Extended-Period Analysis with a Transient Model

Yves R. Filion¹ and Bryan W. Karney, M.ASCE²

Abstract: To design and operate a distribution system, one must understand how it will perform when subjected to external hydraulic loads and demands. This paper presents a hybrid model that efficiently tracks the full range of hydraulic conditions, from steady state to waterhammer, in a system over an extended period by coupling a transient simulator with a reservoir routing scheme. The model’s procedure consists of running waterhammer simulations at the start and end of an extended time step to track the rate of filling of a system’s reservoirs and then using this information to update reservoir levels at the end of the time step. Beyond conventional level-of-service and capacity-assessment applications, the hybrid model can help the engineer link system unsteadiness to its associated costs in terms of design and operation. Extended period and worst-case simulations presented in a case study suggest that the hybrid model has a high routing accuracy and can be used effectively to identify the critical state which will produce the most severe transients in a system.

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Introduction

A common goal in network modeling studies is to predict how a system will behave when it is subjected to different operating states. In many studies, this goal is achieved through what is called extended period or dynamic analysis. Generally speaking, this kind of dynamic analysis is usually performed to estimate pressures, flows, and water levels in a system and its storage reservoirs at specified points in time and under different hydraulic states. With information on the hydraulic performance and requirements of a system over a typical period of 6, 12, 24, or 48 h, the engineer can assess the adequacy of pressures, flows, and velocities in a subject to different external, hydraulic actions ranging from mundane, diurnal fluctuations in demand to the rapid withdrawal of water for a fire or in times of emergency. Moreover, dynamic models are often used to make key planning, design and operation decisions such as sizing pipes, reservoirs, and pumps in a system, timing the operation of pumps, and developing control strategies to maintain an acceptable level of service (Rao and Bree 1977; Bhave 1991). Recently, dynamic models have been coupled with water quality models to track in distribution systems the transport and fate of constituents (e.g., chlorine residuals, organic compounds, and trihalomethane precursors such as humic and fulvic acids commonly found in soils and backfills; Rossman et al. 1993, 1994; Boulos et al. 1994).

 Usually, the extended period analysis of a system is performed with what is more precisely called a quasi-steady state model. This type of model emulates the changing hydraulic conditions in a system by linking together a series of steady states (having different system demands) with a continuity expression which updates water levels at each time step. More precisely, the model does this by first solving the steady state equations of flow (mass conservation and uniqueness of head at a node) to find the steady state heads and flows at the beginning of an extended time step, and then by inserting the resulting values into an integrated differential expression that calculates the net amount of flow into the reservoirs and tanks of a system. These two computational steps are repeated at each extended time step to yield the long-term hydraulic solution of a distribution network. In addition to these basic functions, most commercial models incorporate additional steps in their algorithms to update and control the actions of hydraulic devices such as pumps and valves (Rossman 2000; Boulos 1998).

The first researchers to present a formal extended period procedure were Rao and Bree (1977) who coupled a steady state network solver to a modified Euler integration of the continuity equation for a reservoir. Through this coupling, Rao and Bree showed that his implicit scheme produced estimates of water level which were similar to those generated with a comparable explicit scheme. Bhave (1988) later presented a similar model, showing how an integrated continuity equation for reservoirs could be solved implicitly along with the set of nonlinear steady state equations with known linearization techniques (e.g., Newton-Raphson procedure, linear theory method, etc.). Bhave also showed that his implicit scheme produced estimates of water level which were similar to those generated with a comparable explicit scheme.

The fundamental assumption which underlies all quasi-steady methods is that flows and pressures in a system change slowly with time. Since such models solve equilibrium equations of flow,
this critical assumption is tenable so long as hydraulic conditions are only slightly unsteady and not influenced by any significant inertia and/or compressibility effects. However, this assumption is routinely violated as local control actions are brought to bear on the system, say by bringing pumps on or off line or by quickly adjusting the status of a valve.

When these local changes are imposed on a system, a pressure wave is created and propagates outward in the system from the point of change, informing the rest of the system of the changed hydraulic conditions. This sort of hydraulic “communication” abounds in modern distribution networks and is the cause of frequent episodes of unsteadiness. Mundane hydraulic actions aside, planned or accidental occurrences such as a pump failure or a rapid valve closure can create surge conditions which, in turn, can engender large shear stresses in a pipeline and cause accumulated biofilm and tuberculated material to be more rapidly sloughed off pipe walls and introduced into bulk water.

This paper presents an extended dynamic model capable of simulating a wide range of hydraulic conditions and updating tank levels in a system while stepping through the large part of an extended time period. Briefly explained, the model tracks conditions in a system by simulating short segments of time at the start and end of an extended time step with a waterhammer model and then uses the water level estimates calculated in these segments to “jump” to the next extended time step. This time-stepping procedure is performed with the numerical techniques introduced in this paper. While any conventional transient model can be used to perform an extended period analysis and model the full range of hydraulic conditions in a system, the new approach, with its time-stepping routines, allows one to do this much more quickly and efficiently. Essentially, the approach constitutes a conceptual bridge between conventional extended period analysis, as it is usually performed with a quasi-steady state model, and conventional transient analysis, where a system is typically analyzed over a short period of time under unusual circumstances (e.g., pump failure, sudden valve closure, etc.). By combining the broad modeling capabilities of a transient simulator with the time-stepping efficiency of a quasi-steady state model, this hybrid model constitutes an effective and accurate means of tracking the full spectrum of hydraulic conditions in a system, ranging from steady state to waterhammer, over an extended period.

Among other applications, this novel coupling can help the engineer understand the role of unsteadiness in the design, maintenance, and operation of a water distribution system, effectively broadening the scope of many common types of hydraulic investigations. Specific insights into the role and effects of unsteadiness might include: (i) planning and assessment studies that investigate the level of service in a system; (ii) screening studies that locate zones of low and high operating pressures; and (iii) studies to investigate the behavior of a system when it is operating under normal and/or emergency conditions. The model can also help the engineer set the timing of pumps and reservoirs so as to minimize transients in a system while adjusting to changing demands. Moreover, the ability to accurately track both steady and unsteady conditions in a system over a mid- to long-term period begins to shed light on the complex problem of tracking the quality of water in a system over an extended period of time. Although, in fairness, this approach remains computationally impractical for the analysis of very large systems, it is quite reasonable for the analysis of small- to medium-sized distribution systems.

Extended Dynamic Model

The extended dynamic model (EDM) essentially splits into two components which perform fundamentally different functions: the first is a transient simulator which reproduces the hydraulic conditions in a distribution system with the full equations of transient flow at specified points in time; and the second is a time-stepping scheme which tracks the continuity conditions in the tanks and reservoirs of a system and updates water levels at each extended time step with hydraulic estimates derived from transient simulations.

The model’s transient simulator solves the full one-dimensional, hyperbolic partial differential equations of momentum and continuity in a pipe reach and, in doing so, reproduces the gamut of hydraulic conditions in a pipeline system. By solving these general equations, the transient model can simulate accurately the conditions of steady, quasi-steady, unsteady incompressible, or unsteady compressible flow in a pipeline. The general equations of flow are well described in Chaudhry (1987) and Wylie and Streeter (1993). Since the authors have made no fundamental contribution to this specific area, the equations of transient flow and their solution are not presented here. The specific numerical approach adjusted is discussed in Karney and McInnis (1992).

The extended dynamic model makes use of the transient simulator TransAM (McInnis et al. 1998) (developed through Hydratek Associates) to represent the hydraulic conditions in the pipes and hydraulic devices of a distribution network. TransAM solves the full transient equations of continuity and momentum with the fixed-grid method of characteristics (FGMOC) and a flexible friction linearization scheme. Discretization of the space-time domain is enhanced further through wave speed adjustment and interpolation schemes. The transient model is designed with a high degree of flexibility, allowing one to represent an array of boundary devices including pump stations (with either constant- or variable-speed pumps), nodal or in-line valves with either control or modulating capabilities, air valves, and storage elements like constant-head reservoirs or variable-head tanks. Moreover, the model can simulate a variety of hydraulic scenarios, ranging from power failures and valve actions to variations in demand, and can display the response with an assortment of plotting functions.

Reservoir Equation

The extended period problem essentially consists of solving the reservoir equation of continuity to update the water levels at each extended time step of a simulation. The reservoir routing procedure is carried through at two disparate time scales: the first is the transient time scale and the second is the extended time scale.

At the transient time scale, the continuity conditions in a reservoir are updated in the course of a transient simulation (what we call the front-end and back-end stages). This is done by solving the set of algebraic expressions which describe the continuity, inertia, compressibility, and friction conditions which cause fluid to enter (leave) a system’s reservoir at each transient time step of a simulation (using the usual MOC equations). This storage equation can be represented, in a general way, by the familiar quasi-linear, ordinary differential expression

$$\frac{dZ}{dt} = \frac{Q(t,Z)}{A(Z)} = f(t,Z) \tag{1}$$

where $Z$ = water elevation; $Q(t,Z)$ = net flow into (out) of a tank or reservoir; $A(Z)$ = cross-sectional area of a tank which might
vary with depth; and \( f(t,Z) \) = general quasi-linear, ordinary differential function. The flow function \( f(t,Z) \) written in terms of both time \( t \) and water elevation \( Z \), relates, in an abstract way, the influence of system interactions on the flow at the inlet of a reservoir. Usually, the routing calculations of a transient simulator will be precisely determined, since the transient time step used is typically small and the algebraic MOC equations describing friction, inertia, etc. can accurately describe the filling (emptying) operations of a reservoir.

At the extended time scale, the water level in a tank is updated with the macroscale time-stepping methods of this paper. This extended reservoir routing procedure is largely decoupled from the transient hydraulic calculations and, as such, can be thought to be conceptually “disconnected” from the system. More specifically, this procedure uses the transient model only to calculate average rates of change of water level which are then entered into a separate routing scheme which independently updates water levels at each extended time step. Under these circumstances, Eq. (1) effectively reduces to

\[
\frac{dZ}{dt} = \frac{Q_{net}(t)}{A[Z(t)]} = f(t) \quad (2)
\]

where \( Q_{net}(t) \) = net flow to (from) the reservoir obtained from front- or back-end transient analysis; and \( f(t) \) = simple, ordinary differential relationship. The effective removal of the water elevation term \( Z \) in the flow function \( f(t) \) means that the reservoir routing scheme is physically “detached” from the rest of the system. As such, it “knows” nothing of the physical interactions (i.e., friction, inertia, and compressibility) in the system which are ultimately responsible for producing the history of flows at the inlet of a reservoir or tank. The time-stepping routines used to solve Eq. (2) are elaborated next.

**Reservoir Routing Schemes**

The extended dynamic model advances the hydraulic solution by means of a reservoir routing scheme which tracks the accumulation (depletion) of fluid in a reservoir over an extended time step. Two such schemes are presented in this paper. The first is a predictor-corrector technique which is based on the standard modified Euler approach; the second is a predictor-Adams technique which is loosely based on the well-known family of Adams-Bashforth and Adams-Moulton schemes.

**Predictor-Corrector Scheme**

The predictor-corrector scheme advances the reservoir solution from one time step to the next by extrapolating water levels (with constant, or near-constant cross-sectional area) with average reservoir filling rates calculated in two transient analyses performed at the start and end of an extended time step. The steps involved in the extrapolation are outlined in Fig. 1.

In the first stage of the procedure, transients in a system are simulated over a short time period to find the net change in water elevation in a reservoir (average rate of filling in the reservoir). The centered-difference expression which describes this average rate of change is calculated with

\[
f_1 = \frac{Z_n^1 - Z_n}{\Delta t_1} \quad (3)
\]

where \( Z_n \) = water level at the start of the front-end transient simulation; and \( Z_n^1 \) = water level at the end of the front-end transient simulation lasting \( \Delta t_1 \).

The extended period model then uses the average rate of filling calculated in Eq. (3) to extrapolate the water level to the starting point of the back-end simulation. This prediction is carried out with a first-order linear approximation:

\[
Z_n^2 = Z_n^{1/2} + (\Delta t_1/2 + \Delta t_{in})f_1 \quad (4)
\]

where \( Z_n^{1/2} \) = water level calculated at the midpoint of the front-end simulation; \( \Delta t_{in} \) = length of the extrapolation step; and \( Z_n^2 \) = extrapolated water level used to “start” the back-end simulation.

With the extrapolated water level \( Z_n^2 \), the transient simulator runs a “back-end” simulation to calculate a second average reservoir filling rate. This last calculation takes the form of another finite-difference expression

\[
f_2 = \frac{Z_n^2 - Z_n^1}{\Delta t_2} \quad (5)
\]

where \( Z_n^1 \) = water level at the end of the back-end simulation lasting \( \Delta t_2 \). Following this, the average rates of filling \( f_1 \) and \( f_2 \) are entered into a corrector formula to calculate a value of water level at the end of the current extended time step

\[
Z_{n+1} = Z_n^{1/2} + \frac{f_1 + f_2}{2}(\Delta t - \Delta t_{in}/2) \quad (6)
\]

where \( Z_{n+1} \) = “corrected” water level at end of extended time step; and \( \Delta t \) = length of extended time step. The multiplication on the right side of Eq. (6) computes the trapezoidal area underneath the straight-line segment bounded by end points \( f_1 \) and \( f_2 \).

**Predictor-Adams Scheme**

The predictor-Adams method is similar to the predictor-corrector scheme in that it performs the same basic routing function. In this approach, time-averaged filling rates are entered into a second-order interpolating polynomial which describes the average rate of fluid accumulation or depletion in a reservoir across two extended time steps. Once defined, the polynomial is integrated over a discrete interval of time to find a new water level at the end of the current extended step. The salient steps of this approach are
the three filling rates are calculated, the interpolating polynomial

\[
 f_1^{\ast} = \frac{Z_{n+1} - Z_{n-1}}{\Delta t_1}
\]

where \( Z_{n-1} \) = water level at the start of the previous extended
time step and \( Z_{n+1}^{\ast} \) = water level at the end of the front-end simul-
ation in the previous extended time step.

The time-averaged filling rates \( f_1^{\ast} \) and \( f_1 \) are then used to
evaluate a model for a new water level at a point near the end of the
current step. Note that \( f_1 \) is still calculated with Eq. (3). The extrapolation
is based on a weighting scheme derived from a Taylor expansion

\[
 Z_n^{\ast} = Z_n^{1/2} + f_1^{\ast} + 3 f_1 \Delta t_1 / 4 (\Delta t_1/2 + \Delta t_{int})
\]

The weighting factors 1/4 and 3/4 in this last equation were found
to yield good results through numerical experimentation. Con-
sequently, one could use a variety of weights to calculate the value
of \( Z_n^{\ast} \).

The “predictor” water level \( Z_n^{\ast} \) calculated in Eq. (8) is used to
initiate the back-end transient simulation. The time-averaged flow
\( f_2 \) calculated from this last simulation is approximated with a
backward-difference expression already shown in Eq. (5). Once
the three filling rates are calculated, the interpolating polynomial
is integrated across the interval of time defined by end points
\( \alpha_1 = \Delta t \) and \( \alpha_2 = 2 \Delta t - \Delta t/2 \)

\[
 Z_{n+1}^{1/2} + \int_{\alpha_1}^{\alpha_2} F(f_1^{\ast}, f_1, f_2, t) dt = \int_{\alpha_1}^{\alpha_2} F(f_1^{\ast}, f_1, f_2, t) dt = A c_1 + B c_2 + C c_3
\]

where \( f_1^{\ast} \) and \( f_1, f_2, t \) are equal interpolation polynomial. The inter-
grated polynomial can easily be distorted down to a simple
algebraic expression of the form

\[
 A = f_1^{\ast}
 B = \frac{f_1 - f_1^{\ast}}{\Delta t}
 C = \frac{f_2 - f_1}{\Delta t - \Delta t/2} - \frac{f_1 - f_1^{\ast}}{\Delta t} \frac{1}{2 \Delta t - \Delta t/2}
\]

The time coefficients \( c_1, c_2, \) and \( c_3 \) can be written as

\[
 c_1 = \Delta t - \frac{1}{2} \Delta t_1
 c_2 = \frac{1}{2} \Delta t^2 + \frac{1}{2} \Delta t_1 - \Delta t \Delta t_1
 c_3 = \Delta t^2 (\frac{1}{2} \Delta t - \Delta t_1) + \Delta t_1^2 (\frac{1}{2} \Delta t - \frac{1}{2} \Delta t_1)
\]

Component-Control Algorithm

In addition to the routing techniques presented, the extended hy-
draulic model uses a component-control algorithm to marshal
the transient analysis program through the steps of an extended pe-
riod simulation. This algorithm essentially directs the operations
of demands, pumps, valves, and fluctuating-head reservoirs in an
idealized distribution system, by means of two mechanisms: (i) a
set-point control scheme; and (ii) a time-schedule control scheme.
In the set-point option, the settings of devices are changed when-
ever their hydraulic parameters, or parameters of other devices in
a system, exceed specified limits. Moreover, the time-schedule
option varies the settings of devices according to a predetermined
list of values. The major system components controlled with
the component-control algorithm are listed here.

1. Demand control: including fixed and pressure-dependent de-
mands;
2. Pump control: including fixed-speed and variable speed
pumps, and the action of various pump control valves;
3. Valve control: including all types of discharge and control
valves, as well as orifice and leaks; and
4. Reservoir/tank control: including fluctuating-head supply/
reservoirs and various water level dependent inlet flows.

General Procedure

The basic steps followed during each extended time step of a
simulation are explained next. The discussion highlights how ini-
tial conditions are set prior to each transient simulation and how
reservoir levels are extrapolated from one extended time step to
the next. These steps are also outlined in Fig. 2.

Extended Period Analysis

At the very beginning of an extended period simulation, the first
order of business is to fix the initial “state” of the system. This is
done through the component-control algorithm which sets the ini-
tial water level of tanks, the speed of pumps, the aperture
of valves, and the rate of water consumption. The extended period
model then uses these initial system settings, along with a set of
starting heads and flows in the system, to initiate the first tran-
sient simulation. Once this transient run is complete, the net change
in tank levels in the system is used to calculate average tank filling
(draining) rates over the simulated period. These average filling
(draining) rates are then entered into a time-stepping predictor technique that extrapolates tank levels to a point in time within the current extended time step.

The “predictor” tank levels are then used to initiate the back-end transient simulation. At this point, the component-control algorithm fixes the device settings to their predetermined or set-point values (if need be) and “reuses” the set of heads and flows used at the start of the front-end simulation to initiate the back-end simulation. Once this is done, the back-end transient analysis is run to the end of the current extended step. As before, the net change in tank levels calculated in the back-end transient simulation are used to calculate average tank filling (emptying) rates which, this time, are entered into a time-stepping corrector scheme to compute new tank levels at the beginning of the next extended step. Also, the near-steady heads and flows calculated close to the end of this back-end simulation are recorded to be used again as starting conditions in the front- and back-end simulations of the next extended time step. An entire extended simulation is performed by repeating these steps during each extended time interval.

Implied in this procedure is that transient events (i.e., pump failure or valve closure) must be scheduled to occur at times coinciding with the front- and back-end simulation time steps $\Delta t_1$ and $\Delta t_2$ in any given extended time step. As this is not necessarily the case, the procedure can be altered to track and simulate transient events scheduled to occur between the front- and back-end simulation time steps $\Delta t_1$ and $\Delta t_2$. All that is needed is a routine that identifies those transient events which are scheduled to occur between the front- and back-end transient simulation steps $(\Delta t_1$ and $\Delta t_2)$ and ”tells” the extended dynamic model to temporarily increase the length of either $\Delta t_1$ or $\Delta t_2$ (and thus decrease the length of $\Delta t_{int}$) to properly simulate these events. With this “adaptive” routine, the extended dynamic model could be used in a real-time control feedback system where different short-term transient scenarios are continuously simulated and used to inform the real-time operation of a system.

**Worst-Case Analysis**

Generally speaking, one performs a worst-case analysis by analyzing a transient event repeatedly with a transient simulator for several different hydraulic states in a system. More precisely, a hydraulic state can be defined as being the combination of system demands, reservoir water levels, and setting of system components (i.e., pumps, valves, etc.) which, in concert, set the hydraulic “bearing” of a system. By comparing the maximum and minimum transient pressures simulated for each system state two things are possible: determining the magnitude of the highest and lowest transient pressures and thus the most severe transient response in the system, and also determining what is the “critical” system state that produces the most severe transients in a system and when this “worst” response can be expected to occur.

The series of steps followed to perform a worst-case analysis are similar to those found in the extended period analysis. First, at the start of an extended time step, the component-control algorithm fixes the device settings and starting heads and flows (from previous back-end simulation) in the system. The transient model is then run until the artificial transients (unsteady conditions created when water levels are calculated with the time-stepping method without updating heads and flows in the rest of the system) are attenuated in the system. When the system regains a quasi-steady composure, or once a fixed period of time has elapsed, a transient event (e.g., pump failure, valve closure, etc.) is triggered and the transient model analyzes the pressure surges created in the system over a specified period of time. Beyond this point, everything happens as it did before. The extended period model calculates average tank filling (draining) rates that are used to extrapolate reservoir levels to the start point of the back-end simulation. The component-control algorithm then fixes the new settings of devices and “reuses” the heads and flows used to start the front-end simulation to initiate the back-end transient simulation. Once the back-end transient simulation has reached its end, the calculated change in tank levels is used to find “corrector” values of tank levels at the end of the current extended time step. By repeating this simple procedure at every extended step, one can ascertain, in a systematic manner, the critical combination of demands, pumping conditions, and reservoir levels which produce the “worst” transient response in a system.

**Model Testing**

The extended dynamic model is tested by means of numerical experiments performed on a fictitious pipe system to verify its routing accuracy and its usefulness as a tool for performing worst-case analysis. The focal point of the numerical tests is the small water transmission network shown in Fig. 3. The distribution system draws water from the supply source (perched at an elevation of 130 m) located at node 10 and then transports this water to nodes 11, 14, 15, and 17 at an average rate of 300 L/s. A discharge valve $V_1$ located at node 13 supplies water to a user sporadically during the simulation. To maintain adequate pressures in the system and to provide an emergency store of water, two reservoirs ($R_1$ and $R_2$) are connected to the system at nodes 16 and 12. The physical characteristics of the system are outlined in Table 1.

**Extended Period Analysis**

A group of extended period simulations are performed on the test system to evaluate the changing conditions within it during a 6-h period. In each simulation, the demands in the system are varied while two pumps are turned on and off during periods of high and low flow. More specifically, a single pump is operated during the
first two hours and last hour of the simulation, while two pumps are operated during the middle three hours (between 2:00 and 5:00). The discharge valve \( V_1 \) at node 13 remains closed throughout the 6-h period; the demand varies as is shown in Fig. 4.

In total, five simulations are run on the test system to track the water level fluctuations in reservoir \( R_2 \) located at node 12 in Fig. 3. The full hydraulic solution produced by the transient simulator TransAM \( (0.05 \text{ s time step}) \) labeled "TransAM" is depicted by the solid line in Fig. 5. Since the data points are spaced at 20-min intervals, the transient pressures created by opening and closing the valve and turning pumps on and off at the start of each extended time step have effectively been erased in this plot. Owing to the transient model’s broad modeling capabilities, the full transient solution "TransAM" is used as a standard to compare the relative accuracy of the other runs. The extended dynamic model is then used to run two more simulations labeled "Predictor-Corrector \( (400-200 \text{ s}) \)" and "Predictor-Adams \( (400-200 \text{ s}) \)" in Fig. 5. In the former, a Predictor-corrector routing technique is used while in the latter a Predictor-Adams technique.

Both runs are generated with front- and back-end simulation times of 400 and 200 s, respectively. The two remaining runs labeled “EPANET 1-hour” and “EPANET 15-min” in Fig. 5 are generated with the EPS model EPANET 2.0 \( (\text{Rossman} 2000) \) using extended time steps of 1 h and 15 min.

The results in Fig. 5 suggest, anecdotally, that the extended dynamic model can route accurately the water level in reservoir \( R_2 \) of the system. The two runs labeled "Predictor-Corrector \( (400-200 \text{ s}) \)" and "Predictor-Adams \( (400-200 \text{ s}) \)" both follow the "TransAM" curve closely. This is in large part attributable to the time-averaged filling rates which, in calculating a net difference in water level, neatly summarize the history of interactions (i.e., friction, inertia, compressibility, and boundary) that take place in a system over the course of one or two extended time steps. When entered into a time-stepping scheme, these time-averaged rates produce accurate estimates of water level, as is demonstrated in Fig. 5. To get comparable results with the EPANET model, the extended time step must be made relatively short, much shorter than a typical duration, in this particular example. From a physical perspective, the transient pressures are quickly dissipated due to the time-averaging process.

### Table 1. Physical Attributes of Test System

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<th>Diameter (m)</th>
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</table>

**Fig. 4. Variation in water demand during period of simulation**
cal viewpoint, this holds no surprise since EPANET updates water levels in a reservoir by extrapolating steady, instantaneous values of flow, calculated at the beginning of an extended time step with the nonlinear equations of steady state flow. It stands to reason that the faster conditions change in a system, the shorter the extended time step must be in order to minimize routing errors and maintain an acceptable level of accuracy in the solution. However, if conditions change rapidly, as to create significant transient effects in a system, the quasi-steady state model will incur significant error in its routing solution even if very small time steps are used.

This last point perhaps reveals the model’s main advantage: its ability to represent the whole gamut of flow conditions in a pipeline system with the general equations of transient closed-conduit flow while, at the same time, keeping an accurate account of water levels in the tanks and reservoirs of a system. In this sense, the hybrid model constitutes an extension to the conventional quasi-steady state model, whose hydraulic solver is only able to simulate the restrictive steady state condition. To be fair, the increased capabilities of the hybrid model come at a computational expense. The nature of the MOC solution dictates that the run times associated with the front- and back-end simulations will increase in direct proportion to the length of pipe in a system. This essentially makes the extended dynamic model only presently suitable for analyzing small- to mid-sized systems (say less than a 1,000 pipes or so). Obviously, a system analyst would be more often advised to eschew the use of the proposed model in favor of a more efficient conventional extended period solver if the transient behavior of a system is of no interest and the goal is to reduce the computational burden of simulations run on a system.

While the method has only had limited testing on large networks, such systems pose no intrinsic difficulty and should yield results with an accuracy comparable to that presented in the case study. Essentially, the effectiveness and accuracy of the method largely rests on the capabilities of the underlying transient model. The point is that so long as the chosen transient model can accurately simulate hydraulic conditions in a large distribution network, the extended time-stepping approach should yield accurate results, assuming that the simulation time steps $\Delta t_1$ and $\Delta t_2$ are appropriately chosen.

### Accuracy versus Efficiency

What also adds to the model’s appeal is how easily it allows one to control the balance between the accuracy of a solution and how efficiently it is generated. Indeed, by setting the lengths of the front- and back-end simulations, the model can be either “tuned” for accuracy or efficiency, or (more often) a strategic combination of both. Intuitively, the longer the transient simulations are made, the more accurate the extended hydraulic solution will be and the more computer time will be needed to generate a hydraulic solution. This raises an interesting point: estimating how much computer time has been saved in using the extended dynamic model to “jump” over segments of time as compared to strictly using the transient simulator to generate a full solution is a relatively easy proposition. To determine this time saving, an acceleration factor can be calculated with

$$\text{accel.} = \frac{\Delta t}{\Delta t_1 + \Delta t_2}$$

(17)

where $\Delta t =$ length of extended time step; $\Delta t_1 =$ length of front-end transient simulation; and $\Delta t_2 =$ length of back-end transient simulation. So, according to Eq. (17), we can say that the solutions labeled “Predictor-Corrector (400-200 s)” and “Predictor-Adams (400-200 s)” have been “accelerated” by a factor of 6, since the transient simulation time has been reduced from a maximum length of 3,600 s to only 600 s. Practically, a direct way of controlling time-stepping errors can prove useful when modeling a system at different stages of a project, where the accuracy is often increased as more information becomes available.

### Worst-Case Analysis

The transients created by linearly opening and closing the valve at node 13 are analyzed in a worst-case simulation. The operation of pumps and the pattern of demands in this last simulation remain the same as those specified in the previous extended period simulations. At the start of each extended time step, the valve $V_j$ in Fig. 3 is opened in 5 s and then subsequently closed in 25 s. The plots in Fig. 6 depict the maximum and minimum pressure envelopes which correspond to the maximum- and minimum-hour flow conditions, as well as the “worst” (most extreme) transient pressures along pipes 3, 4, 6, and 11 of the system.

FIG. 6 SUGGESTS THAT HOW A SYSTEM WILL RESPOND TO A PARTICULAR TRANSIENT EVENT (IN THIS CASE, A VALVE OPERATION) LARGELY DEPENDS ON THE SYSTEM’S HYDRAULIC STATE AND WHAT LOADS ARE PLACED ON IT AT THE TIME OF THE EVENT. PROOF OF THIS LIES IN THE DIFFERENCE BETWEEN THE SURGE PRESSURE ENVELOPES WHICH CORRESPOND TO MINIMUM- AND MAXIMUM-HOUR DEMAND FLOWS. CLEARLY IN FIG. 5, THE MINIMUM-HOUR PRESSURES (BOTH POSITIVE AND NEGATIVE) AT TIME 3:00 HRS ARE VISIBLY MORE EXTREME THAN THOSE WHICH ARISE DURING MAXIMUM-HOUR CONDITIONS AT TIME ZERO. THE REASON FOR THIS SUBSTANTIVE DIFFERENCE LIES IN HOW DRastically THE HYDRAULIC CONDITIONS AT THE BOUNDARIES OF THE SYSTEM MUST BE CHANGED IN ORDER TO MEET THE SUDDEN, ALTHOUGH TEMPORARY, FLOW OF WATER AT NODE 13. UNDER MAXIMUM-HOUR CONDITIONS, WHEN BOTH RESERVOIRS ARE BEING EMPLOYED AND A SINGLE PUMP IS OPERATING, THE SYSTEM IS ABLE TO MEET THE SHORT-LIVED INCREASE IN DEMAND SINCE IT NEED ONLY INCREASE THE FLOW FROM ITS RESERVOIRS (AND PUMPING STATION) TO DELIVER WATER TO THE REQUIRED NODE AT THE REQUIRED RATE. HOWEVER, AT TIME 3:00 HRS, WHEN THE TWO RESERVOIRS ARE BEING FILLED AND BOTH PUMPS ARE OPERATING, THE SYSTEM MUST QUICKLY REVERSE THE FLOW TO ITS RESERVOIRS IN ORDER TO MEET THE SUDDEN INCREASE IN DEMAND AT NODE 13, WHEN THE VALVE IS SUDDENLY OPENED. WHEN THE VALVE IS CLOSED SHORTLY THEREAFTER, THE FLOWS MUST AGAIN BE REVERSED SO THAT THE RESERVOIRS CAN RESUME THEIR FILLING OPERATIONS.

Perhaps the most interesting aspect of the test system is the timing of the “worst” transients. According to Fig. 6, the most severe transients are created in the system at time 4:00 hrs, when the system has been subjected to minimum-hour flow demands for roughly an hour (Fig. 4). At this point, two pumps are delivering water to the system while both reservoirs are filling slowly. In fact, these conditions are identical to those found at time 3:00 hrs in Fig. 4, save for a higher water elevation in both reservoirs at time 4:00 hrs. The difference in water levels essentially accounts for the small difference between the transient responses calculated at times 3:00 and 4:00 hrs. If nothing else, this highlights the difficulty of finding the critical state of a system which will lead to the most extreme transients. It should be noted that the negative pressures are unrealistically large in this example and shown only to highlight the severity of transients. Also, the results from the worst-case analysis are anecdotal only and they should not be taken as representative of other systems or other events.

As this example has shown, the extended dynamic model presents us with the opportunity to identify, systematically, and essentially automatically, the critical state of a system which will
create the "worst" transients when it is subjected to a specified transient event, like a power failure, a pipe break, or a valve operation. Often, the analyst's ability to identify the critical hydraulic state of a system will be stymied by the nonlinearities of transient flow, the coupled behavior of looped systems, the unpredictable superposition of transient pressure waves propagating in a system, and the tedium of investigating these conditions manually. To circumvent these complications, rules of thumb and modeling conventions (analyze system under minimum/maximum hour conditions only) have often been employed to decipher the most severe transients in a system. Now, with a systematic means of identifying the critical hydraulic state and the most severe transient pressures in a system, one can do away with these guidelines whose use cannot be justified for all systems (Karney and McNinis 1990, 1994).

Conclusion
This paper presents a hybrid model which, in combining the modeling sophistication of a transient simulator and the time-stepping efficiency of a quasi-steady state model, can simulate steady and unsteady interactions in a system over an extended period. The model's three most important features are: (1) its ability to simu-

Fig. 5. Variation in water elevation in tank 2: results of five modeling runs

Fig. 6. Maximum and minimum transient pressure envelopes corresponding to maximum-hour, minimum-hour, and worst-case conditions
late the full range of flow conditions in the pipes and devices of a system, ranging from steady state to waterhammer, with a transient simulator charged with solving the full equations of transient flow; (2) its accurate routing of tanks and reservoirs in a system with an efficient time-stepping technique, and with the hydraulic estimates derived from short transient simulations; and (3) its ability to identify, systematically, at what time, or under which pattern of demands, the most severe transients will arise in a system, if a system is subjected to a single, or set of, transient event(s) (e.g., power failure, pump trip, sudden valve closure, pipe burst, etc.)

The existence of these three attributes is corroborated through a numerical example. A group of extended period simulations performed on a test system show that the extended dynamic model, with its transient simulator/time-stepping coupling, can represent the full range of conditions in a system and route accurately the system’s reservoirs over an extended period of time. Moreover, a worst-case simulation of the test system shows that the extended dynamic model can identify efficiently and unambiguously the critical state (including the combination of hydraulic device settings, demands, and reservoir levels) and the most severe transient conditions in a system.

From these two fundamental applications stems the double-barreled opportunity to establish a link between unsteadiness in a system and its key design and operation parameters, and to exploit this link to broaden the microeconomic analysis of a water distribution system. With the new extended dynamic model, the engineer can keep a stricter account of the often-ignored costs directly and indirectly associated with transient conditions in a system. A good example of this is the design or capital cost associated with selecting a pipe diameter, pressure rating (wall thickness), and a topological placement which will meet the demand and withstand the transient pressures in a system. Other examples include the electrical cost associated with operating pumps in a system which regularly plays host to unsteady conditions, and the cost of maintaining, replacing, and repairing pipes and other hydraulic devices (e.g., pumps, valves) damaged by surge pressures caused by routine or accidental hydraulic actions at the boundaries of a system.

Acknowledgment

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Notation

The following notation is used in this paper:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>polynomial coefficient used in predictor-Adams method;</td>
</tr>
<tr>
<td>$A(Z)$</td>
<td>reservoir or tank area (varies with depth);</td>
</tr>
<tr>
<td>$B$</td>
<td>polynomial coefficient used in predictor-Adams method;</td>
</tr>
<tr>
<td>$C$</td>
<td>polynomial coefficient used in predictor-Adams method;</td>
</tr>
<tr>
<td>$c_1$</td>
<td>time coefficient used in predictor-Adams method;</td>
</tr>
<tr>
<td>$c_2$</td>
<td>time coefficient used in predictor-Adams method;</td>
</tr>
<tr>
<td>$c_3$</td>
<td>time coefficient used in predictor-Adams method;</td>
</tr>
<tr>
<td>$F(f_1^**, f_1, f_2, t)$</td>
<td>quadratic interpolating polynomial;</td>
</tr>
<tr>
<td>$f(t)$</td>
<td>simple, ordinary differential flow equation;</td>
</tr>
<tr>
<td>$f(t, Z)$</td>
<td>quasi-linear, ordinary differential flow equation;</td>
</tr>
<tr>
<td>$f_1$</td>
<td>average rate of reservoir accumulation (current extended step);</td>
</tr>
<tr>
<td>$f_1^*$</td>
<td>average rate of reservoir accumulation (previous extended step);</td>
</tr>
<tr>
<td>$f_2$</td>
<td>average rate of reservoir accumulation (current extended step);</td>
</tr>
<tr>
<td>$Q_{vol}(t)$</td>
<td>volumetric flow into (out) of reservoir or tank;</td>
</tr>
<tr>
<td>$Q(t, Z)$</td>
<td>volumetric flow into (out) of reservoir or tank;</td>
</tr>
<tr>
<td>$t$</td>
<td>time;</td>
</tr>
<tr>
<td>$Z$</td>
<td>water level in reservoir or tank;</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>water level at start of extended time step;</td>
</tr>
<tr>
<td>$Z_0^{[2]}$</td>
<td>water level at midpoint of front-end simulation;</td>
</tr>
<tr>
<td>$Z_0^1$</td>
<td>water level at start of back-end simulation;</td>
</tr>
<tr>
<td>$Z_{n+1}$</td>
<td>water level at end of extended time step;</td>
</tr>
<tr>
<td>$Z_{n-1}$</td>
<td>water level at start of front-end simulation (previous extended step);</td>
</tr>
<tr>
<td>$Z^1_{n-1}$</td>
<td>water level at end of front-end simulation (previous extended step);</td>
</tr>
<tr>
<td>$\alpha_1, \alpha_2$</td>
<td>integration limits used in predictor-Adams method;</td>
</tr>
<tr>
<td>$\Delta t_{fs}$</td>
<td>duration of extrapolation time step;</td>
</tr>
<tr>
<td>$\Delta t_{ee}$</td>
<td>duration of extended time step;</td>
</tr>
<tr>
<td>$\Delta t_{fs}$</td>
<td>duration of front-end simulation;</td>
</tr>
</tbody>
</table>
| $\Delta t_{ee}$ | duration of back-end simulation.

References


