength, as well as the system's overall reliability, robustness, vulnerability, and resilience.

Yet even this is a tall order and immediately raises three key and intensely practical questions for the analyst: (i) How important is the unsteadiness in flow conditions to understand the system response? (ii) What mathematical models should be used to capture this unsteadiness? And (iii) what spatial and/or temporal discretization is needed to accurately simulate the selected model?

A whole host of intriguing considerations arise from these questions that interconnect with issues including the purpose of the analysis, the physical and economic consequences of both conservatism and risk, and both the experiences and biases of the analyst. This paper briefly explores the related questions of modeling intent, physical and numerical approximations, and how these issues map to the physical behavior of the system. Several specific modeling options are overviewed including nearly steady (or extended period) approaches, rigid water column models (i.e., including inertia effects), and water hammer models (i.e., also including small compressibility effects).

This paper provides a brief summary of these considerations and attempts to provide some tentative and preliminary guidance into how these questions of unsteadiness need to be framed or posed. There are clearly issues of physics at stake, in terms of whether certain phenomena are dominant or trivial, but even this evaluation is contingent on the context of the modeling exercise and what specific questions are being answered.

A couple of indices are helpful, at least in a limited way, for characterizing the system response, and should perhaps be routinely computed to help analysts gauge or judge the unsteadiness in a system. These include the time scale of boundary and flow adjustments relative to the water hammer time scale, and the size of two unsteady head terms (the acceleration head and the Joukowski head) relative to predicted pressure and flow changes. However, in complex systems the rules must always be supplemented with more detailed numerical explorations and exercises in grid refinement.

Keywords: Transient Flow, Unsteadiness, Water Hammer, Rigid Water Column Model, Extended Period Simulation, Numerical Error, Time Step Selection
1. INTRODUCTION

Since accurate mathematical models for simulating water distribution systems (WDSs) are of great value to a water authority for purposes of planning and operation, the selection of a suitable model or models is a critical issue. Partly motivated by water quality considerations, the variation of readily available models range from almost steady or extended period (EP) models to rapid transient or water hammer analysis. Traditionally, EP models have been used for sizing pipes and pumps and planning operations, while surge and water hammer analysis were central to predicting pipe strength and stress and system protection, particularly by considering maximum and minimum pressures for specific design events like power failure and valve closure. More recent concerns, for both types of models, have included issues of water quality, whether transport of chlorine residuals in EP models or cross-connections at devices and contaminant intrusion through leaks for water hammer models.

Analysts must always seek to resolve the tension between two opposing objectives in selecting and using modeling tools: the simplicity of use versus the accuracy of prediction. Since a model is inevitably “other than” reality itself, in one sense the model is invariably an additional complication: a model adds yet another distraction or level of preoccupation that didn’t exist before the model was created. As a direct result, it behooves the modeler to keep this distraction under control, by making the data and conceptual needs of the model as small as possible – good models are generally as simple, clear and easy to use and understand as possible. Yet if the model is too simple, the distortions it introduces in representing the prototype are too great, and the predictions of system behavior will be either misleading or inaccurate. These two tendencies are by and large directly opposed in the sense that simple models are usually less accurate, and vice versa. Thus, the key modeling question is always this: how simply the model can be constructed while still representing enough of reality to make suitable decisions. The best models achieve the best resolution of this accuracy-modeling tension.

Arguments on the trade-off between accuracy and simplicity can be linked to WDS transient modeling through identifying four general categories of unsteady mathematical models in common use:

1. The so-called Joukowski approach that superimposes, generally as a check or confirmation, a simple formula between sudden changes in velocity and pressure changes onto an otherwise steady model. In the past, such an approach was perhaps justified by its simplicity, but is generally considered to be too approximate, and often too conservative, for modern applications, particularly since computers can handle most of the dirty work of intensive computation.

2. Extended period (EP) models allow gradual changes in demand or water level, typically by allowing accumulation or depletion of water levels in tanks, and by allowing demands and the settings of pumps and valves to become functions of time or system state. However, the majority of EP models change flows and pressures at the end of the hydraulic time steps, effectively instantaneously or discretely, with no physical consideration of what such sudden changes would imply for a real system. In this sense, the water itself that is imagined to move through an extended period is simplified, since it exhibits none of the inertial or mass effects that real water possesses.

3. If inertia effects are included, but compressibility effects are excluded, the so-called rigid water column family of models is created. Such models represent the flow in
each pipe as an ordinary differential equation, which is certainly a mathematical
complication compared to the algebraic relations for a pipe in an EP model. Yet,
typically, very rapid changes in rigid models are not realistically represented and
rigid models are often overly conservative of pressure changes. In fact, as the time
of operation decreases, the peak pressure heads can increase without bound; what
actually limits these pressure changes are small (but crucial) compressibility effects.
Water hammer models include the inertia effects of rigid models but the (generally)
head-change-limiting role of compressibility effects. The addition of both
compressibility and inertial effects comes at a price, though, each pipe is now
governed by a pair of partial differential equations. This representation of flow and
pressure changes also introduces a finite propagation velocity, or water hammer
celerity, into the system representation. Typically depending on the dimensionality
of the model, and the representation of friction losses, a whole host of water hammer
representations are possible. The most common of these is the simplest to formulate
and the quickest to solve, but perhaps one of the least accurate ways of representing
losses in the system, by using an essentially steady representation of friction. By
contrast, unsteady friction water hammer models can be more accurate but also the
most difficult and expensive to solve numerically.

Thus, various transient models have evolved in order to achieve economy of design,
construction, operation and maintenance. However, the specific usage of each model is strongly
dependent on the system characteristics and its application; if designers are too simplified or too
complicated in their analysis, frustrations, inefficiencies, uncertainties, and delays multiply. In
this paper, the key differences and advantage for the four transient models are presented,
compared, and contrasted, with respect to three criteria: their physical attributes, the practical
problems they can be used to solve, and the numerical considerations of stability and accuracy.

1.1 Unsteady flow Modeling
Before defining water distribution system modeling, it is important to understand the essence of
the modeling process. Nirmalakhandan (2002) defines modeling as “the process of application
of fundamental knowledge or experience to simulate or describe the performance of a real
system to achieve certain goals.” Models can be cost-effective and efficient tools whenever it is
more feasible to work with a substitute than with the real, often complex, original systems.
Nirmalakhandan also argues that modeling has long been an integral component in organizing,
synthesizing, and rationalizing observations, of and measurements from, real systems and in
understanding their causes and effects.

Loucks (1992) indicated the nature of modeling in the following terms:

Models of real-world systems are always going to be simplified representations of
those systems. What features of the actual system are incorporated into a model,
and what features are not, will depend in part on what the modeler thinks is
important with respect to the issues being discussed or the questions being asked.
… Developing models is an art, requiring a knowledge of the system being
modeled, the client’s objectives, goals and information needs, and some analytical
and programming skills. Models are always based on numerous assumptions, and some of these may be at issue. (p. 215)

Starfield and Cundall (1988) indicated that data-limited problems like rock mechanics cannot use models in a conventional way and argued a need to adopt a distinctive and appropriate methodology, essentially arguing that simplification is a crucial part of rock mechanics modeling. Starfield and Cundall also nicely remind us all that “a model is an aid to thought, rather than a substitute for thinking”. Starfield (1997) experienced similar data-limited problem in wildlife management modeling and suggested that a model is a problem-solving tool, not naively thought of a direct representation of truth. In the same way, Oreskes et al. (1994) found that verification and validation of numerical models of a natural system is fundamentally impossible because the systems are never closed and because model result are always non-unique. Instead, Oreskes et al. (1994) argues that models can only be evaluated in relative terms, and their predictive value is always open to question. Thus, the primary value of models is heuristic: models are simplified representations, useful for guiding further study but not susceptible to final proof. Judgment is inevitably a required component of their effective use.

Stated starkly, Boulos et al. (2004) described the characteristics of a model as follows:

Fundamentally, every model is in some sense a fake – it is a replacement, a stand-in, a surrogate, or a deputy for something else. A model is a thing, an object, a process, or a physical or mental construction that is intended to represent, replace, mimic or demonstrate something else. (Boulos et al., 2004, p. 1-12)

Boulos et al. (2004) cite G.I. Taylor famous line: “a really good model should introduce the minimum amount of complexity while capturing the essence of the relevant physics”. The mapping between the intrinsic accuracy of the physical representation, and the simplicity of the model, can be displayed for unsteady models approximately as shown in Figure 1. As complexity and physics are introduced into the model, we gain one objective (accuracy) but only the expense of the other objective (simplicity). The relative position and orientation of the models displayed in Figure 1 is clearly conceptual only.

Fig. 1 Simplicity and accuracy of five unsteady flow models
Another possible way of representing the relationship of the various models is shown by the Venn diagram of Figure 2. As the quasi-steady equations are special cases of the rigid column approach, all analysis that is performed with a quasi steady approach could likewise be performed, albeit at great computational expense, with a rigid water model. Similarly, all systems that can be modeled with either rigid or quasi steady approaches can technically be modeled with the water hammer or elastic model. Yet, the converse is in general not true, since quasi steady models often grossly distort the representation of true transient models.

Yet, stated in this way, Figure 2 really overstates its case. If we were to show the number of practical questions that can effectively be resolved by the models, or the number of models in use in engineering firms or utilities, the positions on the Venn diagram would reverse. For planning the basic components of system operation, design, and expansion, the less physically correct (but faster) hydraulic models continue to be of great importance. The question that the reflective practitioner would ask is why this is so. The remainder of this document is devoted to providing more clarity on this issue. We start by reviewing the governing equations themselves.

Fig. 2. Shows qualitatively the boundaries of the three methods in terms of physical applicability; however, in terms of practical problem solving, the domains are often reversed.
2. FUNDAMENTAL EQUATIONS AND MODELS

2.1 Conventional Water Hammer Model

To analyze the transient state conditions, mathematical models describing these flows are first formulated. Two equations, a momentum equation and a relation of mass conservation, are generally used to model transient flow in closed conduits (Wylie and Streeter, 1993; Karney and McInnis, 1992). The assumptions here are that the flow is predominately one dimensional, that friction losses are nearly steady, and that the wave speed is much higher than the fluid velocity. If \( x \) is distance along the centerline of the conduit and \( t \) is time, these equations can be written as

Continuity equation: \[ L_1 = H_x + \frac{a^2}{g} V_x = 0 \]  
(1)

Momentum equation: \[ L_2 = V_t + gH_x + \frac{f_p V |V|}{2D} = 0 \]  
(2)

in which \( H = H(x,t) \) = piezometric head; \( V = V(x,t) \) = fluid velocity; \( D \) = inside pipe diameter; \( f = \) Darcy-Weisbach friction factor; \( a = \) celerity of the shock wave; and \( g = \) acceleration due to gravity. To be compatible, \( x \) and \( V \) must be positive in the same direction. Eqs. (1) and (2) are generally applicable if conduit properties (diameter, wave speed, temperature, etc.) are constant, the convective and slope terms are small, and the friction force can be approximated by the Darcy-Weisbach formula for steady flow. Other friction relationships can be substituted into Eqs. (1) and (2) with more or less complication depending on their nature. The two equations shown here are called the water hammer model with steady friction assumption and their solution can be produced numerically using the eigenvalue approach (Jung and Karney, 2004), the method of characteristics (Wylie and Streeter, 1993) or the wave characteristics method (Boulos et al., 2004).

However, the approximation of unsteady pipe friction by the Darcy-Weisbach formula or similar relation for almost steady flow causes insufficient damping for certain unsteady pipe flow events. The difference between actual and modeled results increase as time progresses. Considering so-called inverse transient methods rely on the transient model to predict transient behavior for long periods of time, it is important to be able to accurately model transient behavior (Vitkovsky, 2001). Unsteady friction is described by Vitkovsky (2001)’s model which is based on the model by Brunone et al. (1991) and improved by using energy and momentum concepts from derivations that include the velocity distribution to infer extra terms needed to describe unsteady friction in the familiar one dimensional water hammer model. This modified momentum equation of (2) is expressed as

\[ L_2 = H_x + \frac{1}{g} V_t + \frac{f_p V |V|}{2gD} + \frac{k}{g} \left( V_t + a \cdot SGN(V) \cdot |V_x| \right) = 0 \]  
(3)

where \( k = \) unsteady friction coefficient and ‘SGN’ stands for the sign of the velocity. Eqs. (1) and (3) are called the unsteady friction water hammer, which is more accurate but more difficult to solve numerically and thus more expensive to use. Another approach is to represent the flow using a 2-D model including a radial distribution (e.g., Naser, 2006).
2.2 Rigid Water Column Analysis

In a number of unsteady flow examples, changes in pressures and flows are relatively slow with respect to the time of water hammer wave propagation \((L/a)\) seconds. In many of these cases the change in fluid storage in the pipeline is unimportant and the fluid may be considered incompressible and the pipe walls completely rigid. This leads to the incompressible or rigid liquid column theory, in which the mass density remains constant and the pipe undeformable regardless of pressure. This effectively implies an infinite wave propagation velocity and a common fluid particle velocity along the pipe at any instant (Wylie and Streeter, 1993).

Based on the rigid assumption, either the wave celerity \(a\) tends to infinity in Eq. (1) or the velocity change term \(V_x = 0\), indicating \(V\) is a function of time only. Therefore, the change of \(H\) with \(x\) is also a function of time only; that is, the piezometric head, or hydraulic grade line, is a straight line at any instant between \(H_U\) and \(H_D\) and \(H_x = (H_D - H_U)/L\), where \(L\) = length of pipe, the subscripts \(U\) and \(D\) indicate upstream and downstream, respectively. Equation (2) can be written in terms of the pipeline velocity and the upstream and downstream end pressure, at any instant, as

\[
H_D - H_U + \frac{fLV|V|}{2gD} + L\frac{dV}{dt} = 0
\]

Eq. (4) is called the rigid water column model. The last term \(L/g \cdot dV/dt\) presents the inertia or acceleration head and becomes an index of the role of unsteady effects. For a given simulation model the acceleration head is easily computed and can be compared in magnitude to other head terms to obtain an index of the flow unsteadiness.

2.3 Quasi-Steady Analysis

The quasi-steady flow condition is often used synonymously with extended time period analysis indicating a series of steady flow solutions. Each steady flow solution is determined at a different time for a different system state. In order to simplify a dynamic system with the quasi-steady analysis, the changes in pipe velocity take place extremely slowly \((dV/dt \approx 0)\) so the inertia head in Eq. (3.4) can be negligible such as

\[
H_D - H_U + \frac{fLV|V|}{2gD} = 0
\]

Eq. (5) is called the quasi-steady model and, together with suitable boundary conditions, is the basis of extended period simulation models. This is clearly one of the simplest ways to represent dynamics in a system, though at the expense of neglecting most of the dynamic forces.

Joukowski Approach

The Joukowski approach is the easiest and simplest analysis that superimposes sudden changes onto an otherwise steady model of Eq. (3.5), although the conventional understanding of the origin of this relationship is somewhat complex, as Tijsseling and Anderson (2004) point out. This relation equates the change in head in a pipe to the associated change in fluid velocity:

\[
\Delta H = \frac{a}{g} \Delta V
\]

The simplest relationship is potentially both powerful and accurate but only applicable under a set of highly restricted and often unrealistic circumstances. The two most important restrictions
are that there should be only a small head loss resulting from friction and no wave reflections from any hydraulic devices or boundary conditions in the system. This approach ignores the problem of interaction of the different pipe properties in a WDS. Actual pipes in WDSs are necessarily connected, and water hammer waves are significantly affected by these connections. At pipe junctions and dead-ends, wave reflections and refractions occur, which often magnify or attenuate the surge waves. Moreover, it can not simulate a variety of loadings in the quest for the worst-case scenarios in a distribution network system (Jung, 2005)

3. SELECTING AN UNSTEADY MODEL

The proper selection of unsteady model is strongly depending on the characteristics of system and its dynamic operation. Generally, quasi-steady model can be used for a mild transient flow. By contrast, water hammer models can be applied to a rapid transient flow. Rigid water column model can be applied to the intermediate condition between mild and rapid flows, but are seldom used for water distribution system analysis, since they are only slightly easier to solve than water hammer models but less universally applicable. Moreover, it is not easy to identify the criterion of flow condition, (i.e., how rapid is rapid; how mild is mild).

Fig. 3 gives a rough indication of inertia effect in a single pipeline and provides a general sense of how to select a reasonable unsteady model. Assuming a velocity change \( (dv) \) of 1 m/s, the inertia head. The acceleration head expression \( (L/g \cdot dV/dt) \) can be calculated for different pipe lengths \( L \) and time increments \( dt \).

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<th>Negligible</th>
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<th>Rigid Water Column</th>
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Fig. 3. Proper application of unsteady models based on inertia head

As the time interval increases and \( L \) decreases, the inertia head become negligible so a quasi-steady model represents a reasonably accurate and simple model. Yet, as \( dt \) decreases or \( L \) increases, the inertia head becomes dominate, and at least a rigid water column model is required. However, as the acceleration head continues to increase, the rigid relationship progressively provides a significant over-estimate of the true value of head change, because the limiting effects of compressibility of fluid and deformability of the pipe become important. Thus, very rapid changes in long systems should be estimated using a water hammer model.
4. CASE STUDIES

The purpose of the case studies is to apply and compare quasi-steady, rigid water column and water hammer models. In particular, the examples provide background that partly shows how and when the various models can be applied properly. For simplicity, the water hammer model is generally applied with the steady friction assumption only. The computation time step for three models is fixed as about a tenth of a second (0.1 s).

The case study shown in Fig. 4 represents a single pipeline system with a tank. The system comprises two identical pipes where the length ($L_1$ and $L_2$), diameter ($D_1$ and $D_2$), Darcy-Weisbach friction factor ($F_1$ and $F_2$), initial flow rate ($Q_1$ and $Q_2$) and wave speed of the pipes are 1500 m, 0.5 m, 0.02, 0.2 m$^3$/s, and 1000 m/s, respectively. The pipe is connected to an upstream reservoir with 25 m ($H_r$), a tank in the middle and a valve in the downstream. The valve is initially fully open. The cross-sectional area of the tank ($A_t$) and the valve closing time ($T_v$) are selected in the specific case.

![Fig. 4 Case study system schematic, with two pipes and intermediate surge tank.](image_url)

If the downstream valve is closed essentially instantaneous one obtains a model that is obviously and dominantly unsteady. In this case, a conventional water hammer model captures the oscillation in the system (see Fig. 5) though, it turns out, the decay processes is not well captured. The rigid water column model in this case is problematic, since as the duration of closure approaches zero the acceleration head increases without bound. This is clearly not physical but leaves the analyst with a challenge: what is real? In Figure 5 the value plotted for the rigid column model and for the quasi steady model follow the same convention of showing the head variation in the tank, and clearly this is not the same as what the pipe would experience.

Note also, that the judgment called upon the analyst does not end with the selection of model. The head in the downstream pipe in the water hammer model becomes unrealistically negative during the reverse part of the wave cycle, and this negative pressure needs to be either carefully interpreted, or the model improved yet again, now to include column separation and possible air release. Fortunately, however, this further complication is often not required, since the main role of the water hammer model might be to simply identify that there is a problem, and that instantaneous closures are a very bad thing! Certainly, this is not a bad lesson from this analysis, as sudden changes can unleash powerful and destructive forces into a pipe system.
Fig. 5: Instantaneous valve closure in a system with a small 1 m$^2$ surge tank

Clearly both this discussion and Fig. 5 show that, no matter what values are plotted, the pressures predicted by the three models are dramatically different. Yet even this statement, at least for rapid changes, might be an overgeneralization: the response of the whole system is determined by the properties of the whole system. Depending particularly of the properties of the surge tank (or other storage/release devices) that separates the upstream pipe from the downstream one, the first pipe may be largely “shielded” from the influence of the suddenness of the downstream changes.

However, as mentioned, the number of cycles captured by the water hammer model is not likely a good match to reality. Under such dramatic shock waves events, there are a considerable number of frictional effects in the real system that is not captured by a quasi-steady model. These unsteady friction effects, if they are to be more acutely captured, mean that even a conventional water hammer analysis is not sufficiently refined, and more terms need to be added to the governing equations. And so it goes: as more and more detail needs to be captured, further refinements are progressively added to the model, but the modeling process itself become more demanding and consuming, creating a possible distraction from consideration of the real system itself.

In Fig. 6 the situation is similar to that of case 1 except the surge tank is much bigger and thus provides a greater degree of isolation from the transient. As the plot shows, the response for all three models now closely approximates each other, and the difference, for most modelling purposes, may not be important. This clearly demonstrates the effectiveness of a surge tank in attenuating the pressure peaks and reducing the maximum pressure, but also that it is the overall system characteristics, and the purpose of the modeler, that largely shape the selection of model.
type. This plot also shows that even though the models have different assumptions and physical basis, they converge to more or less the same behavior as the speed of change is reduced, either through a less severe initiating action, or through the designed/inherent properties of the system.

![Water level in a large surge tank after instant valve closure](image)

**Fig. 6.** Water level in a larger 10 m$^2$ tank after instantaneous valve closure

As the area of the surge tank is progressively reduced, though, particularly for the case of instantaneous closure, more and more inertia and compressibility effects become evident even in the response of the tank. **Fig. 7** shows the water level with a tank that is about the same diameter as the pipe. Now the rigid water curve has moved considerably above the quasi-steady curve as acceleration head effects have become significant. Moreover, the waterhammer curve is not smooth, but has peaks and valleys due to the wave propagation in the system. Clearly, the water hammer wave has more detail than is captured by the rigid water column model, which captures much more than the quasi-steady model. Only the water column model captures the constructive and destructive interplay of the pressure waves as they reflect back and forth between the reservoir and the valve. The rigid water column is a simplistic approximation of the situation and thus represents the water level as smooth undulations of high and low water levels. The quasi steady model, in this case, sees almost none of these effects.
As the initiating event is made less sudden, the appearance of inertial and compressibility effects in any system is delayed. As Fig. 8 shows, the behaviour of the models, as the closure time is extended, becomes much more similar even for quite a small surge tank. There are small undulations in the waterhammer and rigid water column model, but these are likely small enough to be neglected for most design and operational purposes. In fact, the specific train of small water hammer waves on this graph depends very strongly on the exact way the valve is initially opened and finally closed and this detail is very difficult to represent accurately in real systems, as many forces and frictional effects are often evident in nearly open and closed valves. Thus, the apparent detail on the water hammer plots may be more apparent than real.

However, what is also noteworthy in Fig. 8 is that the quasi steady and water hammer models are, in some senses at least, more similar than the rigid water model. In other words, compressibility effects, even in this relatively simple system with its relatively long closure time, have served to attenuate the response of the rigid water model. This is a strong tendency, but is not a universally valid generalization, since attenuation at one time might require an over-compensation later. Unfortunately, a priori rules applicable to all systems are difficult to find.
Perhaps the one generalization that is possible to make is this: as system disturbances become less rapid, the importance of both water hammer and inertial effects decrease. The sensitivity plot in Fig. 9 shows that, for rapid closures of the valve, the waterhammer model predicts very high pressures while the rigid water column and the quasi-steady models don’t capture this detail. However as the time of closure is increased, the maximum pressure predicted by all three models gradually converge. Moreover, as more and more storage with a free surface is distributed in the system, there are more locations for water hammer waves to attenuate and fade. Interesting, leaking pipes provide some degree of this relief function, though this is certainly not a valid reason to avoid repairs! Water hammer protection is best designed, not merely available by happenstance.

Real systems have many details and many challenges, in an infinite and sliding scale of complexity and challenge. Not all the details of the system are relevant to all modelling questions, and the analyst must know their purpose, not just the unstated desire for accuracy for its own sake. However, if is important to capture not only the magnitude but the rate of their decay, much more complex models might be justified. Thus, Fig. 10 directly compares a 1-D model to a 2-D one for the same problem; the 2-D model is likely closer to reality, but the question is whether this additional accuracy is worth the computational price. Indeed, this is always the question.
Fig. 9: Sensitivity analysis of pressure change at the tank versus valve closure duration

Fig. 10 – Pressure variation at the middle of the pipe calculated by the standard five-region turbulence model for the 2-D approach (Naser, 2006)
5. CONCLUSIONS

Real pipe systems are subject to many forms of unsteadiness, and the relative importance of these changes is a function of the properties of the system, the purpose of the analysis, and the initiating source of the particular event in question. At one extreme, transient events create quiet undulations in the system that may be only a curiosity to the analyst; at the other end of the spectrum, transient events can create sudden changes in flow conditions and consequently produce high system stresses. If these forces exceed the strength of the conduit, failure can occur, or important water quality events can be initiated.

A variety of unsteady models have been created to meet the various purposes of the modeling activity, ranging from simple “accumulation models” typical of extended period analysis, to sophisticated 2-D or even 3-D models with complex turbulence representation. The most important features of the system that determine its response are the initial conditions, the presence and distribution of storage devices, the length and wavespeed of the lines, and the source of the transient. The most important issues from a modelling perspective are how these physical attributes interact with the purpose of the analysis to determine what physical effects will occur, and whether these are important to the analyst.

9. REFERENCES