Energy and Costs of Leaky Pipes: Toward Comprehensive Picture

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Abstract: Leaky distribution systems are costly in terms of lost water, potentially adverse water quality effects, and the energy consumed in supplying the leaks. To characterize the energy effectiveness of a leaky segment in a single pipe, several dimensionless parameters are analytically derived, which relate the leak size and location to its associated energy burden and water loss. The computer program EPANET is used to simulate the energy costs of leaks on representative distribution networks. In particular, analysis is performed to illustrate the influence of total system demand, leak location, and topological complexity. Furthermore, the connection between water loss and energy costs illustrates the potential importance of energy costs when pipes are leaky. The impact of leaks on water age is also evaluated through simulation and via a dimensionless expression relating leak size and location to residence time.

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Introduction

With high quality water supplies becoming increasingly uncertain, and sustainable energy sources perhaps even more so, there has been growing attention to the problem of unaccounted for water (UFW) and, specifically, to leak detection and control. An obvious issue is the loss of water, which is often expensive to provide and treat. However, leaky pipes are also known to increase pumping energy and system rehabilitation costs and can increase the risk of compromised water quality by allowing intrusion of polluted groundwater. Leaks have been known to undermine roadways by eroding the underlying soil and may even recharge aquifers beneath urban areas at a sufficient rate to pose a risk to building foundations (Price and Reed 1989).

That leaks are costly in terms of money and resources is a well-established idea. One early survey revealed that Chicago was pumping more than twice the water required (Cole 1912), a level still not rare today. A typical range for UFW in Europe is 9–30% (Lai 1991), while rates for Malaysia of 43% (Lai 1991) or for Bangladesh of 56% (Chowdhury et al. 1999) have been reported. In North America, Brothers (2001) suggests that some utilities experience water losses of 20–50%. Leakage is the dominant component of UFW.

Although it has long been acknowledged that leaky distribution systems require more energy in order to maintain desirable service levels, there is a relative absence of literature regarding the energy burden of leaks. Traditionally, leak reduction efforts have focused on reducing the cost of lost water; however, current prices are such that energy costs could be significant for some systems. In many communities, energy consumption by pumps is often the largest component of operating costs for the transmission of water. In addition, the energy wasted in feeding leaks involves an environmental burden related to the many impacts associated with energy production and consumption, including greenhouse gas emissions, acid rain, and resource depletion.

The implicit recognition that important savings can be achieved via improved leak characterization is underscored by the appearance of numerous articles on leak detection and control in recent years (Hunaidi et al. 2000; Vítkovský et al. 2000; Vairavamoorthy and Lumbers 1998). Brothers (2001) recommends that utilities practice “pressure reduction” management in off-peak hours to minimize water loss. Leakage control measures such as excess pressure minimization, while helpful for reducing unnecessary waste, only address the symptoms of the problem. If the externalities associated with leaks were better understood, the impetus to repair and prevent them would likely be greater.

Simple Water Loss and Energy Relations in a Leaky Pipe

Consideration of how leakage increases the energy expenditure of transmitting water through a pipe segment provides a useful departure point for an analysis of leaky networks. When a single leak is concentrated at a fractional distance x along a uniform length of the pipe L, relatively simple equations can be derived that relate energy efficiency to leak location and magnitude. Although elementary, such equations offer a concise description of how leak location and size influence leakage rate and energy requirements. The assumption made throughout this paper is that, whether the system leaks or not, the downstream demands and pressure requirements must be met. Thus, the priority is to evaluate the losses in systems providing an equivalent level of service. Although this approach may not exactly reflect practice in specific communities, this simplified approach, by removing a significant area of variability, greatly facilitates numerical comparisons between different systems and scenarios.
The impact of a leak on energy use is readily ascertained by observing the energy grade line (EGL) in Fig. 1 (Colombo and Karney 2001). The pipe segment has diameter $D$, Darcy-Weisbach friction factor $f$, and a leak situated at $xL$. By assumption, delivery constraints are satisfied if enough water is supplied so that the flow through the leak $Q_l$ is compensated for, and the required flow $Q_d$, is supplied at a prescribed downstream head $H_d$. Thus, the flow upstream of the leak exceeds $Q_d$ by $Q_l$; moreover, the slope of the EGL assumes a discontinuity at $xL$, with the upstream portion following the broken line in Fig. 1. The total head supplied upstream $H_s$ must reflect the modified EGL if pressure at the demand end of the pipe is to be maintained. The leakage flow $Q_l$ can either be expressed as a proportion of demand $aQ_d$ where $a$ is the leakage fraction, or it can be modeled using an orifice function of the form (Rossman 2000)

$$Q_l = C_E A [2g (H_l - H_{gw})]^{a} = C_E H^{a}$$  \hspace{1cm} (1)

where $A =$ leak area; $\Delta H =$ head difference (m) across the leak; $H_l$ and $H_{gw} =$ the heads (m) in the pipe and in the surrounding groundwater, respectively; $C_E =$ discharge coefficient; and $C_E =$ EPANET2’s (Rossman 2000) “emitter coefficient” (in m$^3$/a/s). The emitter exponent $a$ is often assigned a value of 0.5 (the default value used here) to reflect flow through a fixed-size orifice. Clearly, this relation implies that as internal pressure in the pipe builds, $Q_l$ increases and creates a feedback loop that can tax system capacity. An orifice function is usually a more realistic representation than the traditional approach of assigning leaks as fixed demands.

From the orifice expression, it is evident that $a$ and $C_E$ are linearly related; however, the slope that relates them is a nonlinear function of $x$ and the system heads

$$a = C_E (H_l - H_{gw})^{a}/Q_d$$  \hspace{1cm} (2)

$H_l$ is determined from $H_l = H_{gw} + (1 - x) H_f$ in which $H_f$ is the head loss in a pipe without leaks (Fig. 1). If $H_{gw}$ is assumed to be zero (as for unsaturated soil conditions), the resulting expression for the leakage ratio $a/a_0$ can be written as

$$a/a_0 = [1 + (1 - x)h_d]^{a}$$  \hspace{1cm} (3)

where $a_0 = C_E H_{gw}^{a}/Q_d =$ minimum leakage fraction (which occurs when $H_l = H_{gw}$); and $h_d = H_l / H_d =$ relative head loss. Fig. 2 shows how the leakage ratio $a/a_0$ varies with $x$ and $h_f$. Clearly, as the pressure in the pipe decreases, the ratio approaches unity. Thus, as far as water loss is concerned, if a leak must exist, then the downstream end ($x = 1$) of a horizontal pipe (or the point of lowest pressure) is the “best” place to have it. For $x < 1$, $a/a_0$ decreases with decreasing $h_f$, because the pressures at the leak are smaller, thus confirming a common strategy for leakage control.

The Darcy-Weisbach equation $H_f = fLQ^2/2gD^2$ relates the head loss in a leak-free pipe to the flow it conducts. For a pipe with a single leak discharging $aQ_d$ at a point $xL$, the resulting expression for the friction head ratio $h_f$ becomes a linear function of $x$ and a quadratic function of $a$

$$h_f = H_f/H_d = x(1 + a)^{2} + (1 - x) = 1 + ax(a + 2)$$  \hspace{1cm} (4)

Therefore, as $x$ decreases, the additional head loss imposed by the leak also decreases because a greater portion of the pipe segment carries only the design flow. However, if the orifice relation of Eq. (3) is substituted into Eq. (4), the friction head ratio becomes a more complex function of distance, orifice properties, and the relative head loss $h_f$.

The difference between the mechanical energy delivered to the downstream end of the conduit ($E_d$) and that supplied at the source ($E_s$) indicates the energy consumed in feeding the leak. Expressing these energy terms as a dimensionless quantity allows for quick assessment of the energy effectiveness of the leaky pipe. Because overall energy efficiency depends on the supply efficiency (e.g., pump efficiency), an empirical efficiency factor $\eta$ can also be incorporated into the final energy expression

$$E_d/\eta E_s = \eta Q_d H_d = \frac{\eta}{\gamma Q_d (1 + a)(H_d + H_f)} = \frac{\eta}{(1 + a)[1 + (x a(a + 2) + 1)h]}$$  \hspace{1cm} (5)

The extension of Eqs. (4) and (5) to a multileak case, along with a brief discussion of equivalent leak representation, is provided in the Appendix. Fig. 3 shows the response of $E_d/\eta E_s$ to changes in

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**Fig. 1.** Energy grade line (EGL) of a leaky pipe segment

**Fig. 2.** Relative leakage as a function of relative head loss and leak location

**Fig. 3.** Energy ratio as a function of leak location and magnitude ($\eta = 1$, $h_f = 0.5$)
Table 1. Topology of Hypothetical 10-Loop System

<table>
<thead>
<tr>
<th>Pipe Number</th>
<th>Length (m)</th>
<th>Diameter (mm)</th>
<th>C</th>
<th>Node number</th>
<th>Elevation (m)</th>
<th>Demand (MLd)</th>
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leakage fraction for three different values of the fractional distance \( x \). For smaller leakage fractions, the energy ratio changes only slightly with \( x \); however, as \( a \) becomes larger, the dependence upon \( x \) is more noticeable. The descent rates are relatively steep, reflecting the importance of leak size to energy efficiency.

Energy Costs of a Leaky Network

As insightful as the analyses of single pipe systems are, networks generally defy analytical treatment. In order to evaluate the impact of leaks on distribution systems, a variety of steady state EPANET (Rossman 2000) simulations were performed on four hypothetical looped networks. The goal is to find simple relationships that might characterize, at least in a rough way, the interdependence of leakage rate, energy costs, and system complexity.

Hypothetical 10-Loop Distribution System

The topology of the hypothetical 10-loop system is described in Table 1 and in Fig. 4. Some aspects that distinguish this system from a more realistic distribution network are the absence of storage (there are no tanks or reservoirs other than the source reservoir), a fixed demand pattern, and the existence of only one pumping station. Ignoring both demand pattern and storage simplifies analysis and more clearly highlights the specific role of leaks; moreover, because average conditions dominate in the estimation of long-term energy consumption, their omission is not especially problematic. Naturally, a variety of additional operational considerations will also come into play when determining how leakage is managed and how it influences the overall economic performance of a real system.

Leaks at specific nodes are represented in EPANET using emitters that are governed by the orifice relationship of Eq. (1). A leak at a particular node represents the existence of leaks in some or all of the incident pipes, thus extending the equivalent leak concept in the Appendix. For this system, leaks have been defined at nodes N5-8, 10, 11, and each leak is assigned the same value of \( C_E \). The leakage is then determined by assigning a new pump curve so that the resulting pressure distribution closely resembles the pressure distribution of the no leak scenario. Specifically, the pump curve is modified until the pressure at the most downstream node, N16, is nearly equal to its original "no-leak" value (i.e., 35 ± 0.1 m). In this way, the system may be considered as "pressure compensated." The daily energy cost is calculated by EPANET.

Fig. 4 compares the nodal pressures both with no leaks and with 25% leakage (pressure values in parentheses). Without any leaks, the total flow through the network is equal to the total demand of 24 MLd and all nodal pressures are at least 35 m. When the leaks at the specified nodes have emitter coefficients associated with a 25% leakage scenario, and the original pump station curve still applies, the total flow through the system increases to 28.5 MLd. All flow requirements are still satisfied, but pressures do fall significantly. Although satisfaction of nodal demands is a typical modeling requirement, Germanopoulos (1985) correctly indicates that this assumption may not be realistic when system pressures drop too low. The feedback effect of the orifice relationship of Eq. (1) is apparent when the pump curve is adjusted to restore pressures and, thus, service conditions. In compensating for the leaks, the magnitude of the losses increases so that the total system flow becomes 30 MLd; the extra 1.5 MLd represents the pressure-dependent demand exerted by the leaks.

Role of System Demand and Orifice Hydraulics

The orifice function of Eq. (1) is defined by two parameters—the emitter coefficient \( C_E \) and the emitter exponent \( \alpha \). \( C_E \) generally reflects the size and shape of a leak and is often adjusted when modeling leaks of different magnitudes. Although the value of \( \alpha \) is usually set at 0.5, other values have been suggested. For example, Goodwin (1980) reports an exponent value of 1.18, with the higher value suggesting an "elasticity" factor that describes how a leak's effective area responds to internal pipe pressure. The emitter exponent may also reflect the flow regime through the
leakage volumes are associated with the same approaches unity, the rate of this decrease diminishes and higher

\[ \frac{dQ_t}{dh} = \alpha C_E h^{\alpha - 1} \]  

(6)

For \( \alpha < 1 \), \( Q_t \) increases with \( h \) at a decreasing rate; however, as \( \alpha \) approaches unity, the rate of this decrease diminishes and higher leakage volumes are associated with the same \( K_D \). When \( \alpha = 1 \), \( Q_t \) is linearly related to \( h \) and the associated curve in Fig. 5 would be flat; that is, \( L_k \) would be independent of \( K_D \). All three curves descend at a decreasing rate for large \( K_D \). The inverse relationship exists because the increase in total system demand outpaces the increase in leaked volume as \( K_D \) grows. The higher pressures associated with larger \( K_D \) in the pressure compensated system lead to greater leakage volumes, and the rate of descent of each curve is reduced. It should be noted that the increased pumping required to meet higher flows causes excessively high pressures in the upstream portion of the system. Ordinarily, the system capacity would be upgraded to avoid this, and the current analysis is unrealistic in this sense.

The role of \( C_E \) can be assessed by comparing curves A and B, which represent leaks with the same exponent value (\( \alpha = 0.5 \)) but different values of emitter coefficient. The relative position of these curves exposes the essential linear relationship between \( L_k \) and \( C_E \) when \( \alpha, K_D \) and pressures are held constant. For example, at \( K_D = 0.8 \), a move from A to B represents an increase of 50% in \( C_E \) from 0.1 to 0.15, which also corresponds to a 50% increase in \( L_k \) from 20 to 30%.

### Relevance of Leak Location

Analysis of the single leaky pipe indicated that leak location affects energy consumption in that, as the specified leak is moved further downstream, its impact is more dramatically felt. Although direct extrapolation of this analytical result is not feasible for distribution networks, it is logical to expect that leaks situated at the most downstream portions of a network will often involve a larger energy cost, because the larger flows must be transmitted through a greater portion of the system. A rudimentary analysis was performed in which a single leak with \( C_E = 0.2 \) MLD/m^1/2 was placed at four different nodes (N4, 5, 11, and 16) of the hypothetical network, and the resulting pressure distributions were compared to the no-leak case for the average day regime. Table 2 shows the percent reduction in pressure of each node, relative to the no-leak case, for a single leak at each of the four test nodes. The results, though significant, are not surprising—a leak present at any node causes every node to respond. However, the nodes most severely affected are those adjacent to, or in the vicinity of, the leaky node. Moreover, leaks at “downstream” nodes like N11 and N16 also cause a greater degree of pressure reduction, both in terms of the magnitude of reduction and the number of nodes with reductions over a given quantity.

### System Complexity and Energy Cost Response

How the complexity in a system influences the relationship between leakage and cost is an interesting question for which there

<table>
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<th>Pressure (m)</th>
<th>Percent Reduction in Nodal Pressure (%)</th>
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</thead>
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<td>1</td>
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<tr>
<td>16</td>
<td>35.3</td>
<td>4.8</td>
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### Table 2. Pressure Reduction Due to a Single Leak at Selected Nodes


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is no comprehensive answer. Nonetheless, simulations of systems with different degrees of complexity can provide an idea of the influence of system structure on the energy costs of leaky systems.

The number of loops is used here as a crude indicator of a system’s complexity. Four different systems are compared in Fig. 6. These include the 10-loop system already discussed, modified forms of the 20-loop network provided in Walski et al. (1987) and of the 2 loop system in the EPANET 2 User’s Manual (Rossman 2000) tutorial, and a single pipe (a “zero-loop” system). The chief modification to the Walski and EPANET tutorial systems is the elimination of tanks and an exclusive focus on average conditions. The single pipe ($L = 4$ km, $D = 300$ mm, and $C = 100$) simulations were based on a downstream demand of 30 l/s at 40 m pressure and a single leak at the demand node. The Walski network (Fig. 7) was simulated with two different leak distributions. The system referred to as Walski #1 was assigned leaks at the central nodes N60, 80, 90, 100, 150, and 160. For Walski #2, several of the leaks were relocated to the periphery of the network (N55, 90, 120, 140, 150, and 170). All leaks had the same $C_e$ and simulations were conducted according to the methodology described earlier.

A family of second-order polynomials of the form $ay^2 + by + c$ were fit to the curves in Fig. 6 with excellent accuracy. For example, the energy cost curve of the 10-loop system is well described by the function $z = 0.012y + 1.03y$, where $z$ and $y$ are percent increases in energy cost and leakage, respectively. Despite each curve following the shape of a quadratic function, there is no simple “rule of thumb” for relating energy cost to leakage for water distribution systems. What is clear, however, is that leaks are definitely costly with all curves well above the 1:1 datum despite their obvious differences; the relative increase in energy costs can be expected to significantly exceed the associated leakage rate. The effect of leak distribution in a network is evident from the disparity between the two Walski curves.

**Leaks and Water Quality**

Another important aspect of leaks is their influence on water quality. Of the many possible impacts that might be considered, two are discussed here. One is the negative impact of leaks as entry points for potentially contaminated groundwater, pathogens, and soil constituents when a pipe experiences a hydraulic transient. The other, the largely positive role leaks play in reducing water age, is discussed first.

**Water Age**

Water age is a popular indicator of the general water quality in a distribution system. Like all surrogate parameters, it must be viewed in the light of its limitations. Theoretically, if there were no exchange of matter with the outside environment, interaction with pipe material or breakthrough from the treatment plant, it would not matter if the residence time in the system were 10 hours or 10 years. Of course, these assumptions are invalid, and it is well documented that tuberculation, disinfectant residual decay and biofilm interactions deteriorate quality within distribution systems over time. Although comprehensive relationships between residence time and water quality transformations are difficult to establish, particularly as they depend on the specific properties of individual systems, the general correlation between water age and degradation is well accepted.

A key factor affecting water age is velocity. LeChevallier (1990) indicates that higher velocities influence water quality by promoting greater transport of disinfectants throughout the system and increasing the likelihood of biofilm detachment due to higher shear stresses. In essence, higher velocities provide a more or less continuous “flushing” of the pipes. Donlan and Pipes (1988) found an inverse relationship between maximum velocity and heterotrophic plate count (HPC) on cast iron test cylinders exposed to drinking water at several sites within a distribution system. Elton et al. (1995) plotted the number of customer taste complaints on the distribution system map of Gloucester, England and found a clear correlation between customer dissatisfaction and water age extremes. In general, lower water age is preferable from a water quality standpoint.

The effect of leaks on residence time is easily demonstrated by revisiting the leaky pipe of Fig. 1. The total residence time $t$ for water travelling the full length of the conduit is the sum of the advection times on either side of the leak. These times differ according to the flow and the location of the leak $xL$. The sum of residence times gives

$$t = \frac{xL}{(1+a)Q_d/A} + \frac{(1-x)L}{(1+a)Q_d/A}$$

(7)

which can be nondimensionalized to yield an expression for the relative water age

$$\frac{t}{t_o} = \frac{1+a(1-x)}{1+a}$$

(8)

where $t_o = AL/Q_d$ is residence time when no leak is present.
The relative water age is the ratio of the leaky-pipe advection time \( t \) to the advection time \( t_0 \) when the pipe has no leak. Fig. 8 shows how \( t/t_0 \) varies with the dimensionless parameters \( a \) and \( x \). The value of \( t/t_0 \) is greater than 0 but does not exceed 1, i.e., no leak when \( a = 0 \) for any combination of values for \( a \) and \( x \) and decreases with increasing leak size (larger \( a \)) and downstream location (larger \( x \)). As \( a \) increases, the sensitivity of \( t/t_0 \) to \( x \) also increases.

Derivation of a concise expression to describe the effect of leakage on water age for a looped network is not possible. Boulos et al. (1992) presents a general algorithm for calculating the water age in a multisource nonleaky network that could be adapted to account for leaks. Programs are well suited to calculate water ages for a network and, thus, EPANET was used to conduct a water age analysis for the 10-loop base network. Both average day (24 MLD) and minimum day (demand multiplier 0.6) demand regimes were considered, and for each regime, the leaks were assigned \( C_E \) values of 0.1 and 0.15 MLD/m\(^{1/2}\). The pressure distribution for minimum day is necessarily different from the average day, and a more relaxed criterion of 30 m pressure head at the node with the lowest pressure was used in order facilitate comparison. Table 3 shows the percentage reduction in the water age at each node as a result of two different leakage amounts for each demand scenario. Fig. 9 shows the water age at nodes 1, 5, 8, 11, and 16 when \( C_E = 0, 0.1 \) and 0.15 for the minimum day scenario.

Not surprisingly, several of the leaky nodes (N5–8) exhibit some of the largest reductions in water age. The tendency of leaks to decrease residence times is clearly evident for both scenarios in Table 3. Of particular interest is the discrepancy between demand regimes. For a given \( C_E \), the leakage is greater during the minimum day demand period, because the lost volume of water comprises a larger share of total demand (14.4 MLD) and, as a result, the proportional impact of leaks is more significant than for average day conditions. This is the same type of relationship depicted in Fig. 5 between the average \( (K_D = 1) \) and maximum day \( (K_D = 1.5) \) scenarios. Leakage figures are typically based on average day conditions; yet, a 17% average day leakage is associated with roughly 28% leakage during the minimum day period. Overall, leaks reduce water age and may entail a water quality benefit that becomes more obvious during periods of reduced flows. This is somewhat analogous to the difficulty encountered with air quality in air-tight buildings and their need for greater ventilation.

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**Table 3. Water Age Analysis for 10-Loop Network for Average and Minimum Day Demand**

| Node Number | **AVERAGE DAY** | | | **MINIMUM DAY** | | |
|-------------|----------------|---|---|----------------|---|
|              | Age (hrs)       | % Reduction in Age | | Age (hrs) | % Reduction in Age |
|              | No leak | \( C_E = 0.1 \) | \( C_E = 0.15 \) | No leak | \( C_E = 0.1 \) | \( C_E = 0.15 \) |
| 1            | 0.22   | 18.6 | 25.3 | 0.37   | 28.2 | 37.4 |
| 2            | 0.61   | 13.0 | 17.6 | 1.02   | 20.0 | 42.7 |
| 3            | 0.05   | 12.5 | 18.8 | 0.08   | 21.3 | 28.8 |
| 4            | 0.22   | 18.9 | 25.7 | 0.37   | 28.1 | 37.3 |
| 5            | 0.31   | 20.3 | 27.4 | 0.52   | 30.2 | 39.5 |
| 6            | 0.51   | 19.9 | 26.6 | 0.85   | 27.3 | 25.5 |
| 7            | 0.43   | 16.4 | 17.4 | 0.72   | 29.9 | 39.1 |
| 8            | 0.26   | 14.1 | 20.2 | 0.43   | 19.5 | 27.2 |
| 9            | 0.51   | 11.1 | 15.6 | 0.85   | 22.1 | 30.4 |
| 10           | 0.96   | 6.3  | 8.2  | 1.34   | 18.0 | 24.7 |
| 11           | 0.53   | 10.2 | 14.6 | 1.61   | 8.6  | 13.6 |
| 12           | 0.74   | 11.2 | 15.7 | 0.88   | 16.6 | 23.2 |
| 13           | 0.74   | 11.2 | 15.7 | 1.24   | 18.1 | 25.1 |
| 14           | 0.91   | 11.0 | 15.6 | 1.52   | 18.2 | 25.4 |
| 15           | 1.71   | 5.0  | 11.3 | 2.86   | 8.4  | 13.4 |

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**Fig. 8.** Relative water age as a function of leak location and magnitude

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**Fig. 9.** Water age at selected nodes for minimum day demands

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Leaks may entail some benefit in reducing water age, but they also create a serious water quality risk under transient conditions. In essence, hydraulic transients are pressure waves that propagate in the pipe system as a response to relatively rapid flow adjustments. Transients have the potential to burst a pipe (high pressure) or to cause regions of low pressure or vacuum conditions. Specifically, leaks may enhance the likelihood that foreign matter is drawn into a pipe when a low-pressure transient event occurs. Matter drawn into the pipe might include potentially toxic pollutants, pathogens, and soil constituents. Pathogens pose a direct health risk by increasing the likelihood of waterborne disease and certain soil compounds, though not directly toxic, may act as disinfectant byproduct precursors. The extent of transient intrusion depends on the severity and duration of internal pressure changes, the external groundwater pressure, and the orifice parameters of the leak. Funk et al. (1999) defines an intrusion potential factor that incorporates these elements and is a function of the head difference across the leak. Because the leak is a two-way orifice, an intrusion volume, determined simply from the head gradient across the leak, could be misleading unless contaminant concentrations are considered in light of this bidirectionality.

There may be several mitigating factors in the intrusion scenario. The water in the immediate vicinity of the leak may be of superior quality relative to the surrounding groundwater, as it is likely to be treated drinking water that originally leaked from the pipe. Transient phenomena often involve some kind of oscillation between high and low pressures. When a sudden outflow of water occurs due to the high internal pressures of a water hammer, the immediate area around the leak is “flooded” with treated water.

Fernandes and Karney (2001), using a two-way orifice and a tank as a boundary condition, showed that much of the water that is drawn into the pipe may be immediately released as a result of the same transient event. Of course, there is the question of the time period for water exchange. Water that has escaped when the internal pipe pressure was high (or due to leaks under steady state conditions) may have sufficient time while outside the pipe to solubilize contaminants or mix with ambient groundwater that already contains impurities. When this water is returned to the pipe, it may be thought of as the same water, except with possibly diminished quality.

The transient intrusion phenomenon is worthy of further investigation. A better understanding of intrusion potential and volumes, as well as the role of water exchange for mitigating adverse intrusion effects, is required so that this process can be accounted for within the context of broader pipe network modelling.

Cost of Leaks in a Broader Perspective

Fig. 10 depicts three conceptual maintenance cost curves of a pipe (or portion of a distribution system) adapted from Kleiner et al. (1998). The “assumed” curve represents the increase in annual maintenance cost for a pipe due to the general degradation of its capacity with age (i.e., lower C values). Because this curve does not account for leaks, a pipe’s service life may be erroneously overestimated when based on it. The presence of leaks implies that annual maintenance costs are higher than anticipated and will cross the replacement cost threshold earlier than planned. The threshold replacement cost serves as a criterion by which the decision to replace a pipe is made. The time gap between the transgression of this threshold and the design service life constitutes the “delay period” during which costs run over budget. If the onset of this period could be estimated, leak repair or other rehabilitation measures might be implemented to minimize the extra cost. Conversely, the pipe represented by the “improved” curve is initially more expensive due to better fabrication and materials choices, but also more leak resistant. A pipe that conforms to it may offer planners surplus time in which to recoup the extra capital costs associated with its manufacture and application. The opportunity to save money during the pipe’s extended service life implies that funds can be allocated to repair or replace other pipes and infrastructure. Improved leak characterization and cost assessment will help determine the nature of the improved curve.

The difference in cost between the “actual” and “assumed” curves of Fig. 10 comprises both lost water and energy costs. Although lost water costs have dominated concern regarding leak expense, consideration of the trade-off between water and energy costs has tended to be ignored. The extra daily cost of system operation due to lost water through a leaky pipe $P_w$ can be computed from $P_w = \frac{3600}{1000} \frac{Q_w T}{7300}$ as

$$P_w = k_w C E [H_d + (1 - x)H_f] \times 3600 T$$  \hspace{1cm} (9)

where $k_w =$ unit price of water in $/m^3$; and $T =$ analysis duration (i.e., 24 hours). The extra daily energy cost $P_E$ is the product of the unit price of electricity ($/kWh$), the difference in supply energy $\Delta E$ between the leak and no-leak cases, and the analysis duration $T$ and is given simply as $P_E = k_E \Delta E T$. From Eqs. (4) and (5) the expression is resolved into system heads and the non-dimensional parameters $a$ and $x$

$$P_E = k_E Q_d [(1 + a) (H_d + H_f) - (H_d + H_f)] T$$  \hspace{1cm} (10)

$$= k_E Q_d [(1 + a) [H_d + H_f [1 + a x (a + 2)]] - (H_d + H_f)] T$$  \hspace{1cm} (11)

where $a$ is computed according to Eq. (2).

These equations were tested for a typical water main ($L = 2$ km, $D = 254$ mm) with demand constraints $Q_d = 0.07 m^3/s$ and $H_d = 25 m$ and leak parameters $C_E = 0.001 m^{0.5}/s$ and $\alpha = 0.5$. Fig. 11 plots the extra daily water and energy costs as a function of leak location $x$ when the pipe is assigned Hazen-Williams roughness coefficients of $C = 130$ (dashed curves) and $C = 80$ (solid curves). These $C$ values reflect the tendency for pipes to leak when they are new (due to manufacture and/or installation defects) and when they are old, and they are included to illustrate how leakage costs are distributed between lost water and wasted energy according to the pipe’s friction characteristics. Unit prices for water and electricity are loosely based on summer 2001 prices in the City of Toronto. A
price of $0.10/kWh is chosen for electricity. Residential customers typically pay about $0.50/m³ for water, and, therefore, the City pays a fraction of this amount as its marginal cost for a cubic meter of water. A value of $0.05/m³ is chosen in order that the extra daily costs of the pipe due to lost water and energy are commensurate. Fig. 11 shows that, at these prices, extra daily water costs dominate those for energy at all possible leak locations.

The role of leak location is clearly evident in Fig. 11 and exhibits the relationships depicted in Figs. 2 and 3. As \( x \) increases, energy costs follow while water costs decline. When \( C \) is changed from 130 to 80, both water and energy costs increase at all values of \( x \) (except at \( x = 1 \) where water costs are equivalent because \( H_i = H_d \)); however, their associated curves approach each other more quickly. These changes reflect the fact that rougher pipes dissipate energy (and, thus, pressure) more effectively as water travels downstream. As a result, leakage decreases more quickly with \( x \) while supply energy must be boosted to overcome greater friction. The relative flatness of the \( C = 130 \) energy cost curve reflects the compensatory nature of leakage costs when a leak is modelled using an orifice. Close to the upstream end of the pipe pressures are higher and, therefore, more water is lost. Energy consumption is lower here, because most of the pipe carries only the design flow. Near to the downstream end, pressures are lower and less water is lost; however, most of the pipe carries a larger flow, and thus, friction losses are greater. Overall, more of the total extra operating cost due to the leak is comprised of energy wastage for a rougher pipe. In fact, for this basic example, when the pipe is assigned \( C = 80 \) and the leak is located in the middle of the pipe (i.e., \( x = 0.5 \)), the yearly cost of lost water is about $10,400, while the cost of wasted energy is approximately $5,900. Consequently, even if the marginal cost of water is higher, the leak is still expensive from an energy perspective alone. Clearly, from a financial perspective, the relative importance of water or energy costs depends on their relative prices.

Although the results of the paper need to be placed in a broader economic context before changes in operational procedure can be prescribed, the preceding discussion illustrates that the motivation for leak repair certainly exists. Locating and repairing leaks requires both monetary expense and the commission of resources. Consequently, planners must make decisions regarding which leaks to repair based on a variety of other concerns. Walski (1993) indicates that water distribution systems must satisfy many objectives, some of which compete against energy use minimization, and that there are several ways (other than repairing leaks) in which to save on pump energy costs. While such tactics as improving pump efficiency and taking advantage of time-of-day pricing are effective for reducing the financial burden of leaks, they do not fully eliminate either their environmental burden or the opportunity cost of failing to fix them. Operational approaches such as maintaining lower tank levels or minimizing excess pressures (especially in off peak hours) can also reduce the amount of water and energy wastage, although their application may be impractical.

Despite the obvious benefits of repairing leaks, it is nonetheless interesting to briefly consider some of the “hidden” costs of leak repair. One such “benefit” is diminished water age during periods of reduced flows. Another is the possible attenuation of hydraulic transients, because leaks can provide a means of dissipating excess pressures in much the same way as pressure relief valves. Consequently, leaky systems may experience fewer crippling pipe breaks and other damage when experiencing transients. Thus, an interesting trade-off between leaks and pipe breaks could exist. Although transient analysis is beyond the scope of this paper, it is obvious that the impact of leaks on the performance and economics of distribution networks is multifaceted.

**Conclusions**

Leaks are expensive for a variety of reasons, including the loss of water and treatment chemicals, the increased risk of water quality deterioration, unnecessary capacity expansion, and the increased energy expenditure required to feed the leaks. Given current typical prices, lost water costs upstage those associated with energy wastage. Moreover, if either water or energy prices continue to rise, the importance of leak repair will become even more pronounced. For both pipe segments and distribution networks, leaks are shown to substantially increase energy costs. These costs depend on a variety of factors including demand regime, the spatial distribution of leakage, and system complexity. In general, percentage increase in energy cost appears to be a second-order polynomial function of leakage. Although system topology has an impact on energy response, no simple “rule of thumb” seems to account for it.

The externalities associated with leaks are several and varied. Although the overall impact of leaks on water quality appears to be negative, leaks do reduce water age, especially during periods of reduced demand and, by acting as relief mechanisms, may mitigate pressures during transient events. A comprehensive picture of the role of leaks should consider these interesting attributes so that a prioritization scheme may be introduced into leak detection and repair strategies.

Leaks, and the reasons for controlling them, are not new issues. The ubiquity of pipes in contemporary infrastructure is such that leaks exist virtually everywhere and perhaps in a greater number than actually realized. A recently reprinted statement, originally made over a century ago, is still to the point: “There is no water-supply in which some unnecessary waste does not exist, and there are few supplies, if any, in which the saving of a substantial proportion of that waste would not bring pecuniary advantage to the Water Authority” (Hope 1996).

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Appendix: Formulations for a Pipe with Multiple Leaks

Energy Ratio for a Single Pipe with Multiple Leaks

When \(N\) leaks are present, the EGL is resolved into \(N+1\) distinct segments reflecting the same number of different flows passing through the pipe. Downstream of each leak, the flow decreases until, after the last leak, it becomes equal to the design flow \(Q_d\). Consistent with declining flow is a reduction in the EGL slope (i.e., unit head loss) after each successive leak until the slope for the pipe segment between the last leak and the downstream end is equal to that for the overall segment \(H_f/L\) when no leak exists. The friction head ratio \(h_f\) of a pipe segment with multiple leaks is obtained by extending the single leak relationship.

The relative head loss for the single leak is given by Eq. (4); the first term in Eq. (4) can be expanded into a series of \(N\) terms to account for the \(N\) leaks

\[
h_f = H_f' / H_f = \sum_{m=1}^{N} \left( x_m - x_m-1 \right) \left( 1 + \sum_{i=m}^{N} a_i \right)^2 + (1 - x_N)
\]

(12)

where \(x_m\) = fractional distance from the supply end to the \(m\)th leak, \(a_i\) = leakage fraction of the \(i\)th leak; and \(x_e = 0\). The energy ratio \(E_d/E_s\) for a single pipe with multiple leaks is then easily determined by making the appropriate adjustments to the corresponding single leak relationship

\[
E_d/E_s = \eta \left( 1 + \sum_{m=1}^{N} a_m \right) (H_d + H_f')
\]

(13)

Equivalent Leak Concept

In practice, quality information concerning the number and severity of leaks is difficult to acquire. The fact that most water distribution pipes are buried is the most obvious reason. In addition, uncertainty regarding true demand (i.e., actual \(Q_d\)) exists despite vastly improved water accounting procedures and technologies over the past few decades. Although it is easier to assume a single concentrated leak for a pipe segment, there may be more than one leak, all with different properties, along the conduit.

A pipe segment with two leaks and their associated equivalent leak is presented in Fig. 12. Leaks 1 and 2, which are located at \(x_L\) and \(x_2\), respectively, are responsible for a total loss of \((a_1 + a_2)Q_d\). The equivalent leak that represents them must satisfy two criteria: (1) water loss equivalence; and (2) energy equivalence. The first criterion requires that the leakage fraction of the equivalent leak \(a_e\) is equal to the sum of the leakage fractions of both leaks: \(a_e = a_1 + a_2\). Energy equivalence requires that the total head loss \(H_f'\) is the same regardless of which leak representation (EGL path) is chosen. The presence of the two leaks implies that the EGL follows the path defined by ABDE. The \(H_f'\) experienced is the sum of the individual loss terms: \(H_f' = H_{AB} + H_{BD} + H_{DE}\). This is also equal to the sum of the head loss terms of the two segments \((H_{AC} + H_{CE})\) associated with the single equivalent leak (path ACE). \(H_{CE}\) is determined from the Darcy-Weisbach equation using the design flow \(Q_d\) applied over the reach \((1 - x_e)L\), where \(x_e\) is the fractional location of the equivalent leak. \(H_{AC}\) is evaluated similarly except with the larger flow of \((1 + a_1 + a_2)Q_d\) applied over the reach \(x_eL\). If one has knowledge of the leakage fractions and the friction head ratio, \(h_f x_e\) can be determined as

\[
x_e = \frac{h_f - 1}{(1 + a_e)^2 - 1}
\]

(14)

Thus, the equivalent leak associated with leaks 1 and 2 has a magnitude of \(a_e\) and is located at \(x_e\). The same approach can be used to derive an expression for the emitter coefficient and \(x_e\) of an equivalent leak given knowledge of the emitter coefficients for the original leaks and the relationship between \(a\) and \(C_E\). The resulting expression is slightly more complex, but fundamentally the same.

Notation

The following symbols are used in this paper:

- \(A\) = cross-sectional area of pipe;
- \(a\) = leakage fraction;
- \(a/a_o\) = leakage ratio;
- \(a_e\) = leakage fraction for equivalent leak;
- \(a_i\) = leakage fraction for leak \(i\);
- \(C\) = Hazen-Williams roughness coefficient;
- \(C_d\) = discharge coefficient of orifice function;
- \(C_E\) = emitter coefficient (m\(^{5/2}\)/s or ML/d/m\(^{1/2}\));
- \(D\) = pipe diameter (m);
- \(E_d\) = flow energy received at the downstream end of the pipe (kWh);
- \(E_s\) = energy ratio;
- \(E_x\) = supply energy (kWh);
- \(H_d\) = demand head (m);
- \(H_f\) = head loss due to friction when no leak is present (m);
- \(H_f'\) = head loss due to friction when one or more leaks are present (m);
- \(h_f\) = friction head ratio;
- \(h_f\) = relative head loss;
- \(K_D\) = demand multiplier;
- \(k_w\) = unit price of water ($/m\(^3\)));
- \(L\) = length of pipe (m);
- \(L_k\) = leakage (%);
- \(m\) = current leak number;
- \(N\) = total number of leaks along a pipe segment;
- \(P_E\) = extra daily energy costs ($);
- \(P_W\) = daily cost of lost water;
- \(Q_d\) = demand flow (m\(^3\)/s);
- \(Q_l\) = flow through leak (m\(^3\)/s);
- \(T\) = analysis duration (hr);
\[ t = \text{residence time for a leaky pipe (s)}; \]
\[ t_p = \text{residence time for a pipe without leaks (s)}; \]
\[ x = \text{fractional location of leak}; \]
\[ x_m = \text{fractional location of leak m}; \]
\[ x_N = \text{fractional location of most downstream leak (leak N)}; \]
\[ y = \text{percent leakage}; \]
\[ z = \text{percent increase in energy costs}; \]
\[ \alpha = \text{emitter exponent (usually 0.5)}; \]
\[ \gamma = \text{specific weight of water (9.81 KN/m}^3); \]
\[ \Delta E_s = \text{difference in supply energy between leak and no-leak cases (kWh)}; \]
\[ \eta = \text{wire-to-water pump efficiency}. \]

References


