Suspended sediment concentration and deformation of riverbed in a frazil jammed reach

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Abstract: The presence of ice in rivers affects hydrodynamic conditions through changes in both the river's boundary conditions and its thermal regime. Therefore, the characteristics of sediment transport and the deformation of the river channel in ice-covered rivers are quite different from those experiencing conventional open channel flow. The variables of ice behavior, ice jamming extent, sediment transport, and deformation of the riverbed during ice periods are interrelated on the basis of both physical arguments and field experiments of river ice jams in the Hequ Reach of the Yellow River. The characteristics of sediment concentration in water, frazil ice, and ice cover are described. Analyses have been made on the mechanism of the evolution of frazil jam and the associated adjustments in the riverbed. It has been found that the evolution of the ice jam and the deformation of the riverbed reinforce each other. The interrelationship between the particular features of evolution of ice jam and deformation of riverbed is summarized here in the form of regression relationships relating the hydraulic parameters of water under ice jams to the deformation-extent of the riverbed and the jamming-extent.

Key words: deformation of riverbed, evolution of frazil jam, frazil jam, suspended load, sediment concentration.

Résumé : La présence de glace dans des rivières affecte les conditions hydrodynamiques par le biais des conditions frontières et du régime thermique de la rivière. De ce fait, les caractéristiques du transport de sédiment et de la déformation du canal de la rivière pour des cas de rivières avec couvert de glace sont bien différentes de celles sous l'effet d'un écoulement à surface libre conventionnel. Les variables du comportement de la glace, de l'étendue de l'embâcle de glace, du transport de sédiment et de la déformation du lit de la rivière sont reliées sur la base d'arguments physiques et d'expériences sur le terrain sur des embâcles de glace en rivière dans le tronçon Hequ de la rivière Jaune. Les caractéristiques de la concentration en sédiment dans l'eau, du frasil, et du couvert de glace sont décrites. Des analyses ont été faites sur le mécanisme de l'évolution de l'embâcle de frasil et de l'ajustement correspondant du lit de la rivière. Il a été trouvé que l'évolution de l'embâcle de glace et de la déformation du lit de la rivière encouragent l'une et l'autre. Les inter-relations entre les caractéristiques particulières de l'évolution de l'embâcle de glace et de la déformation du lit de la rivière sont résumées ici sous la forme de relations de régression reliant les paramètres hydrauliques de l'eau sous l'embâcle de glace à l'étendue de la déformation du lit de la rivière et à l'étendue de l'embâcle.

Mots clés : déformation du lit de la rivière, évolution de l'embâcle de frasil, embâcle de frasil, charge suspendue, concentration en sédiment.

[Traduit par la Rédaction]

1. Introduction

The Yellow River in China is notorious for the enormous amount of sediment it carries. The total average annual sediment discharge to the China Sea is estimated at 1.94×10^9 t, of which 59% comes from the Yellow River (Yang et al. 1996). A concentration of 911 kg/m^3 was measured on September 7, 1977, at the Sanmenxia gauge station near the entrance of the lower Yellow River (Yang et al. 1996). For the about 700 km Lower Reach alone, the average annual sediment discharge is 1.6×10^9 t with an average annual sediment concentration of 30 kg/m^3. Because of its small bed slope, the decreasing flow velocity leads to 4×10^8 t/a of sediment being deposited in the Lower Reach. Thus, the riverbed, which is already 5–7 m (maximum 10 m) higher than the adjacent land behind its dikes, is rising at an average rate of 0.1 m each year (River Sediment Engineering 1981).

River ice is critical for many reasons, including its impact on both global climatic variations (Beltaos 1998; Prowse 1993) and river hydraulics. A major consequence of ice cover formation on northern rivers is the jamming that occurs during spring breakup of the cover and, to a lesser degree, during the freeze-up period. Because of their large aggregate thickness and hydraulic resistance relative to those of a simple sheet ice, ice jams tend to cause unusual deformation of the riverbed, high water stages, and other effects. This has repercussions in many operational and design problems such as the overturning moment on river structures due

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to moving ice, forces on ice booms, spring flooding and associated stage–frequency relationships, and riverbed scour due to surges from released jams, to mention but a few (Beltaos 1983). In northern countries, many rivers become ice-covered during the winter months. Inasmuch as the hydrological and thermodynamic regimes as well as the boundary conditions of rivers are greatly influenced by the ice regime, the characteristics of sediment transport in river and the deformation of riverbed are significantly different from those of open channel flow. Based on a long-term field measurements of frazil jams and riverbed deformation in the Hequ Reach of the Yellow River, the characteristics of sediment transport and the riverbed deformation are discussed in the present paper. The empirical relationships between the hydraulic parameters and the riverbed deformation as well as the frazil jam evolution are established from the data through regression analysis.

2. Geographic location and morphology of Hequ Reach

The Hequ Reach (Fig. 1) of the Yellow River is located at a latitude between 39° and 40°N and at a longitude between 110° and 112°E. It extends from Longkou Gorge (near section 1) to Tianqiao Power Dam (section 22) over a distance of 70 km. Upstream from Longkou Gorge, there exists a 100–200 km reach of open water with numerous rapids caused by locally high flow velocity. As a result of cold air temperature in winter, an enormous amount of frazil ice is generated in this long open water reach, which leads to the formation of a large frazil jam in the Hequ Reach of the Yellow River.

The riverbed of Hequ Reach is alluvial, broad, and shallow. The bed material is mainly fine sand, with presence of some stretches of pebbles in the main channel. Other reaches contain fine sand and are subject to rather intensive deformation during the entire ice period.

Each year between 1982 and 1992 the river experienced ice jam effects for over 100 days. Frazil ice jams of tremendous size have often been formed upstream of Shiyaobu (near section 10), leading to high water levels that in 1982 caused a serious ice flood disaster. Since then, measurements of the ice jam profile along this river reach have been made each winter. By the winter of 1986, using the geodetic theory, beside Hequ gauge station (section 9), 21 additional measurement sections, including two water gauges at every section, have been installed in this reach to support a data and labour intensive field monitoring program. This approach was used to measure the water levels, the water depth (using sonar), the thicknesses of ice cover and frazil jams, current velocities under ice jams (using a velocity modified flow measurement meter and current meter), sediment concentration during jamming periods, morphology of riverbed, water and air temperatures, etc. The backwater from the reservoir reaches section 17, a distance of 49.73 km from section 1. As the ice cover in the backwater region is usually thin and unstable, only occasional measurements were made in this region. The measurements of ice cover thickness and jam profiles were surveyed once every 5 days for cross sections 1–18. Water levels at all of these cross sections, and water discharge and water temperature at the Hequ gauge station (section 9), were measured daily. Sediment concentrations in water for cross section 9 (Hequ gauge station) and the sediment concentration in the ice cover and the frazil ice (cross sections 2–17) are measured at a frequency of once every 5 days (1986–1988). The related measurements at the Hequ Reach of the Yellow River were conducted according to protocols established by the Hydrological Survey Standard of China.

The time-integrating suspended sediment sampler was used to measure the sediment concentration in the water. To measure the sediment concentration in the ice and frazil ice, each ice sample along the measured section was identified and weighed. After the sample was melted and dried, the sediment concentration in the ice could be calculated.

3. Characteristics of sediment transport

Water levels and current velocities are two important variables influencing the flow state in a river. During the ice period, especially when the river is frozen up, inasmuch as the wetted perimeter increases and the hydraulic radius of cross section decreases, the flow resistance increases. In addition, the accumulation of frazil ice under the ice sheet occupies a portion of the flow cross section. This leads to a relative increase in both water level and channel storage in a frazil ice jammed reach compared to an equivalent open water discharge. In alluvial channels, in which the mobile bed forms affect the flow properties and vice versa, the effect of ice covers (ice jams) may be even more complicated. Changes in the bed shear stress will alter the bed load. The quite radical change in the diffusivity distribution will certainly have a significant effect on the suspended load as well (Lau and Krishnappan 1985).

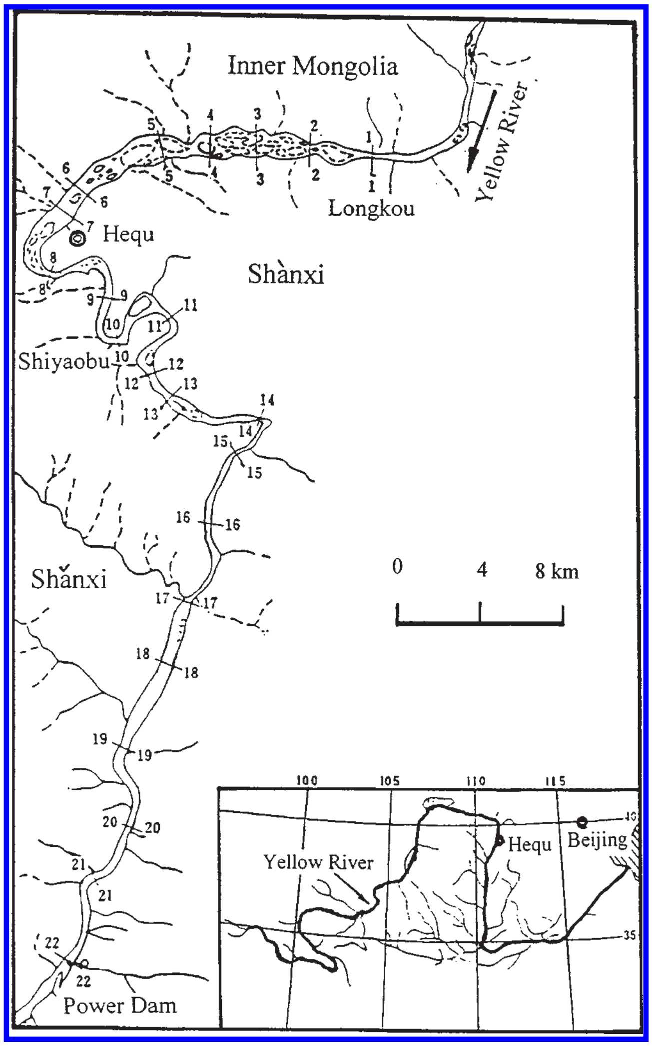
An important but poorly understood impact of ice breakup pertains to increased sediment loads and deformation of the riverbed. During breakup, higher sediment concentrations can occur for a variety of reasons: increased exposure of river banks to erosion, caused elevated water levels; higher flow velocities during ice jam releases; and accelerated erosion resulting from the interaction between ice and the bed banks (Beltaos 1998; Prowse 1993).

Table 1 summarizes the characteristics of stream flow and sediment regime from 1954 to 1983 at the Yimen (Fugu) gauge station downstream of the studied reach. It has been found that during the four-month flood season (July to October), the discharged water makes up 58.1% of the annual discharge. However, the corresponding sediment transport accounts for 85.3% of the annual total sediment transport. Thus, only 14.7% of the sediment is transported during the non-flood season. Compared to the sediment transport in open channel flow, transport in a frazil jammed reach is quite different as discussed in the following sections.

3.1. Sediment concentration in ice

In open channel flow, sediment is transported by the current, meaning that the sediment carrier is water. However, sediment transport in a jammed river is conducted through both water and ice (or frazil). In addition to the suspended sediment in current, there exists sediment contained in the frazil ice or ice sheet. This kind of suspended sediment frequently exists in rivers of northern countries. Its movement

Fig. 1. Hequ Reach of the Yellow River.



and physical features depend on ice motions directly and on the current velocity indirectly. The diameter of the entrained sediment carried by ice varies dramatically, ranging from minute clay particles to large diameter stones. At sections 12–14 of the Hequ Reach, 0.2–0.5 kg pebbles have been observed in frazil jams and are almost certainly associated with so-called anchor ice. If the turbulence of a water body is sufficiently intense to bring supercooled water to the bottom, ice may be attached or nucleated on pebbles or other underwater objects to produce this anchor ice. Ashton (1986, pp. 282–284) discussed this phenomenon. It has been found that frazil attachment is the most important process in anchor ice formation. Anchor ice is formed initially with a strong bond. Later, after the bond has been weakened by heat transfer from the water, the bond between anchor ice and the river is reduced to the point that buoyancy alone can float the anchor ice off the bottom. Because anchor ice can be thick, the buoyancy force can be substantial, allowing heavy objects, such as substantial boulders or gravel, to be lifted by the released anchor ice.

Based on the data observed at the Hequ Reach, it has been found that the sediment concentration of the ice sheet is usually between 0.8 and 2.0 kg/m^3, but reached 9.8 kg/m^3 at section 18 and 6.0 kg/m^3 at section 16. Generally, the sediment concentration of frazil ice is between 1.0 and 10.0 kg/m^3. It has been found that the sediment concentration of frazil ice during the formation of frazil ice jam is large with the highest recorded sediment concentration of 25.2 kg/m^3, much larger than the largest sediment concentration of current under frazil jams (7.0 kg/m^3). Interestingly, the upper layer frazil has a larger sediment concentration than the lower layer.

By the end of stable jamming periods or during breakup, there are many small sand dunes on the surface of the frazil jams. These formations are ice blocks which have sand coat. On the one hand, with the increase in temperature, the ice block that contains more sediment begins to melt and the sediment separates out. On the other hand, the soil erosion of the Loess Plateau through wind also influences the formation of this sand coat. The thickness of this sand coat of the sand dunes at Hequ Reach is usually between 3 and 20 mm. The thickest sand coat is over 50 mm. With the increase in temperature, growth of these sand dunes accelerates the melting of ice cover and promotes the breakup of the jammed river.

3.2. Sediment concentration of current

Generally, for a certain open channel reach, the larger the discharge, the higher the sediment concentration. In fact, the sediment concentration depends mainly on the current velocity as well as the characteristics of bed material.

Prowse (1993) discussed the dependence of sediment concentration on discharge during breakup on the basis of the data at Liard River near its mouth in the N.W.T. (Fig. 2). Obviously, the sediment concentration of the current increases with the discharge during breakup. Beltaos (1998) also described the interrelationship between sediment concentration and discharge during breakup based on the data at the Saint John River (Fig. 3). He found that both sediment concentration and discharge first rise and then decline during breakup. When plotted against each other, these graphs exhibit a distinctly “looped” character. At the same value of discharge, the value of sediment concentration are much higher while the flow rises than when it declines. This looped character is similar to those of the Volga River in Russia under open channel condition during flood period (the result published in Russia in 1952, translated in Chinese in the *Handbook of Sediment*, 1992). In fact, during the dynamic breakup periods of river ice jams, the water level and the discharge increase is often dramatic. This hydrologic process is somewhat similar to the flooding phenomenon under open channel conditions. The net result is that the sediment concentration becomes higher during breakup periods.

On the basis of the measurements of sediment concentrations associated with the current during jamming periods at the Hequ Reach of the Yellow River, the following trends may be discerned:

(*i*) The larger the discharge, the larger is the value of sediment concentration. Figure 4 shows the relationship between sediment concentration and discharge at the Hequ gauging station during the ice period 1986–1987 along with data from the Saint John River in Canada published by Beltaos (1998). It has been found that the sediment concentration during breakup and jam-formation periods is usually much higher than those observed during the stable jamming periods.

Table 1. Water and sediment data at Yimen (Fugu) gauge station on the Yellow River from 1954 to 1983.

					% (yearly)		% (flood season)	
Items	V_W ($\times 10^6$ m^3)	V_S ($\times 10^6$ m^3)	Q (m^3/s)	C_S (kg/m^3)	V_W	V_S	V_W	V_S
Flood season								
July – Oct.	15 230	266	1433	17.50	58.1	85.3		
July – Aug.	7 170	178	1338	24.80			47.1	66.9
Sept. – Oct.	8 060	88	1529	10.90			52.9	33.1
Non-flood season	11 000	46	526	4.18	41.9	14.7		
Yearly	26 230	312	832	11.90				

Note: V_W, volume of water; V_S, volume of sediment; Q, discharge; and C_S, sediment concentration.

Fig. 2. Discharge and suspended sediment concentration of the Liard River near the mouth, N.W.T., Canada, 1987 (Prowse 1993).

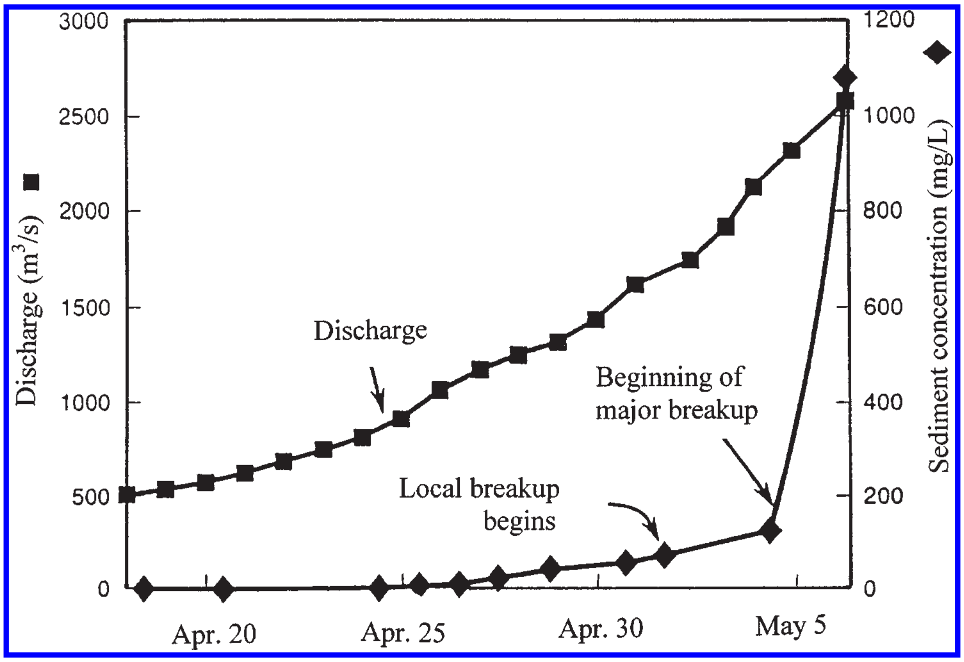


Fig. 3. Typical variation of sediment concentration with discharge, Saint John River at Clair, Canada, April 1994 (Beltaos 1998).

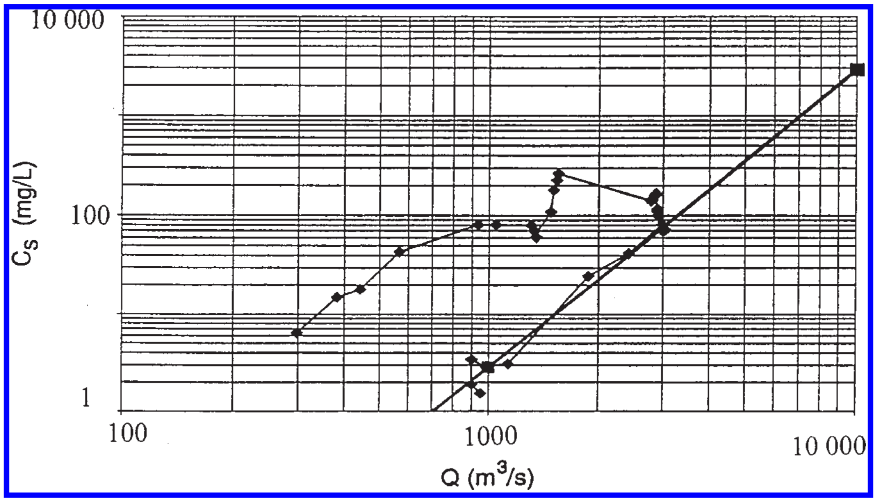
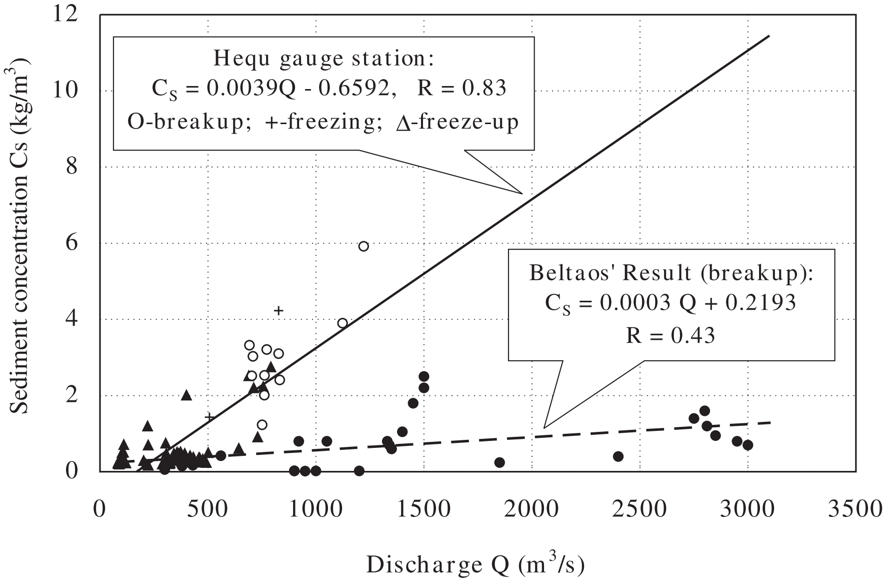


Fig. 4. Sediment concentration with discharge, Hequ Reach (1986–1987).



(*ii*) As shown in Table 1, the average sediment concentration during the four-month flood season from 1954 to 1983 at the Yimen gauge station is 17.5 kg/m^3, but only 4.18 kg/m^3 during non-flood season. However, during stable jamming period (usually lasting about 3 months from the end of December to mid-March), the sediment concentration of the water is much less, and its value increases dramatically during river breakup. For example, based on the measurements of sediment concentration at the Hequ gauging station during the ice period from December to mid-March 1986–1987, the sediment concentration of water is only about 0.5 kg/m^3 during stable jamming period. However, it exceeds 6.0 kg/m^3 during river breakup at the same station.

Reasons for this phenomenon can be explained in several ways. In winter, the precipitation in this area falls as snow. The presence of snow cover diminishes the soil erosion of the Loess Plateau, which normally causes the large sediment concentration of the Yellow River. In addition, the wind-blown sediment is intercepted by the ice cover and cannot fall into water. With the decrease in temperature, many small branch rivers which carry sediment to the Yellow River under open channel conditions become completely frozen. Thus, the sediment transported to the Yellow River through branch rivers during the jamming period decreases dramatically. For example, the Huang-Pu-Chuan river (near section 17) is well known to contribute high sediment loads. Based on observations at Huang-Pu-Chuan gauge station from 1953 to 1971, there are 109 days during which the sediment concentration of the current is over 500 kg/m^3; on 33 days the concentration exceeds 800 kg/m^3 and on 6 days it exceeds 1000 kg/m^3. However, during jamming period, there is little water in this branch river causing a corresponding reduction of sediment to the Yellow River during ice periods.

(*iii*) From November to March, especially in January and February, the suspended load in water is coarser than that in October and April. Figure 5 shows the comparison of grain size distribution of sediment in the ice period (1979–1980) at the Hequ gauging station. This is possibly due to different origins of sediment: the load delivered during the ice period primarily originates from the bed material, in the virtual absence of surface runoff, with very small contributions from the banks by winds. A part of the finer sediment is also contained in frazil and ice cover. Thus, the sediment remaining in water becomes somewhat coarser. In March, surface runoff and soil erosion through wind will bring some sediment into the river during the thaw. In addition, melting of the ice cover returns the finer sediment it contains to the stream; the grain size of sediment tends to become finer.

4. Deformation of riverbed and evolution of frazil jam

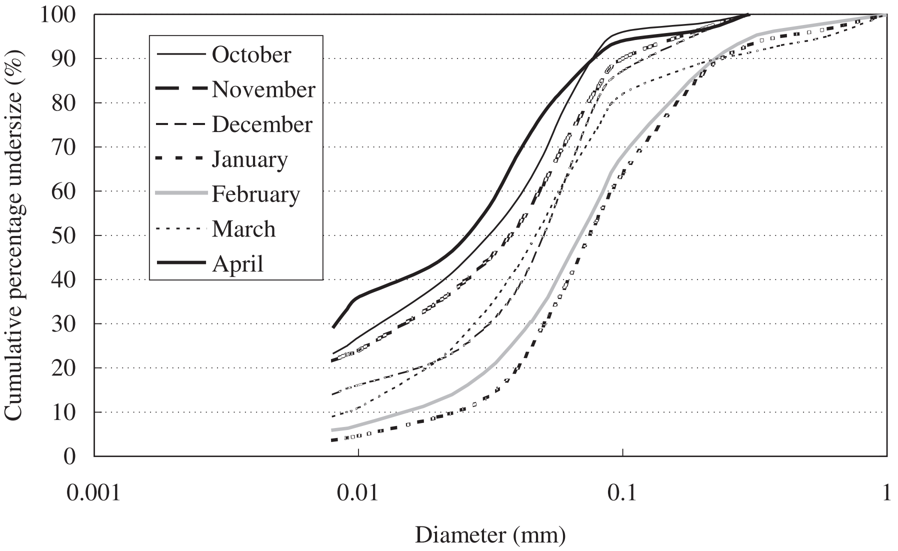
The movement of sediment in water arises from two different mechanisms; this difference creates both bed-load and suspended load transport. However, because of the variation of current velocities, bed-load can be transformed to suspended load and vice versa. During the ice period, the presence of ice cover (jams) alters the flow conditions and the boundary conditions of the channel. Therefore, the sediment movement is much more complicated than that under open channel conditions.

4.1. Mechanism

It is seen from the observed data at the Hequ Reach that the following regularities prevail with respect to variation in water levels, frazil jam evolution, and riverbed deformation.

(*i*) During the formation and upstream propagation of frazil ice jam, frazil ice enters the ice-covered section at the leading edge and accumulates on the underside of the ice cover. The accumulation of frazil jam leads to the scour of riverbed. The frazil jam propagates upstream following the progression of the leading edge (Sui 1988; Sun and Sui 1990). As a result, the scour of

Fig. 5. Comparison of grain size distribution of sediment in ice period (1979–1980) at Hequ gauge station.



the riverbed and the rise in water level also progress upstream.

(*ii*) After the formation of the initial frazil jam along the river reach, frazil ice plunges under the foremost upstream edge of the ice jams (jam head) (Beltaos 1983) and accumulates there. The head region of the ice jam becomes thicker, causing a further scour of the riverbed and a rising water level. By contrast, erosion of the frazil jam at toe region (jam-toe) (Beltaos 1983) causes deposition of sediment at the riverbed and a decrease in the associated water level at this location, as shown in Figs. 6 and 7.

(*iii*) As shown in Figs. 6 and 7, during the midwinter, with the unceasingly replenishment of frazil ice from the upstream, the accumulation under the head of the jam intensifies. The increased accumulation leads to reduction of cross section for passing the stream flow, and consequent scour of riverbed and rise in upstream water levels. With the increase in local flow velocity caused by the reduction in the flow cross section, frazil ice from the jam head is transported toward toe. This leads to the scour of riverbed and increase in water levels at the jam toe. This sequence is repeated throughout the ice period, causing the frazil jam to grow.

(*iv*) The air temperature drops to its lowest value in mid-January. During this period, ice discharge reaches a peak. Thus, the jam thickness near the jam head, water level, and ice transport capacity reach their maximum values. After mid- January, the air temperature begins to rise and the frazil production in the upstream open water reach decreases. Both the frazil ice supply and the frazil accumulation under jam head region decrease. The existing frazil accumulation continues its migration to the toe of the jam. This leads to deposition on the riverbed and a decrease in the water level in the head portion; however, scour of the riverbed and an increase in water level occur in the toe portion of the study reach.

These field observations strongly imply that the frazil formations under the ice and the deformation of riverbed reinforce each other. To further test this hypothesis, laboratory experiments were undertaken to explore the dependence of jam accumulation on the form of riverbed. For simplicity, the simulated riverbed was designated as a fixed-bed of sinusoidal shape. Figure 6*c* is based on these experiments and clearly shows that the resulting frazil ice formation under the ice does indeed closely mimic the simulated shape of the riverbed.

4.2. Characteristics

(*i*) Scour and deposition on the riverbed depend on the growth and diminution of the ice jam, the characteristics of riverbed material, the current velocity, and other factors. As shown in Fig. 7, as the jam grows, the riverbed tends to be scoured, and the riverbed is deposited again as jamming diminishes. During the entire ice period, cycles of scour and deposition occur repetitively. Figure 7 shows the deformation of the riverbed and evolution of ice jam, in which Q is discharge, A_s is the increase or decrease in cross-sectional area relative to a specified reference section (as measured on January 1, 1987) due to scour (negative) or deposition (positive), and I is the percentage of the cross-sectional area to pass the stream flow that is occupied by the ice jam.

(*ii*) Evolution of the ice jam as well as scour and deposition at the riverbed render the effective cross section for the passage of flow to become one of the least energy consumption. Stream flow in the ice-jammed section is actually closed conduit flow, and the water level is consequently much higher than that at the same discharge in free flow. The increment of water level is generally 2–4 m, but may exceed 6 m or more, thus causing the extent of deformation of the riverbed in the ice-jammed reach to exceed significantly a free flow with the same discharge as shown in Fig. 7 (Sui 1988; Sun et al. 1990). The slope of the water surface influenced by the evolution of the frazil jam and the deformation of the riverbed will both tend to be those associated with least energy consumption for the given effective cross section to pass the required discharge.

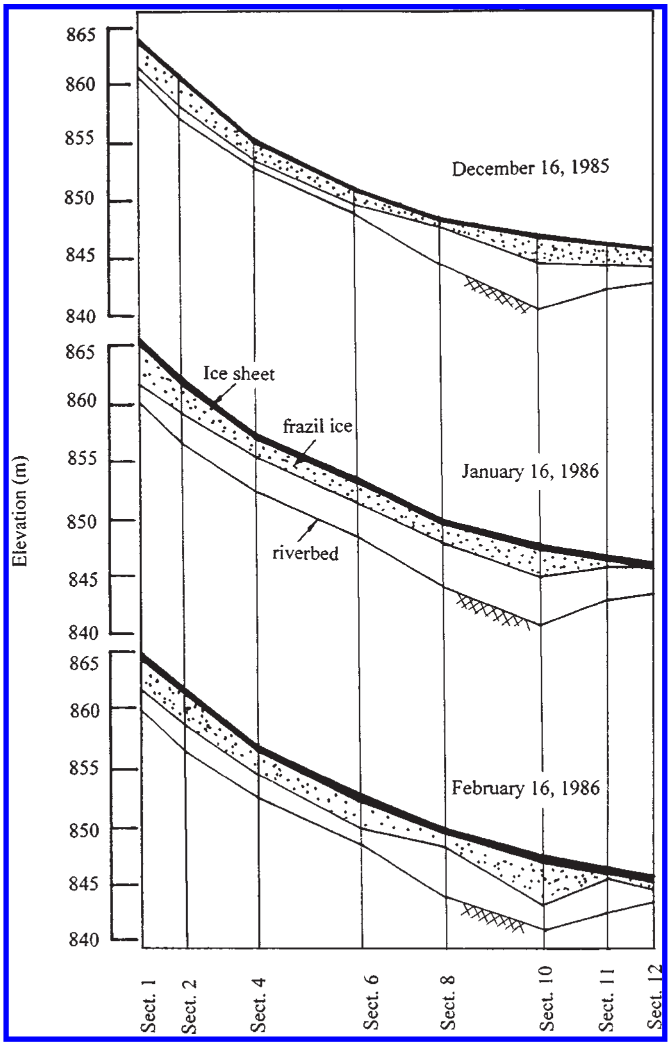


Fig. 6*a*. Longitudinal evolution of ice jam and riverbed at Hequ Reach during stable jamming period (1985–1986).

4.3. Analyses of field data

In winter, after the formation of a river ice jam, the hydraulic and boundary conditions of the river are different from those of a conventional open channel flow. The factors affecting the evolution of the frazil jam and the deformation of riverbed include the oncoming stream flow and its variation, sediment delivery capacity, size distribution of sediment and its variation, hydraulic slopes in the reach (J), river morphology and geology, ice discharge per unit width of river (q_i m^3/(s·m)), composition of frazil and drift ice, and many other minor factors. Oncoming ice discharge and composition of frazil and drift ice directly affect the size and shape of ice jam as well as the extent of the deformation of the riverbed. If the flow conditions remain the same, the larger the ice concentration, the more serious is the ice jam that develops, and the more seriously the riverbed is scoured and the higher the water level. This chain of events leads to an increase in flow velocity, aggravation of erosion at the frazil jam, and deformation of the riverbed.

Conceptually at least, the thickness of frazil jam and the deformation of riverbed may be evaluated by means of a general formula for computation:

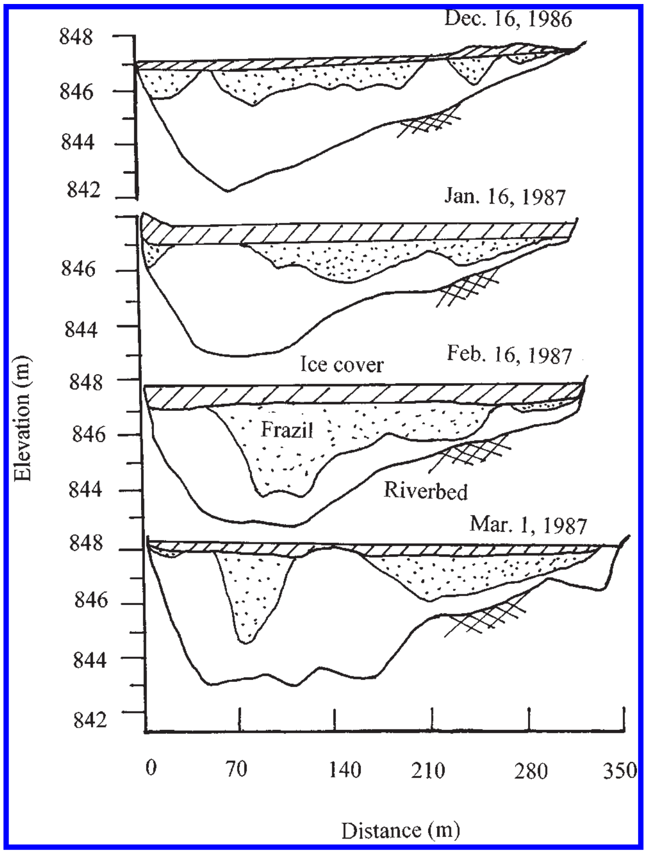


Fig. 6*b*. Evolution of cross section of river (at section 9 of Hequ Reach) during stable jamming period (1986–1987).

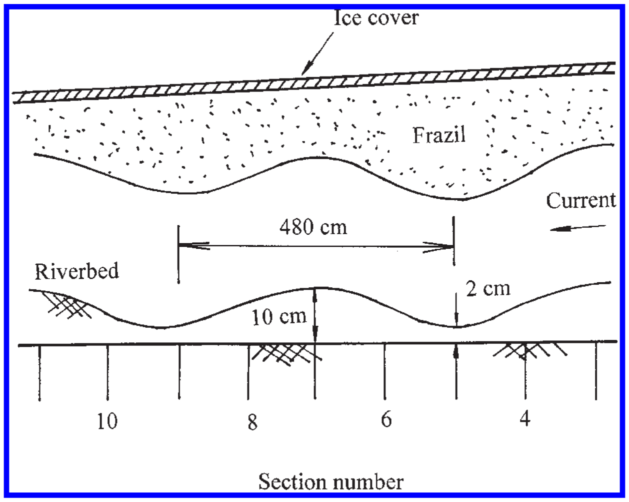


Fig. 6*c*. Waved accumulation of simulated ice jam in a waved bed flume (sinusoidal function) in laboratory (Hefei University of Technology, China, October 28, 1990).

[1] $h_i = f_1(v, h, J, n_b, n_i, g, \rho, \rho', \rho_s, \sigma_{di}, d_{50i}, q_i, B)$

[2*a*] $h_s = f_2(v, h, J, n_b, n_i, g, \rho, \rho', \rho_s, \sigma_{ds}, d_{50s}, q_s, B)$

or, alternatively,

[2*b*] $A_s = f_3(v, h, J, n_b, n_i, g, \rho, \rho', \rho_s, \sigma_{ds}, d_{50s}, q_s, B)$

[3] $A_s = h_s B$

in which h_s and A_s are, respectively, the mean thickness (m) and area (m^2) of deformation relative to the reference cross section of riverbed (measured on January 1, 1987); v is the mean velocity of flow under ice jam (m/s); h is the mean effective depth of water under jam (m); n_b, n_i, and n are the roughness coefficient of riverbed, ice jam, and the composite cross section, respectively; ρ, ρ', and ρ_s are the density of water, frazil ice, and sediment (kg/m^3), respectively; d_{50i} and d_{50s} are the median diameter of frazil and sediment particles (m), respectively; σ_{di} and σ_{ds} are the mean square deviation of grain size of frazil and sediment particles, respectively; q_s is the sediment discharge per unit width of river (m^3/(s·m)); and B is the width of river (m). A series of steps is now used to simplify this equation bearing in mind that only a first-order relation is being sought.

From the Manning and Chezy equations, we have

$$[4]\qquad v = v(J, h, n)$$

The well-known Sabaneev equation may be used to estimate the composite roughness of cross section (Beltaos 1983):

$$[5]\qquad n = \left(\frac{n_b^{3/2} + n_i^{3/2}}{2}\right)^{2/3}$$

In addition,

$$[6]\qquad q_i = q_i(v, d_{50i}, \sigma_{di}, h)$$

$$[7]\qquad q_s = q_s(v, d_{50s}, \sigma_{ds}, h)$$

With the above equations, having eliminated non-independent and minor variables in eqs. [1], [2*a*], and [2*b*], we have

$$[8]\qquad h_i = f_4(v, h, g, \rho', \rho, d_{50i})$$

$$[9a]\qquad h_s = f_5(v, h, g, \rho_s, \rho, d_{50s})$$

or

$$[9b]\qquad A_s = f_6(v, h, g, \rho_s, \rho, d_{50s})$$

Through dimensional analysis, the following simplified equations can be obtained:

$$[10]\qquad \frac{h_i}{h} = f_7\left(\frac{v}{\sqrt{gh}}, \frac{\rho'}{\rho}, \frac{d_{50i}}{h}\right)$$

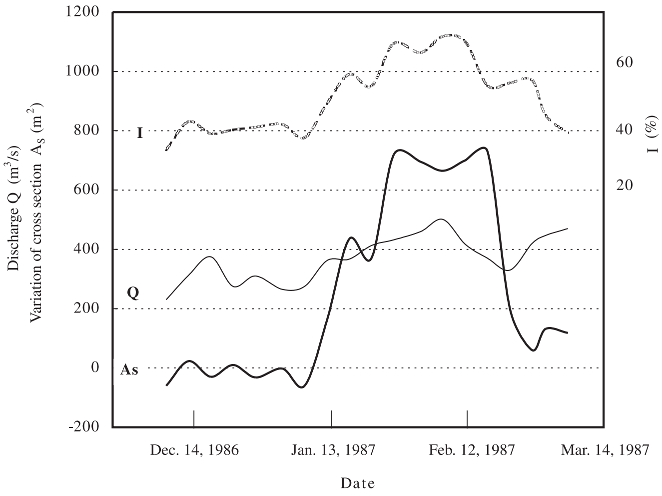
$$[11]\qquad \frac{h_s}{h} = f_8\left(\frac{v}{\sqrt{gh}}, \frac{\rho_s}{\rho}, \frac{d_{50s}}{h}\right)$$

As the second terms on the right-hand side in eqs. [10] and [11] only vary over very limited range within the reach and may thus be taken as constant. This further simplification results in

$$[12]\qquad \frac{h_i}{h} = K_1\left(\frac{v}{\sqrt{gh}}\right)^{\alpha_1}\left(\frac{d_{50i}}{h}\right)^{\beta_1}$$

$$[13]\qquad \frac{h_s}{h} = K_2\left(\frac{v}{\sqrt{gh}}\right)^{\alpha_2}\left(\frac{d_{50s}}{h}\right)^{\beta_2}$$

Fig. 7. Results of evolution of riverbed and ice jam (section 6).



or

$$[14]\qquad \frac{A_s}{A} = K_3\left(\frac{v}{\sqrt{gh}}\right)^{\alpha_3}\left(\frac{d_{50s}}{h}\right)^{\beta_3}$$

To avoid difficulty in the regression analysis due to the presence of both positive and negative values of h_s and A_s, let

$$[15]\qquad A_s' = 1000 - A_s$$

The equation thus assigns A_s' as the reduced cross-sectional area taking the value of 1000 m^2 as the base area. Thus, eq. [14] may be converted into

$$[16]\qquad \frac{A_s'}{A} = K_4\left(\frac{v}{\sqrt{gh}}\right)^{\alpha_4}\left(\frac{d_{50s}}{h}\right)^{\beta_4}$$

As the sediment in this river reach mainly comprises fine sand, the frazil particles are generally also uniform (with mean particle diameters between 7 and 11 mm and the major portion of ice particles being of this size (Shen and Wang 1995)), d_{50s}/h and d_{50i}/h need not to be taken as variables for the time being and the grain size of frazil particles and bed material being taken as unchanging along the path of flow. Regression analysis is carried out based on the observed data at 10 sections from section 2 to 11, with the results as follows.

To summarize the previous steps, the evolution of the frazil jams at the Hequ Reach of the Yellow River can be approximated as

$$[17]\qquad \frac{h_i}{h} = 7.0237\left(\frac{v}{\sqrt{gh}}\right)^{1.019}$$

Similarly, the deformation of riverbed under the frazil jam condition can now be written as

$$[18]\qquad \frac{A_s'}{A} = 4.3334\left(\frac{v}{\sqrt{gh}}\right)^{0.5942}$$

Fig. 8. Relationship between evolution of frazil jam and hydraulic parameters.

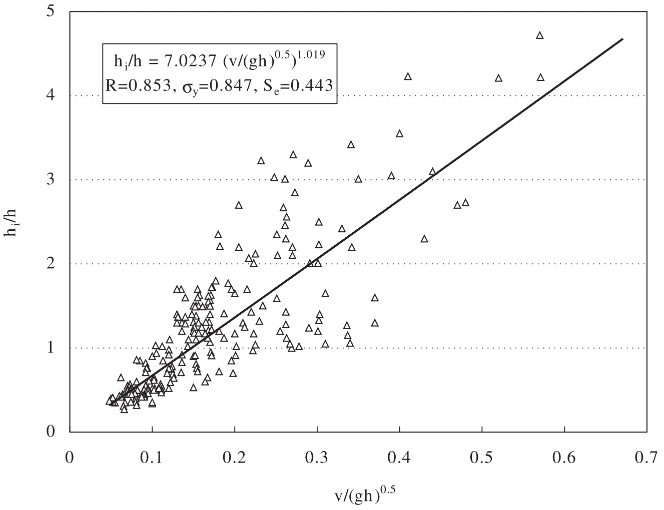
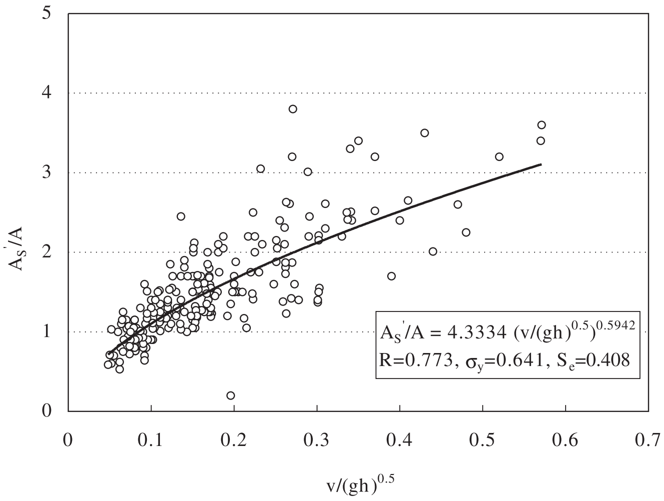


Fig. 9. Relationship between deformation of riverbed and hydraulic parameters.



The regression coefficient is 0.85 for frazil jam evolution and 0.77 for riverbed deformation, and the correlation was found to be predominant. The results of computation are shown in Figs. 8 and 9.

The correlation coefficient R, the standard deviation σ_y, and the standard error of estimate of the correlation S_e are used to indicate the fitting goodness of the regression. Based on the sample data, the standard error of estimate S_e and the standard deviation σ_y are calculated through the following two relationships (Ponce 1989):

$$[19] \qquad S_e = \sigma_y \sqrt{\frac{n-1}{n-2}(1-R^2)}$$

$$[20] \qquad \sigma_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - Y)^2}{n-1}}$$

In which, σ_y is the standard deviation of y-series (here h_i/h or A_s'/A); R is the correlation coefficient; n is the number of values; and Y is the average of the sample data. The calculated results are shown in Figs. 8 and 9. Not surprising, it is found that the correlation is not overwhelming. However, the uncertainty of the measurements, the channel irregularities, and the nonuniform particle sizes (bed material and frazil particles) certainly account for some of the scattering shown in Figs. 8 and 9. The tendency of the curves reflects the general dependence the jamming extent of the cross section (h_i/h) and the deforming extent of riverbed (A_s'/A) on the Froude number $v/(gh)^{0.5}$.

5. Discussion of results

Equation [18] can be converted into a form for computing the thickness of the deposit or the depth of scour in a broad and shallow river section. The result is

$$[21] \qquad \frac{h_s}{h} = -4.3334\left(\frac{v}{\sqrt{gh}}\right)^{0.5942} + \frac{1000}{Bh}$$

The condition that balances scour and deposition in a riverbed during the ice period is of particular interest. Assume that a relative state of equilibrium prevails with respect to scour and deposition (e.g., no scour and deposition with respect to the reference section); setting $h_s/h = 0$, we have from eq. [21] that

$$[22] \qquad F_c = \frac{v}{\sqrt{gh}} = \left(\frac{1000}{4.3334Bh}\right)^{1.683}$$

This approach leads to the following three conditions:
(*i*) when $F > F_c$, scour prevails;
(*ii*) if $F < F_c$, deposition is manifest; and
(*iii*) if $F = F_c$, the riverbed is relatively stable.

The so-called stable channel above is clearly relative, whereas the deformation of the riverbed is certain to take place. Even if $F = F_c$, a given section will be in part scoured and in part subject to deposition; the balance or equilibrium simply means that these two parts are of roughly equal magnitude.

6. Conclusions

The interrelationships between sediment concentrations in the current and frazil ice and ice cover in the Hequ Reach of the Yellow River are discussed in the present paper. Based on the field observations, it has been confirmed that the sediment concentration is dependent to a large extent on the current discharge. The larger the discharge (current velocity) under frazil jam, the larger is the value of sediment concentration. From November to March, especially in January and February, suspended load is coarser than in October and April. By implication, the suspended load in the current during stable jamming period is coarser than that during formation of ice jams and breakup period. During the stable jamming period, the sediment concentration is much smaller than that with an equivalent flow under open channel conditions. Generally, the sediment concentration of frazil ice is between 1.0 and 10.0 kg/m^3. It has been found that the sediment concentration of frazil ice during the formation of frazil ice jam is much larger than the sediment concentration of current under frazil jams. Interestingly, the upper layer frazil has a larger sediment concentration than those of lower layer. Analyses have been made on the evolution of frazil jam and deformation of riverbed. It has been found that the deformation of riverbed and the evolution of frazil jam reinforce each other. Preliminary empirical interrelationships between the hydraulic parameters of water under ice jams (Froude number of water) and the particular features of evolution of ice jam (jamming extent) and the deformation of riverbed (deforming extent) have been established.

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