Multi-Objective Design Optimization of Branched Pipeline Systems: Analytical Probabilistic Assessment of Fire Flow Failure

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ABSTRACT

This paper develops a multi-objective optimization approach that incorporates the probability of fire flow failure in branched water distribution networks. An analytical probabilistic model is developed to quantify the probability of fire flow failure in branched networks and incorporated into the non-dominated sorting genetic algorithm (NSGA-II). The optimization approach seeks to minimize two conflicting objectives: capital cost and the probability of fire flow failure. Capital cost and fire flow failure probability are balanced through the selection of the pipe diameters and the number of pumps in the system. The probability of hydraulic failure under fire flow conditions is solved analytically in a branched network. The non-dominated sorting genetic algorithm is used to produce a set of Pareto-optimal solutions in the objective space of pipe and pump cost and fire flow failure probability.

INTRODUCTION

Many optimization algorithms have been applied to solve water distribution system (WDS) design and management problems such as i) finding low-cost solutions in the design and/or expansion of new and existing water distribution networks (Simpson et al., 1994; Dandy et al., 1996; Eusuff and Lansey, 2003; Kadu et al., 2008), ii) minimizing energy consumption in pump operation (Ostfeld and Tubaltzev, 2008), iii) calibrating a hydraulic network model against field data (Jung et al., 2008), and iv) identifying the most severe transient loadings through simulation and choosing the most suitable surge protection strategy (Jung et al., 2009). The underlying assumption of these approaches is that, if the correct data is used and the proper methodology is selected, the resulting answer will be correct. However, this assumption is often questionable and these problems are not deterministic. For example, as the critical loading conditions for WDS design, typically the greater of peak-hour demand or peak-day demand and a chosen fire flow, are often subject to uncertainty, the traditional system design based on the assumed deterministic design loads could be inappropriate under probabilistic demand conditions, either by being either over-or under-designed. Over-design leads to easily meeting the required conditions but at too great a cost, while under-design means the specified conditions are not met.

Engineers dealing with WDS design and management are often expected to explicitly consider uncertainties involved, to make assessment of the performance reliability of their system designs, to quantify the potential risk of failure due to uncertainties, and then to assess the associated consequences. A significant amount of research has been completed in risk-based optimization. Tung (1986) first formulated the least-cost, chance-constrained design problem by considering uncertainty in demands, minimum required pressure head at nodes, and uncertainty in pipe roughness with known probability density functions (PDFs). Lansey et al. (1989) and Xu and Goulter (1999) applied the generalized reduced gradient method (GRG2) in combination with the first-order, second moment (FOSM) and first-order reliability method (FORM) approaches to solve the nonlinear chance-constrained problem. To overcome local minima and maxima search problems in the GRG2 algorithm, Tolson et al. (2004) combined a simple genetic algorithm (SGA) with FORM to increase the likelihood of finding the global solution to the nonlinear least-cost, chance-constrained design problem. To eliminate the need for derivatives in FORM, Babayan et al. (2005, 2006, and 2007) solved the nonlinear least-cost, chance-constrained design problem by combining an SGA with a new integration-based method. In a concurrent research effort, Kapelan et al. (2004) and Babayan et al. (2006) developed a robust chance constrained GA (rccGA) to perform uncertainty propagation of demand and pipe roughness to evaluate the hydraulic robustness of a network through stochastic sampling rather then through the quasi-analytical evaluation of a multiple integral. Kapelan et al. (2006) developed a risk measure to incorporate the consequences of hydraulic failure into the network optimization problem. Filion et al. (2007) developed a risk measure that computes the expected annual damages sustained during low- and high-pressure emergency events (including fires) in a water network with Monte Carlo Simulation (MCS). Building on the work of Jung and Karney (2008 and 2009) and Filion et al. (2007), Filion and Jung (2010) developed a Particle Swarm Optimization (PSO) algorithm that incorporates a new integration-based method to estimate the expected damages sustained during fire flow conditions in the least-cost design problem. The integration-based measure developed by Filion and Jung (2010) eliminates the need to perform computationally-expensive MCS to evaluate expected annual fire damages in networks.

This paper develops a multi-objective optimization approach that incorporates the probability of fire flow failure in branched water distribution networks. An analytical probabilistic model is developed to quantify the probability of fire flow failure in branched networks and incorporated into the non-dominated sorting genetic algorithm (NSGA-II). The optimization approach seeks to minimize two conflicting objectives: capital cost and the probability of fire flow failure. Capital cost and fire flow failure probability are balanced through the selection of the pipe diameters and the number of pumps in the system. In this study, the probability of fire flow failure is solved analytically in a branched network. The non-dominated sorting genetic algorithm (NSGA-II) is used to produce a set of Pareto-optimal solutions in the objective space of pipe and pump cost and fire flow failure probability. Lastly, the NSGA-II algorithm is tested on a branched network to generate Pareto-optimal solutions.

PROBABILITY OF FIRE FLOW FAILURE IN A BRANCHED NETWORK: ANALYTICAL PROBABILISTIC APPROACH

An analytical probabilistic approach is developed to estimate the probability of fire flow failure in a branched distribution network. A prototypical branched pipeline is indicated in Figure 1. In this system, pumps draw from a water source (e.g., reservoir, clearwell) with a known water level of H_s and apply a pumping head of h_p to convey water to downstream demand locations. The maximum day demand at node j is Q_j, and the needed fire flow at node j is Q_{fj} (Figure 1) and both the available pressure head (H_j) and the minimum required pressure head (H_{fj}) at node j during the fire flow are indicated. The needed fire flow is modeled as a random variable to reflect its uncertainty. It is further assumed that a single fire demand occurs at any time in the system and that the maximum day demand exists at the same time. In reality, there is a joint probability distribution of domestic and fire demands but this more complex problem is a topic for subsequent research.

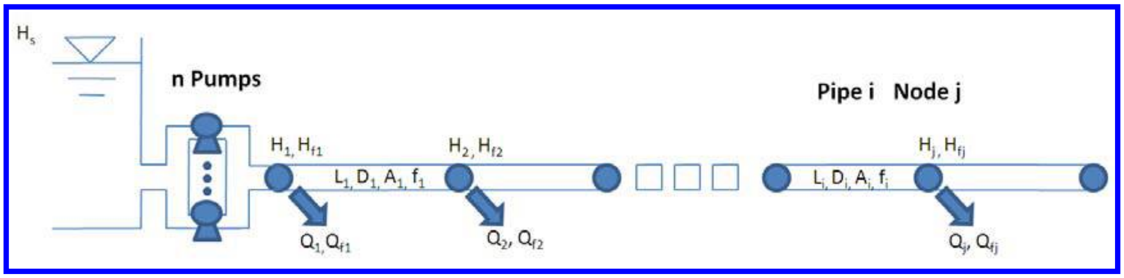

Figure 1. Branched network with source, parallel pump installation, water mains and demand locations.

The available pressure head H_j during a fire at node j with needed fire flow Q_{fj} is calculated as

$$\begin{aligned} H_j &= H_s + h_p - h_{L1} \cdots - h_{Li} \\ &= H_s + A - B/n^2 (Q_{fj} + \sum_{k=1}^{N_n} Q_k)^2 - K_1 (Q_{fj} + \sum_{k=1}^{N_n} Q_k)^2 \cdots - K_i (Q_{fj} + \sum_{k=j}^{N_n} Q_k)^2 \end{aligned} \quad (1)$$

where h_p = head supplied by n identical parallel pumps; A and B = coefficients of the quadratic pump characteristic curve expressed as $hp = A - B(Q/n)^2$; n = number of identical parallel pumps; N_n = number of nodes in the network; h_{Li} = head loss in pipe i; $K_i = f_i L_i / 2gD_i A_i^2$ represents the hydraulic resistance of pipe i; f_i, L_i, D_i and A_i are Darcy-Weisbach friction factor, pipe length, the pipe diameter and the pipe cross-sectional area of pipe i, respectively. The pumping head value of h_p can be further extended to any configuration (series/parallel or combination) of pumps with identical pump curves.

The probability of fire flow failure is defined as the probability that the available pressure head at node *j* (H_j) is below the minimum required pressure head (H_{fj}) during a fire at node *j*. This is computed by evaluating the cumulative distribution function (CDF) of available pressure head at the minimum required residual pressure head H_{fj}, such that

$$F_{H_j}(H_{fj}) = P[H_j \le H_{fj}] \tag{2}$$

Combining Equations (1) with (2) yields

$$P[-aQ_{fj}^2 - 2bQ_{fj} + c \le H_{fj}] \tag{3}$$

where $a = B/n^2 + \sum_{l=1}^{i} K_l$,

$$b = B/n^2 \sum_{k=1}^{N_n} Q_k + \sum_{l=1,\, m=2}^{i,\, j} K_l \sum_{k=m}^{N_n} Q_k \text{ and}$$

$$c = H_s + A - B/n^2 \left(\sum_{k=1}^{N_n} Q_k \right)^2 - \sum_{l=1,\, m=2}^{i,\, j} K_l \left(\sum_{k=m}^{N_n} Q_k \right)^2$$

Since Q_{fj} is positive always, the CDF of pressure head is analytically solved as

$$\begin{aligned} F_{H_j}(H_{fj}) &= 1 - P\left(Q_{fj} \le \frac{-b + \sqrt{b^2 - a(H_{fj} - c)}}{a} \right) \\ &= 1 - F_{Q_{fj}}\left(\frac{-b + \sqrt{b^2 - a(H_{fj} - c)}}{a} \right) \end{aligned} \tag{4}$$

The CDF of pressure head is used to compute analytically the probability of fire flow failure at node *j* of a branched network. This is possible because the flow and headloss in each pipe depends in a straightforward way on all downstream demands and the fire flow in the system. In looped networks, multiple flow paths are possible and cannot be determined by simple inspection; the analytical approach is thus not yet applied to looped networks.

MULTI-OBJECTIVE OPTIMIZATION FORMULATION

The multi-objective optimization of a branched network under probabilistic fire flow conditions is developed next. The first objective in (5) seeks to minimize the cost of pipes and pumps through the selection of the pipe diameters and the number of pumps as the decision variables. The second objective seeks to minimize the probability of fire flow failure. The fire flow failure probabilities at all fire flow nodes are calculated with (4) and summed together. The summation is normalized by the number of fire flow nodes and called the "average fire flow failure probability" which can range from 0.0 to 1.0. A fire flow probability of zero represents a condition of no fire flow failure; the higher the probability, the higher the likelihood of failure.

$$\text{Minimize } \sum_{i=1}^{N_p} c(D_i) + n \cdot c_p \qquad \text{(Pipe cost + Pump cost)} \tag{5}$$

$$\text{Minimize } \frac{1}{N_n} \sum_{j=1}^{N_n} F_{H_j}(H_{fj}) \qquad \text{(Average Fire Flow Failure Probability)} \tag{6}$$

Subject to:

$$\sum Q_{in} - \sum Q_{out} = Q_j \,,\ j = 1, \ldots, N_n \tag{7}$$

$$D_i \in \{\mathbf{D}\},\ i = 1, \ldots, N_p \tag{8}$$

$$H_j \geq H_{\min j} \,,\ j = 1, \ldots, N_n \tag{9}$$

$$V_i \leq V_i^{\max}, \quad i = 1, \ldots, N_p \tag{10}$$

where D_i = discrete pipe diameters selected from the set of commercially-available pipe sizes $\{\mathbf{D}\}$; $c(D_i)$ = cost of pipe i with diameter D_i; N_p = number of pipes; c_p = cost of single pump; $H_{min\ j}$ = minimum required pressure during maximum hour demand condition. Equation (7) represents the continuity at all $j = 1, \ldots, N_n$ nodes. Equation (9) requires that the nodal pressure H for any node j be equal to or greater than a specified minimum pressure $H_{\min}$ for the maximum hour demand condition. A maximum allowable fluid velocity (e.g., 3 m/s) can be applied through Equation (10) to prevent pipe wall scouring.

CASE STUDY

The multi-objective optimization approach was tested on the branched network illustrated in Figure 2. This network represents an actual water system and consists of 28 pipes, 29 junctions, parallel pumps and one source. This example was originally taken from the EPANET user's manual (Rossman, 1993) and skeletonized into a branch system. The 28 pipes selected for the optimization run were lumped into 13 distinct groups (Table 1) based on similar characteristics (e.g., location) to reduce the number of decision variables (13 pipe groups and a number of parallel pumps). The rated flow, Q, and head gain, hp, of a single pump are set to 20 L/s and 100 m so the pump characteristic curve of the n parallel pumps is defined as $hp = 133 - 0.083(Q/n)^2$. The needed fire flow to control a fire in a single-family residential unit is 32 L/s with a duration of 2 hours (AWWA 1999). Needed fire flow is uncertain and is considered to be normally distributed (without loss of generality) with a mean of 32 L/s and a standard deviation of 8 L/s (coefficient of variation of 25%). It was assumed that the reservoir storage volume is adequately sized for the 2-hour required fire flow duration. The minimum nodal pressure required during a fire flow event is 14 m, the maximum hour demand peaking factor is set to 3.4, and the minimum pressure required at all nodes during maximum day demand and fire condition is also 14 m.

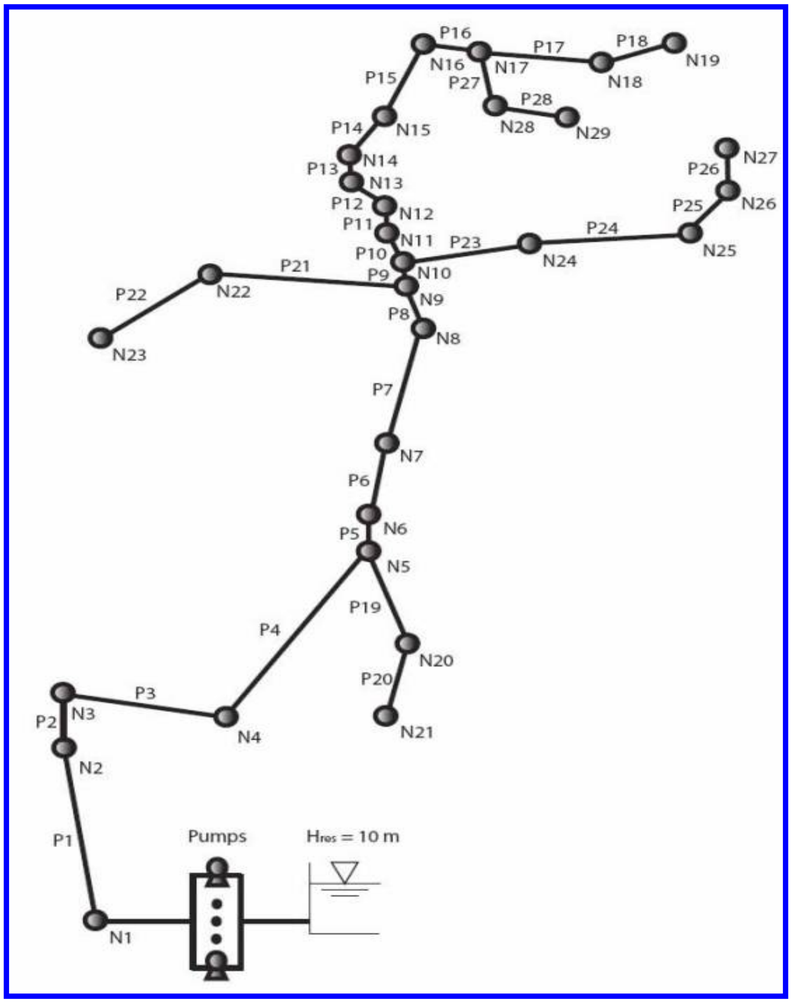

Figure 2. Branch-skeletonized network.

The NSGA-II population size was set to 200 and the number of generations was set to 400. The probability of mutation was set to 0.025, the probability of uniform crossover to 0.9, and the length of each chromosome to 24. Pipe sizes were chosen from the commercially-available diameters with unit pipe costs indicated in Figure 3. Each pump was assumed to cost $5,000. For this problem, 16 pipe sizes for 13 pipe groups and 16 maximum allowed number of pumps make up a solution space of 16^{14} possible combinations. At the beginning of an optimization run, the NSGA-II initializes the population of solutions, where each solution comprises pipe diameters and the number of pumps in the pumping station. The NSGA-II then calculates the cost of pipes and pumps and the fire flow failure probability of each solution as well as the constraint-violation errors to generate a new population in the next generation.

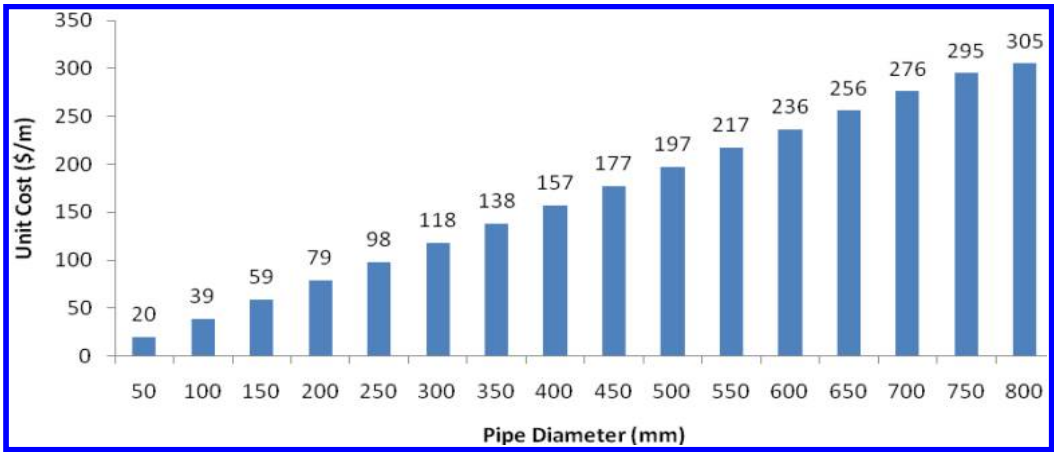

Figure 3. Commercially-available pipe diameters and unit costs

The solutions obtained after 400 generations are indicated in Figure 4. The x-axis plots the total cost of pipes and pumps in Equation (5) and the y-axis plots the fire flow failure probability in Equation (6). Table 1 indicates the optimal pipe diameters, number of pumps, total cost (pipe and pump) and fire flow failure probability for the 8 representative Pareto-optimal solutions indicated in Figure 4. The interaction among the two objectives gives rise to a set of Pareto-optimal solutions. Each solution on the Pareto-optimal front of Figure 4 and Table 1 is not dominated by any other solution. In going from one solution to another, it is not possible to improve on the objective of minimizing the total cost without compromising the other objective of minimizing the fire flow failure probability. This trade-off relationship provides useful information on the cost-effectiveness of adding pipe and/or pump capacity to reduce the probability of fire flow failures. The results in Table 1 indicate that when the total cost is increased from $372,000 to $451,000 (Solution #1 to #4), the 21% additional investment in pipe and pump can achieve a 51% reduction in fire flow failure probability (0.61 to 0.30). Similarly, when the pipe cost is increased from $451,000 to $687,000 (Solution #4 to #8), the 52% additional investment in pipe and pump can reduce the fire flow probability to zero or near-zero. The results in Table 1 also suggest that the optimization approach allocates large pipe diameters (250-300 mm) near the water source to meet maximum hour demand conditions and then adds pipe capacity in downstream locations to reduce the probability of fire flow failures in low-pressure sites located far away from the water source. This interaction between total cost and fire flow failure probability is important for water utilities in making master planning decisions concerning the sizing of distribution mains in new development or redevelopment scenarios. Solutions #7 and #8 also suggest that a minimum 150 mm pipe diameter is required to reduce the fire flow failure probability to a near-zero level. The results support the industry practice of adopting a minimum 150 mm pipe diameter for local distribution mains to provide fire flow protection.

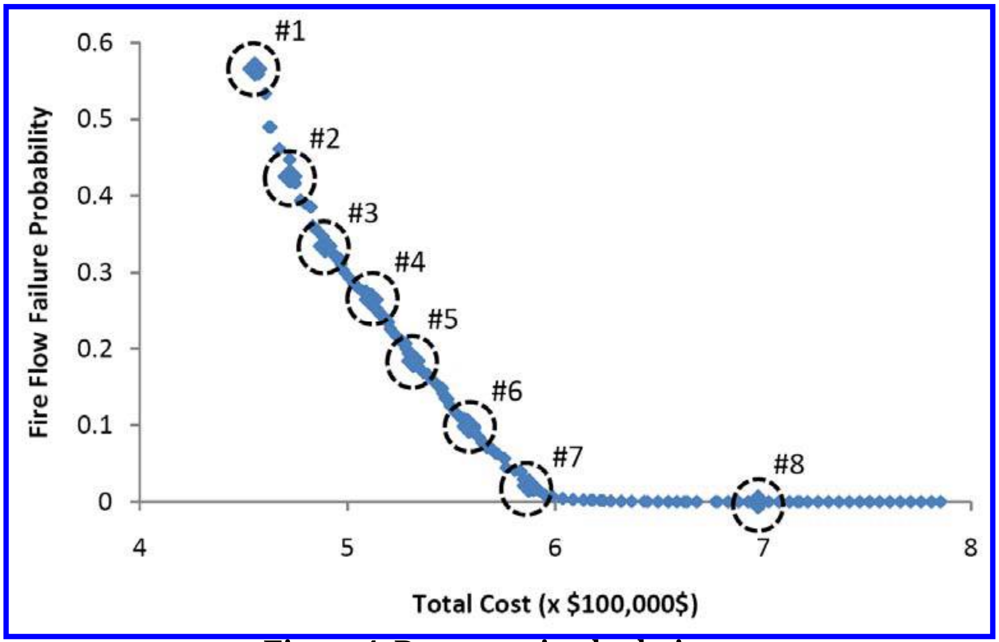

Figure 4. Pareto-optimal solutions.

Table 1. Optimal pipe diameters, number of pumps, total cost (pipe and pump) and fire flow failure probability for 8 Pareto-optimal solutions.

Pipe diameter (mm)	Solutions							
	1	2	3	4	5	6	7	8
P1, P2	250	250	250	250	250	250	250	300
P3, P4	250	250	250	250	250	250	250	250
P5, P6	150	200	200	200	200	200	200	200
P7, P8	150	150	200	200	200	200	200	200
P9	150	150	150	150	200	200	150	200
P10, P11, P12, P13	150	150	150	150	150	150	150	200
P14, P15, P16	150	150	150	150	150	150	150	200
P17, P18	50	50	150	100	150	150	150	200
P19, P20	50	50	50	50	50	50	150	150
P21, P22	100	100	100	100	100	150	150	200
P23, P24	100	100	100	150	150	150	150	200
P25, P26	50	50	50	50	150	150	150	200
P27, P28	50	50	50	100	100	150	150	200
Number of pumps	4	6	4	5	4	4	5	8
Cost (x $100,000)	3.72	3.90	4.19	4.51	4.96	5.32	5.70	6.87
Fire flow failure probability	0.61	0.52	0.41	0.30	0.18	0.08	0.02	0.00

CONCLUSION

This paper develops a multi-objective optimization approach that incorporates the probability of fire flow failure in branched water distribution networks. An analytical probabilistic model quantifies the probability of fire flow failure in branched networks and incorporated into the non-dominated sorting genetic algorithm (NSGA-II). The optimization approach seeks to minimize two conflicting objectives: capital cost and the probability of fire flow failure. Capital cost and fire flow failure probability are balanced through the selection of the pipe diameters and the number of pumps in the system. The optimization program was applied to a 28-pipe skeletonized branched network to generate Pareto-optimal solutions for total cost and fire flow failure probability. The case study demonstrated that fronts of Pareto-optimal solutions provides useful information to decision-makers on the cost-effectiveness of adding pipe and/or pump capacity to reduce fire flow failure probability in the water distribution network. Optimization results supported the industry practice of using a minimum 150 mm size for local distribution mains to provide fire flow protection.

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