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Charts for water hammer in pipelines resulting from valve closure from full opening only

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Maximum pressure head rises, which result from total closure of the valve from an initially fully open position, are calculated and plotted for the valve end and for the midpoint of a simple pipeline. Uniform, equal-percentage, optimum, and parabolic closure arrangements are analysed. Basic parameters such as pipeline constant, relative closure time, and pipe wall friction are considered with closures from full valve opening only. The results of this paper can be used to draw the maximum hydraulic grade line along the pipe with good accuracy for the closure arrangements considered. It is found that the equal-percentage closure arrangement yields consistently less pressure head rise than does the parabolic closure arrangement. Further, the optimum closure arrangement yields consistently less head rise than the equal-percentage one. Uniform closure produces pressure head rise that usually lies between those produced by the parabolic and the equal-percentage closure arrangements, except for the range of low pressure head rise combined with low or zero friction, where the rise due to uniform closure approaches that produced by optimum closure.

L'augmentation maximale de pression résultant de la fermeture totale de la vanne, initialement complètement ouverte, est calculée et mise en graphique pour les points situés à l'extrémité, c'est-à-dire à la vanne, et au milieu d'une conduite simple. L'analyse porte sur différentes conditions de fermeture: uniforme, à pourcentage égal, optimale et parabolique. Les paramètres fondamentaux tels que la constante de la conduite, le temps relatif de fermeture et le frottement à la paroi sont considérés pour des fermetures produites dans le seul cas d'une vanne qui était d'abord complètement ouverte. Les résultats de cette publication peuvent être utilisés pour tracer la ligne piézométrique maximale le long de la conduite, pour une fermeture donnée, avec une précision considérée comme bonne. On trouve que la fermeture à pourcentage égal amène une augmentation de pression nettement moindre que la fermeture parabolique. De plus, la condition optimale de fermeture amène une augmentation de pression nettement moindre que celle à pourcentage égal. La fermeture uniforme produit une augmentation de pression qui se situe entre celle produite par la fermeture parabolique et celle à pourcentage égal, sauf dans la zone de faible augmentation de pression alliée à un frottement faible ou nul, où l'augmentation due à une fermeture uniforme se rapproche de celle due à une fermeture optimale.

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Introduction

In a given pipeline the pressure changes depend primarily on the water velocity, valve closure time, and closure arrangement. For many installations the provision for rapid flow shutoff is of particular importance, especially in emergency conditions. These cases require a short valve closure time and therefore the closure arrangement has great importance in reducing the maximum pressure head rise. A suitable valve closure arrangement often permits a reduction of pipe wall thickness.

A sketch of a typical installation is shown in Fig. 1, where a pipeline of constant diameter and constant wall thickness conveys water from a distant reservoir in which the water surface elevation stays constant during the transient conditions.

During the closure of the valve, the pressure head along the pipe rises and reaches a maximum. This maximum can occur during or at the end of the closure operation. The magnitude of the maximum pressure head and the instant when it occurs largely depend on the valve opening versus time relation. While the derivations that follow are based on the constant head reservoir at the upstream boundary of the pipeline, the results are also indicative for other supply sources, e.g. for centrifugal pumps in steady operation.

In this paper, four theoretical closure arrangements, most commonly referred to in technical literature as the uniform, the parabolic, the equal-percentage, and the optimum valve closure arrangements, are analysed numerically for the maximum pressure head rise at the valve end and at the midpoint of the pipeline. Graphs are drawn to show the maximum pressure head rise in terms of the pipeline constant, relative closure time, and pipe wall friction for closures from full valve opening. Closures from partial valve openings are not considered in this paper.

Closures from partial openings can greatly increase the maximum pressure head rise and must therefore always be allowed for in the design of installations where the possibility for such closures exists. For a comprehensive treatment of closure arrangements

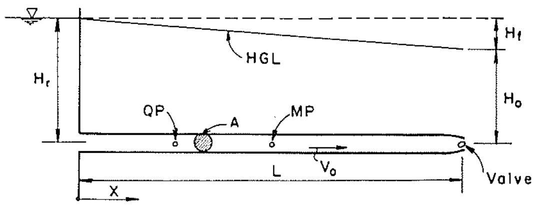

FIG. 1. Sketch of installation.

where closures from both full and partial openings are considered, the reader is referred to the publication by Ruus and El-Fitiany (1980).

In many installations with shutoff valves, however, the occurrence of closures from partial valve openings is exceedingly remote. Increased stresses could be allowed in the pipe wall when considering such conditions, while only normal stresses would be appropriate for the regularly occurring closure from full valve opening.

Valve closure arrangements are often classified on the basis of the duration of the closure movement. Instantaneous closure refers to a closure arrangement where the time of closure approaches zero, whereas the term sudden closure refers to a closure time of less than $2L/a$ s. In general, the shorter the closure time, the greater the pressure head rise. However, the very maximum pressure head rise occurs at the valve end for all closures occurring in $2L/a$ s or less.

Valve closures occurring within a short closure time are usually termed rapid, while those extending over long times are called slow closures. In this paper, the term rapid closure is used for all closures where the elasticity of the water and pipe wall is significant, whereas the term slow closure is restricted to closures where rigid water column theory would yield a reasonable approximation of the head rise. For the uniform closure arrangement, this limit is approximately $5(2L/a)$ s. For valve closures of longer duration, the curves on the double-logarithm plot become straight lines (see, for example, Fig. 7). In this paper, elastic water column theory has been used throughout the calculations.

Main parameters and assumptions

The main parameters of the pipeline are given below in nondimensional form. The pipeline constant:

$$[1] \quad \rho = aV_0/2gH_r$$

in which a = water hammer wave velocity, V_0 = initial steady state velocity in the pipe, g = acceleration of gravity, and H_r = static head at the valve.

The pipe wall friction is considered in the nondimensional form:

$$[2] \quad h_f = H_f/H_r$$

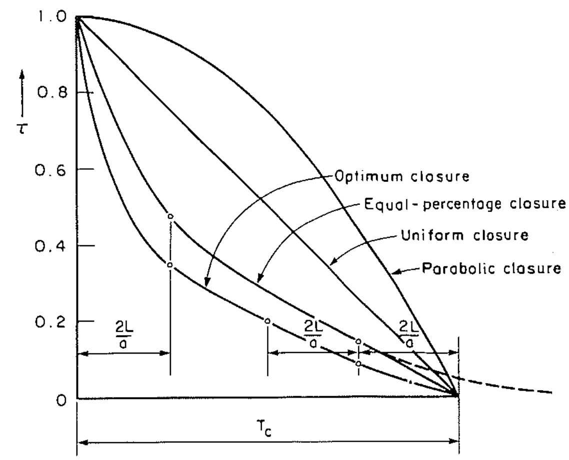

FIG. 2. Parabolic, uniform, equal-percentage, and optimum closure curves.

in which $H_f = f(L/D)(V_0^2/2g)$ by Darcy–Weisbach, where f = friction factor, L = length of pipe, D = pipe diameter, $V_0^2/2g$ = velocity head, and h_f = relative head loss in initial steady state condition.

The relative time of closure:

$$[3] \quad t_c = \frac{T_c}{2L/a}$$

in which T_c = total time of closure in seconds.

The relative valve opening:

$$[4] \quad \tau = \frac{C_d A_g}{C_{d0} A_{g0}}$$

in which A_g = area of valve opening and C_d = coefficient of discharge. The subscript $_0$ refers to the initial steady state condition.

The maximum relative pressure head rise Δh_{max} used in calculations to determine the required total closure time is given in the nondimensional form:

$$[5] \quad \Delta h_{max} = \frac{H_{max} - H_r}{H_r}$$

in which H_{max} = maximum pressure head.

Note that ρ and h_f are based on the static head H_r. This allows an easy evaluation of the effect of pipe wall friction on the pressure head rise. Note also that the maximum pressure head rise is based on the static head H_r and is measured above the reservoir level.

In the analyses, it is assumed that (a) the pipe diameter and wall thickness are constant, (b) the valve is located at the downstream end of the pipe, (c) the reservoir level remains constant, (d) no air pockets exist in the pipe, (e) the pipe friction associated with a given unsteady velocity follows the quadratic law, (f) the velocity head is negligible, and (g) only one closure

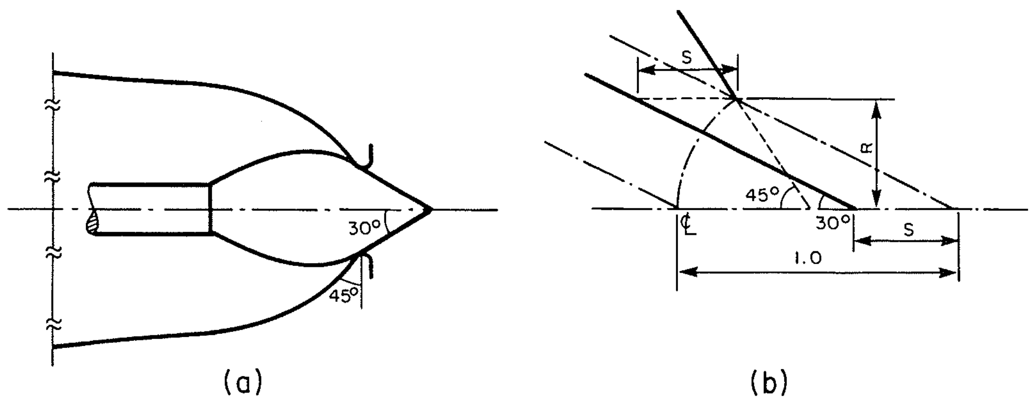

FIG. 3. Straight conical needle valve: (*a*) closed position; (*b*) definition sketch for valve travel.

operation takes place, during which the valve is closed from a fully open to a totally closed position.

Valve closure arrangements

Uniform valve closure arrangement

Uniform, linear, or straight-line closure is a simple and uniquely defined closure arrangement. It is expressed by an equation relating time and the relative valve opening:

[6] $\tau = 1 - T/T_c$

in which T = time in seconds. For uniform valve closure (see Fig. 2), the rate of closure is constant and depends only on the total time of closure, T_c.

Parabolic valve closure arrangement

Parabolic closure is a simple and uniquely defined closure arrangement. It is based on a linear equation relating time and the relative valve travel:

[7] $s = 1 - T/T_c$

in which s = relative valve travel, measured from the fully closed position of valve. For a conical needle valve with the angle of the generatrix equal to 30° and the angle of the tangent of nozzle outlet equal to 45° (see Fig. 3), the relationship between the relative effective valve opening τ and the relative valve travel s can be expressed by the fourth-order polynomial (Knapp 1960):

[8] $$\tau = \frac{2s - s^2}{0.6377}[1 - 0.3623(2s - s^2)]$$

This relationship is hereafter referred to as the parabolic closure.

Equal-percentage valve closure arrangement

The equal-percentage valve closure implies that a geometrical relationship exists between the relative effective valve opening and time (see Fig. 2). If, for example, the relative opening after the first $2L/a$ s interval is $\tau = 0.9$, then the openings after the subsequent $2L/a$ s intervals are $\tau = (0.9)^2 = 0.81$, $\tau = (0.9)^3 = 0.729$, etc. Such a relationship is expressed by the equation

[9] $\tau = 10^{-m(T/T_c)}$

in which m = exponential parameter. The exponential parameter m depends on h_f, ρ, t_c, and Δh_{max} as well as whether or not closures from partial valve openings are considered.

The equal-percentage closure arrangement is not unique. Closures at many different rates, represented by many different closure curves are possible. Note also that the curve described by [9] has a long exponential tail at small openings, as indicated in Fig. 2. This is of no practical value in reducing pressure head rise. Therefore, to achieve a clearly defined closure time and a clearly defined rate of closure, the curve described by [9] is truncated and then provided with a straight line extension during the last $2L/a$ s of closure.

In Fig. 4, the curves (a), (b), and (c) are all equal-percentage valve closure curves combined with straight portions. The curvature of these lines is, however, different.

Two pronounced peaks of pressure head rise occur when a valve is closed according to each of the curves (a), (b), and (c). The first peak occurs at the end of the first $2L/a$ s interval or a little later, the second peak occurs at the end of the straight-line portion of the curve, i.e. at the end of the last $2L/a$ s interval. A closure according to curve (a) will give a smaller maximum pressure head rise at the end of the first $2L/a$ s interval than that at the end of the last $2L/a$ s interval. Conversely, a closure arrangement according to curve (c) will give a larger pressure head peak at the end of the first $2L/a$ s interval and a smaller peak at the end of the last $2L/a$ s interval. If the curve (b) in Fig. 4 is used in calculations, then the corresponding closure operation

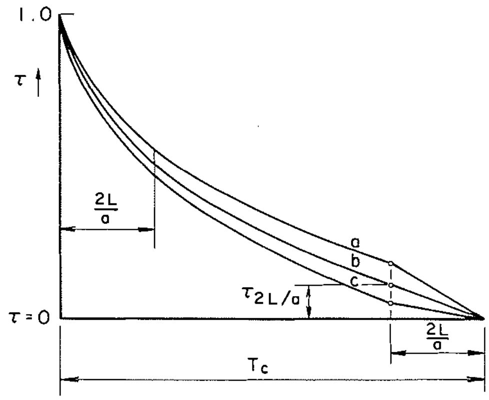

FIG. 4. Equal-percentage valve closures.

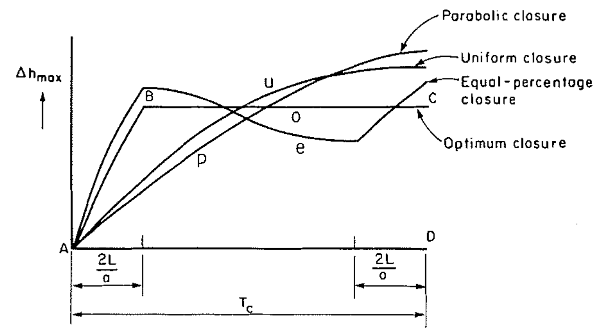

FIG. 5. Pressure head rise due to parabolic, uniform, equal-percentage and optimum closures.

yields two maximum, equal pressure head peaks at the valve, one at the end of the first $2L/a$ s interval or a little later, and the other at the end of the last $2L/a$ s interval. This is done by changing the exponent m so that for a given combination of h_f, ρ, and t_c these two peaks will be equal. The same curve based on the rise of the valve end is used to find the pressure head rise at midpoint and at the quarter point.

Optimum valve closure arrangement

The optimum closure is defined as the one, out of many possible arrangements, that for a given combination of ρ, t_c, and h_f yields the least value for the maximum pressure head rise. The derivation of an optimum closure arrangement is straightforward if pipe friction is disregarded. For a comprehensive treatment of the optimum closure arrangement, the reader is referred to the publications by Ruus (1966) and Wylie and Streeter (1983).

The optimum closure arrangement is not unique. Pipe wall friction h_f, pipeline constant ρ, and maximum prescribed pressure head rise Δh_{max} influence the shape of the optimum closure curve, so that for each combination of these variables, there is a different optimum closure curve. In particular, the shape of the optimum closure curve, derived for conditions where closures from partial openings are excluded, is different from a curve where such closures are included. This means that different optimum closures are possible, depending on whether or not the closures from partial openings are considered. The difference disappears for $\rho > 3.5$ when friction is ignored.

A curve representing the theoretical optimum closure arrangement while considering pipe wall friction can have a rather complicated shape. The analyses herein are therefore based on a simplified practical optimum closure curve which closely fits the theoretical one. The method of characteristics is used to calculate the resulting pressure head rise and to derive and verify the adequacy of the closure curve in each individual case analysed. The closure curve is optimized for pump end and the same curve is used to find pressure head rise at midpoint.

Pressure head rise

The typical variation of the pressure head rise at the valve end during a closure according to the four closure arrangements is indicated in Fig. 5. The velocity of water in initial steady state condition, the pipeline constant, the total closure time, and the pipe wall friction are identical at these closures. Therefore, the areas under the curves (e), (u), (o), and (p) are also equal. Note that the uniform closure arrangement produces the maximum pressure head rise toward the end of the closure. This maximum head rise is even more pronounced at the parabolic closure arrangement.

The pressure head rise diagram shown for equal-percentage valve closure in Fig. 5 has two equal peaks. The first one occurs at instant $2L/a$ s or soon thereafter and the second one occurs at the end of the closure of valve. The optimum valve closure arrangement yields a constant pressure head rise from instant $2L/a$ s to the end of closure. During the first $2L/a$ s, the pressure head rise increases linearly from a zero value to the maximum for a pipeline without friction.

Calculations and results

Two partial differential equations describing the unsteady flow are transformed into four total differential equations by the method of characteristics. A computer program is written to solve these differential equations with appropriate boundary conditions for a range of values of ρ, t_c, and h_f usually encountered in practice, using the approach of specified time intervals (Wylie and Streeter 1983). In the calculations for the maximum pressure head rise, the pipe is divided into eight equal reaches of Δx. A time interval $\Delta t = \Delta x/a$ is used throughout the calculations. For each separate combination of ρ, t_c, and h_f, the maximum relative pressure head rise Δh_{max} is calculated at the valve end and at the

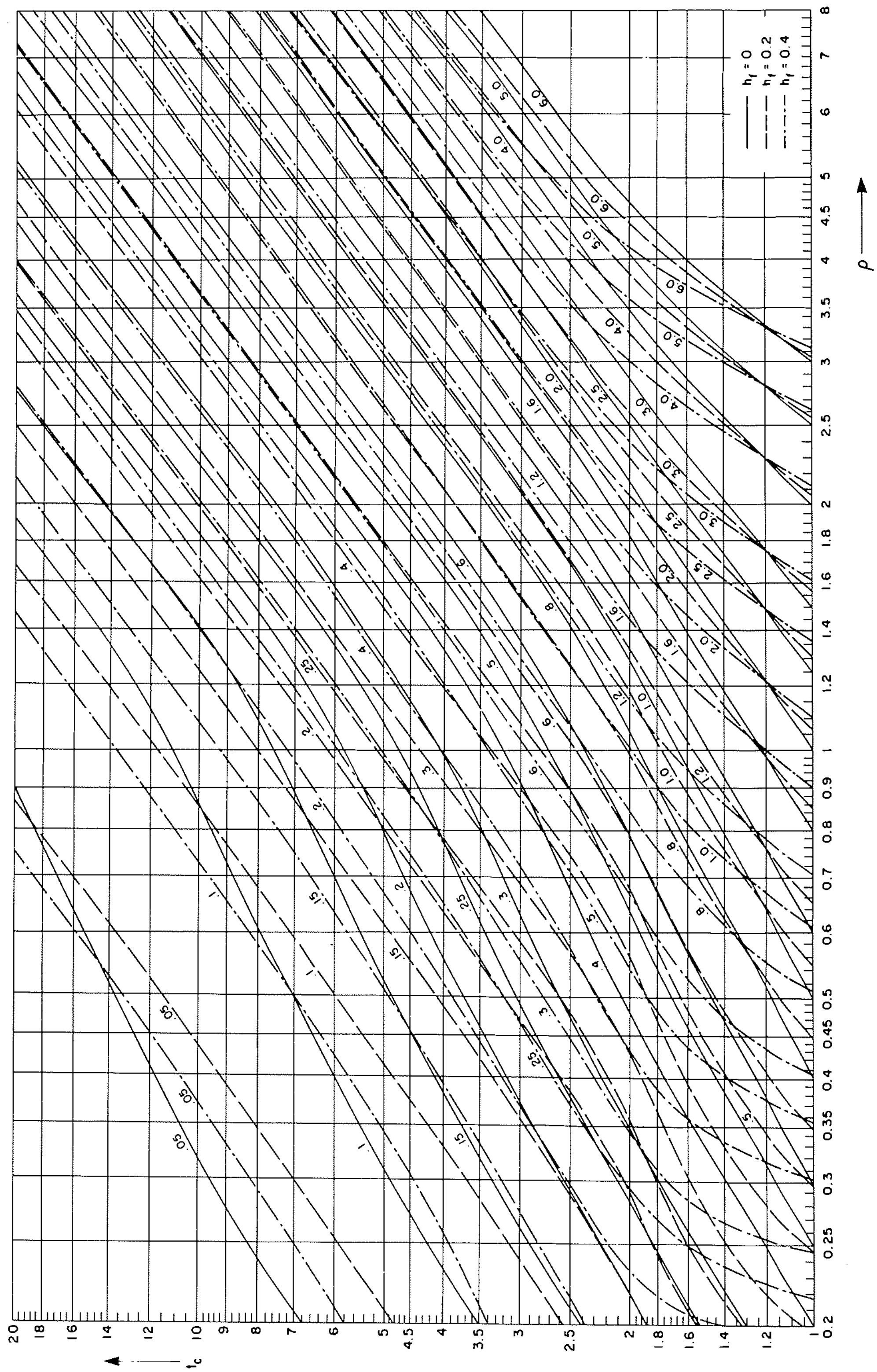

FIG. 6. Maximum pressure head rise at the valve: uniform valve closure from full opening only.

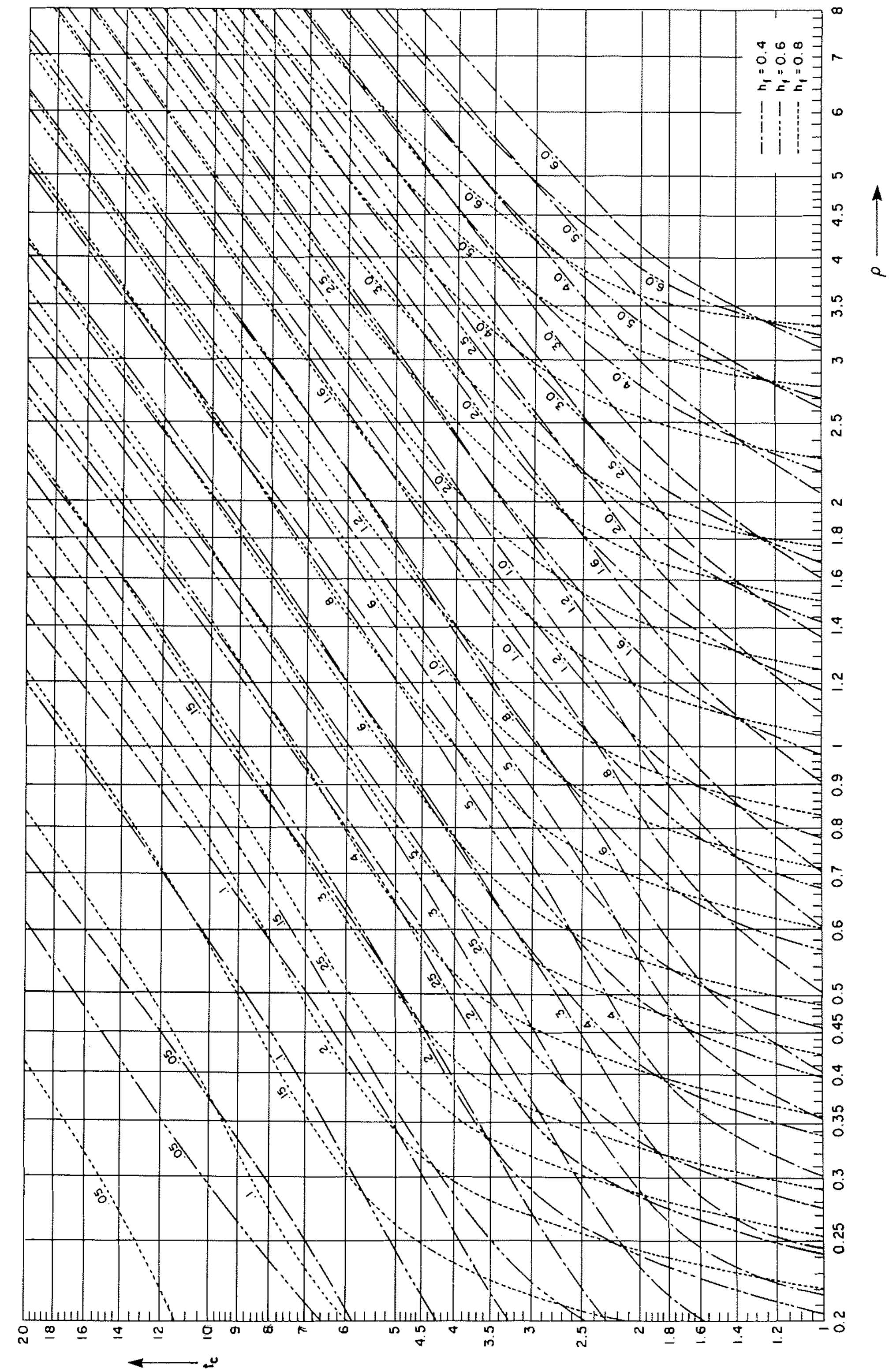

FIG. 7. Maximum pressure head rise at the valve: uniform valve closure from full opening only.

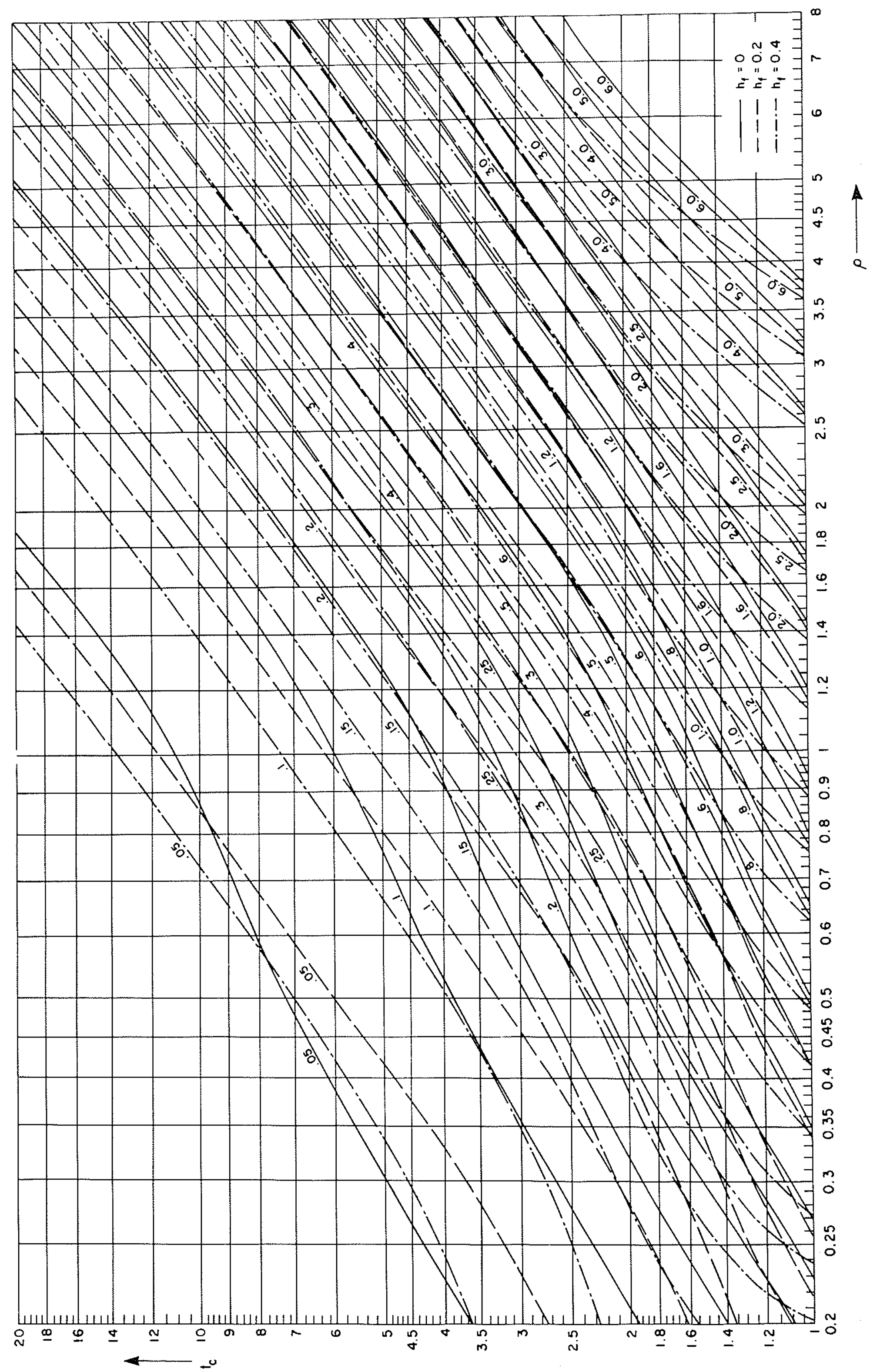

FIG. 8. Maximum pressure head rise at the midpoint: uniform valve closure from full opening only.

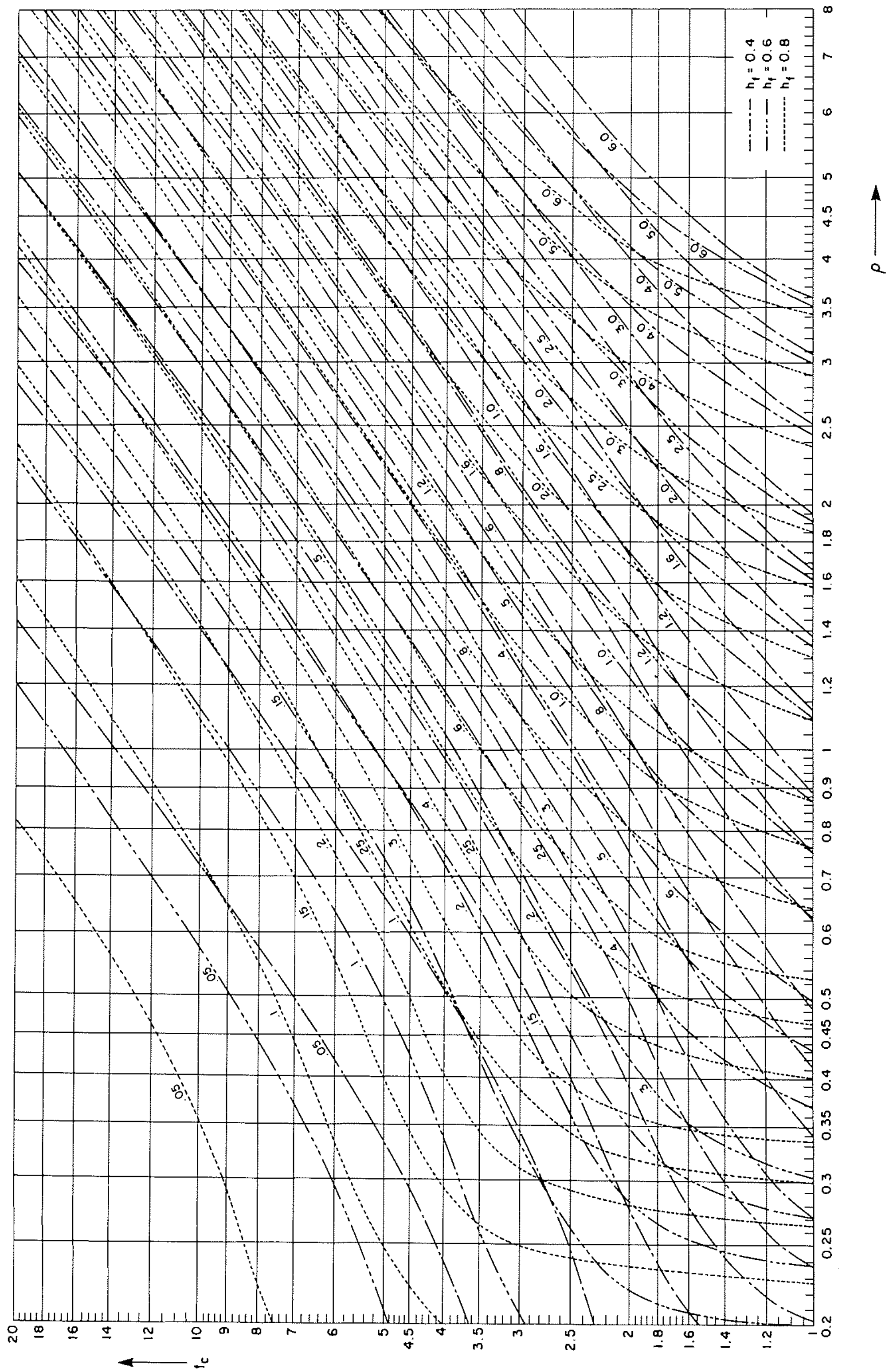

FIG. 9. Maximum pressure head rise at the midpoint: uniform valve closure from full opening only.

midpoint of the pipeline. This allows the maximum hydraulic grade line to be drawn with good accuracy. Note that closures from full valve opening only are considered.

In the calculations, each of the relative values of pipe wall friction h_f = 0.0, 0.2, 0.4, 0.6, and 0.8 is considered together with 10 values of relative closure time t_c ranging from 1.0 to 22.4, and with values of maximum pressure head rise Δh_{max} = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.2, 1.6, 2.0, 2.5, 3.0, 4.0, 5.0, and 6.0 to find the corresponding pipeline constant ρ for each of the four closure arrangements.

The results of calculations based on closures from full valve openings only are shown in Figs. 6–22. Separate curves are drawn in these figures for pipe wall friction h_f = 0.0, 0.2, 0.4, 0.6, and 0.8. Intermediate pipe wall friction values can be considered by interpolation between these curves. The numbers on the curves indicate the maximum pressure head rise at the valve end and at the midpoint of the pipeline.

Figures 6–9 show the pressure head rise resulting from the uniform valve closure. Pipe friction tends to increase the maximum pressure head rise for this closure arrangement except for very short closure times. In the range of small pressure rises, this tendency is not apparent.

The increase of the pressure head rise caused by wall friction may be surprising to the reader because, in general, pipe friction reduces the pressure head rise. Note, however, that any substantial reduction caused by friction always occurs in the early stage of closure when valve opening and velocity are still nearly at maximum. For linear closure, the maximum pressure head rise usually occurs at or near the end of the closure and therefore head rise reduction cannot be expected.

Why, however, is a substantial increase in pressure head rise caused by friction? To explain this, let us consider two otherwise identical pipelines, one with large wall friction, the other without friction, under equal reservoir head. Both pipes are discharging equal amounts of water. In the initial steady state condition, the first pipeline will require a substantially larger valve opening than the second pipeline because of the reduced head at the valve. Suppose now that both valves are closed uniformly during the same time interval and compare the resulting velocities at the beginning of the last $2L/a$ s. For the pipeline without friction at small (10–15%) relative valve openings, the relative velocity is only marginally greater than the relative valve opening. For the pipeline with large initial friction, the corresponding relative velocity is substantially greater than the relative valve opening. This is because the friction loss decreases rapidly as the flow is reduced, resulting in a greatly increased head at the valve. This greater head results in an increased velocity in the pipe. Therefore, during the last closure increment the large relative velocity differential results in substantial differential in pressure head rise.

Figures 10–14 show the pressure head rise resulting from equal-percentage closure for the valve end, midpoint, and quarter point. The information for the quarter point is included for this closure arrangement because of the substantial nonlinearity found in the pressure head variation along the pipe for low wall friction. For this closure arrangement, the exponent m is shown in Figs. 15 and 16 in terms of pipeline constant ρ and relative head rise Δh_{max}. Separate curves for exponent m are drawn in this figure for each of the relative pipe friction values h_f = 0.0, 0.2, 0.4, 0.6, and 0.8. These curves are highly regular and nearly straight towards high values of ρ and m, in the semilogarithmic plot. They become more and more irregular towards lower values of ρ, Δh_{max}, and m. The explanation for this can be found in Fig. 10 or 11, which indicates a low t_c value for such conditions. When t_c approaches unity, the pressure head rise becomes independent of the exponent m and becomes independent even of the closure arrangement. Large irregularity of the exponent m also results in irregularity of the curves indicating pressure head rise in Figs. 10–14. For this reason, the curves are omitted above Δh_{max} = 2.5.

Figure 10 shows that pipe wall friction largely reduces the pressure head rise at equal-percentage closure arrangement in the usual range of interest.

Figures 17–20 show the pressure head rise resulting from optimum closure for the valve end and for the midpoint of the pipeline. Friction clearly reduces the pressure head rise at optimum closure.

The closure curve for the equal-percentage closure arrangement is drawn in the following manner. The parameter m is first read from Fig. 15 or 16 for the given values of ρ and h_f and the selected value of Δh_{max}. Interpolation is needed between the curves for pipe friction h_f and the pressure head rise Δh_{max}. The required total closure time is then obtained from Fig. 10 or 11. Thereafter the effective valve opening τ is calculated from [9] and plotted for the range from $T = 0$ to $T = T_c - (2L/a)$. The point corresponding to the valve opening at $T = T_c - (2L/a)$ is joined with the point at $\tau = 0$, $T = T_c$ by a straight line representing the uniform closure during the last $2L/a$ s.

For all combinations of the parameters h_f, Δh_{max}, and ρ used in the calculations for the required relative closure time t_c, the effective valve opening time relation, necessary for the construction of the optimum closure curve, is given in table form in the Appendix.[1]

[1]The Appendix is available, at a nominal charge, from the Depository of Unpublished Data, CISTI, National Research Council of Canada, Ottawa, Ont., Canada K1A 0S2.

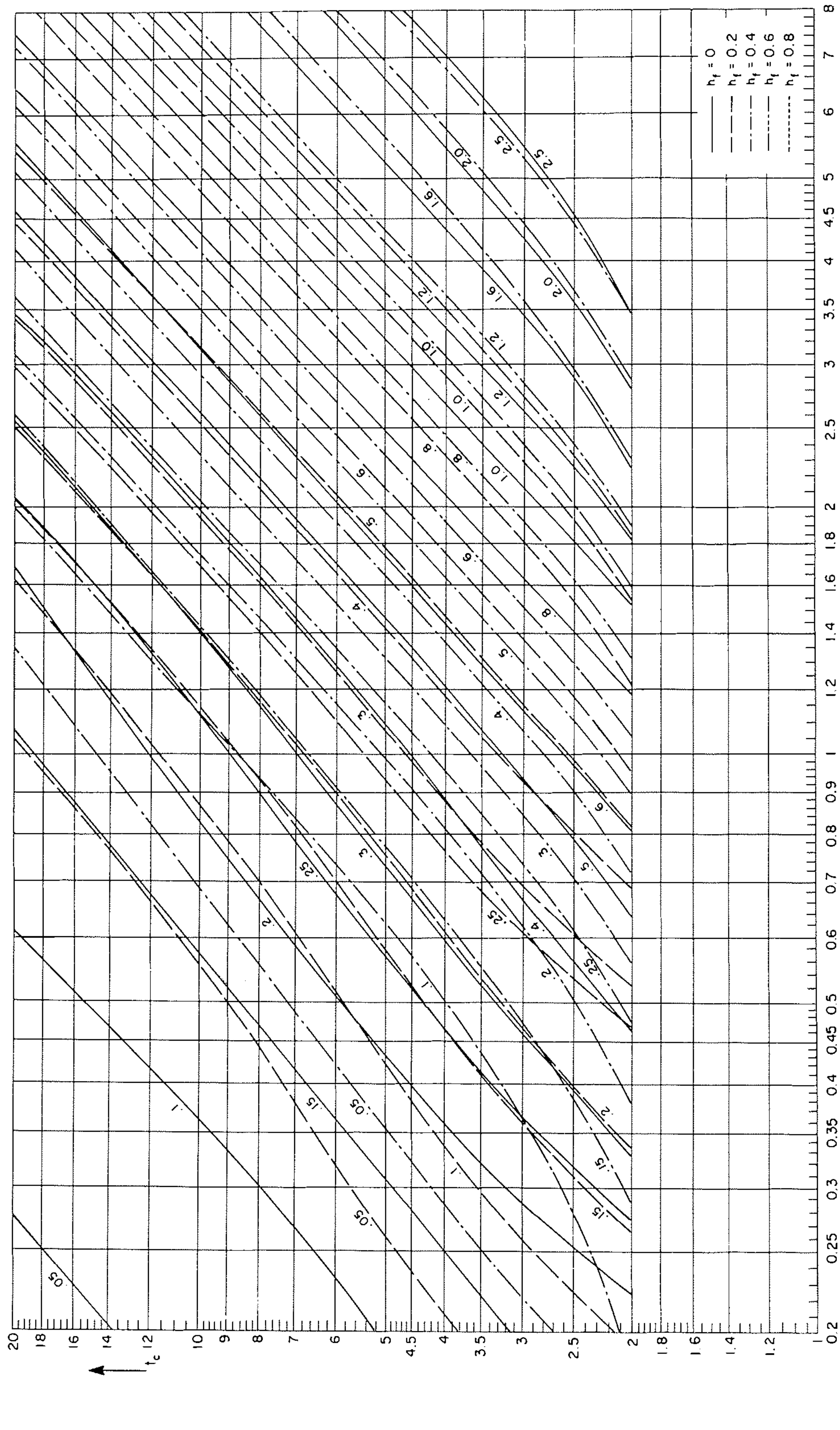

FIG. 10. Maximum pressure head rise at the valve: equal-percentage valve closure from full opening only.

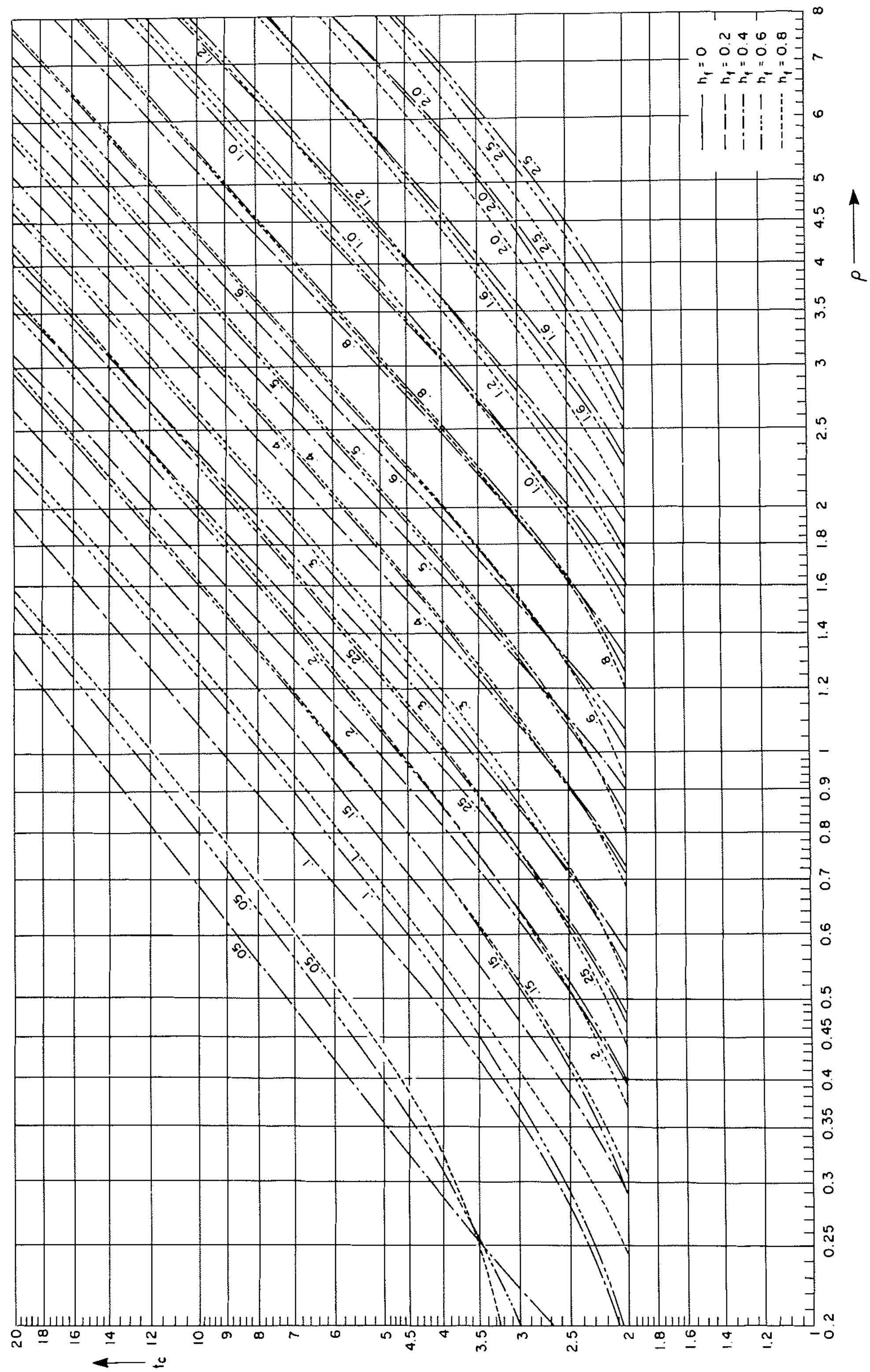

FIG. 11. Maximum pressure head rise at the valve: equal-percentage valve closure from full opening only.

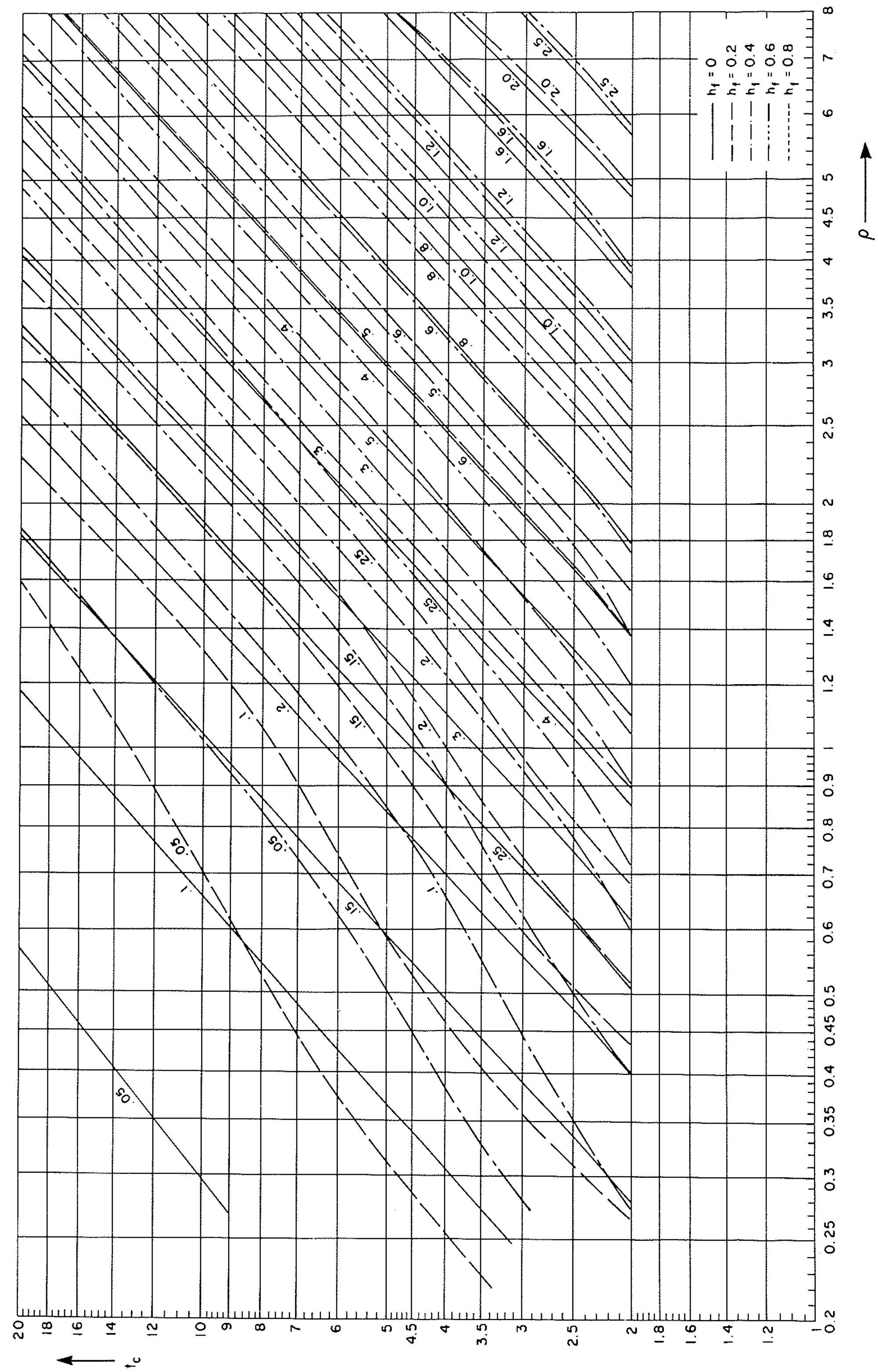

FIG. 12. Maximum pressure head rise at the midpoint: equal-percentage closure from full opening only.

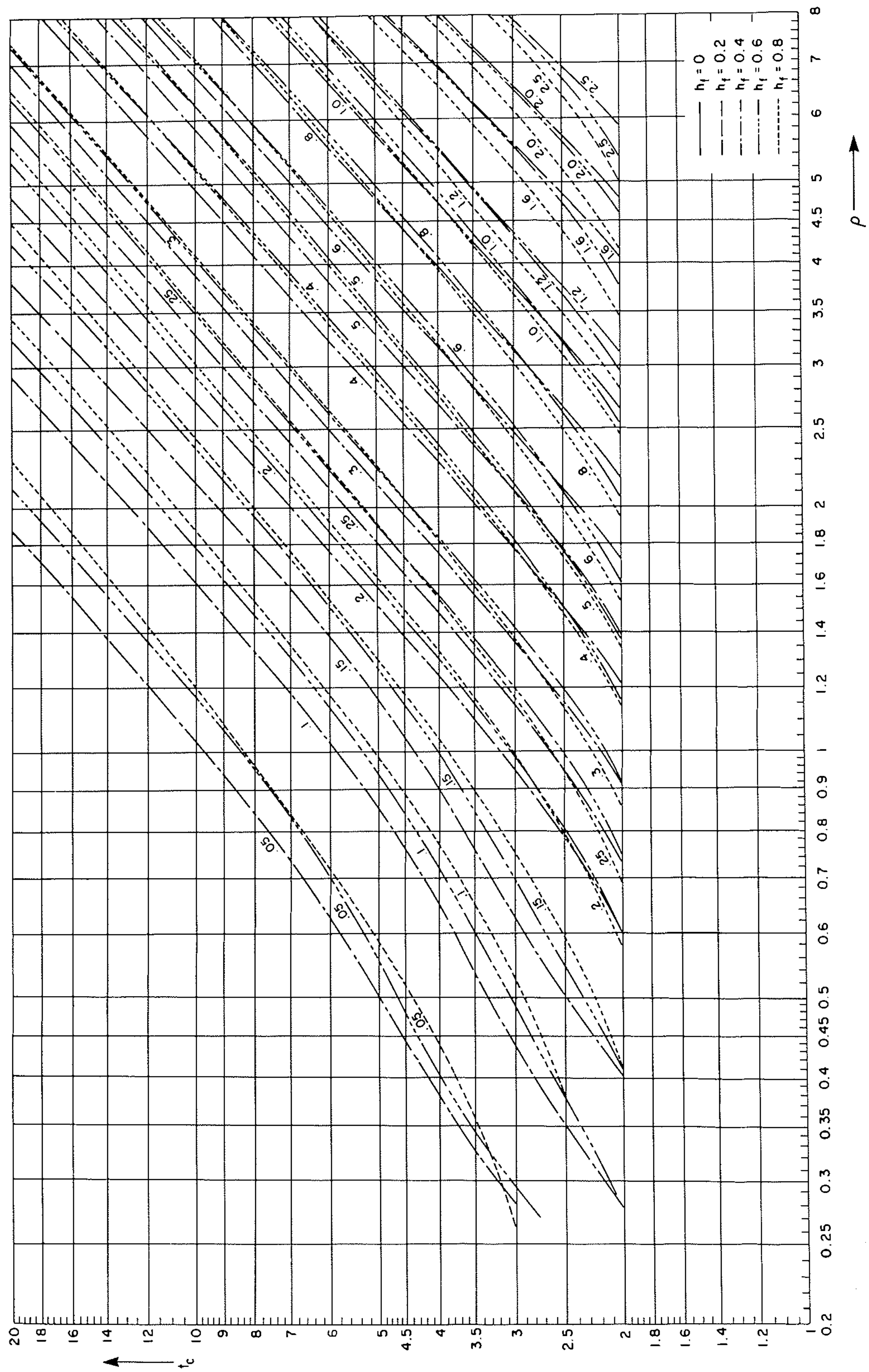

FIG. 13. Maximum pressure head rise at the midpoint: equal-percentage closure from full opening only.

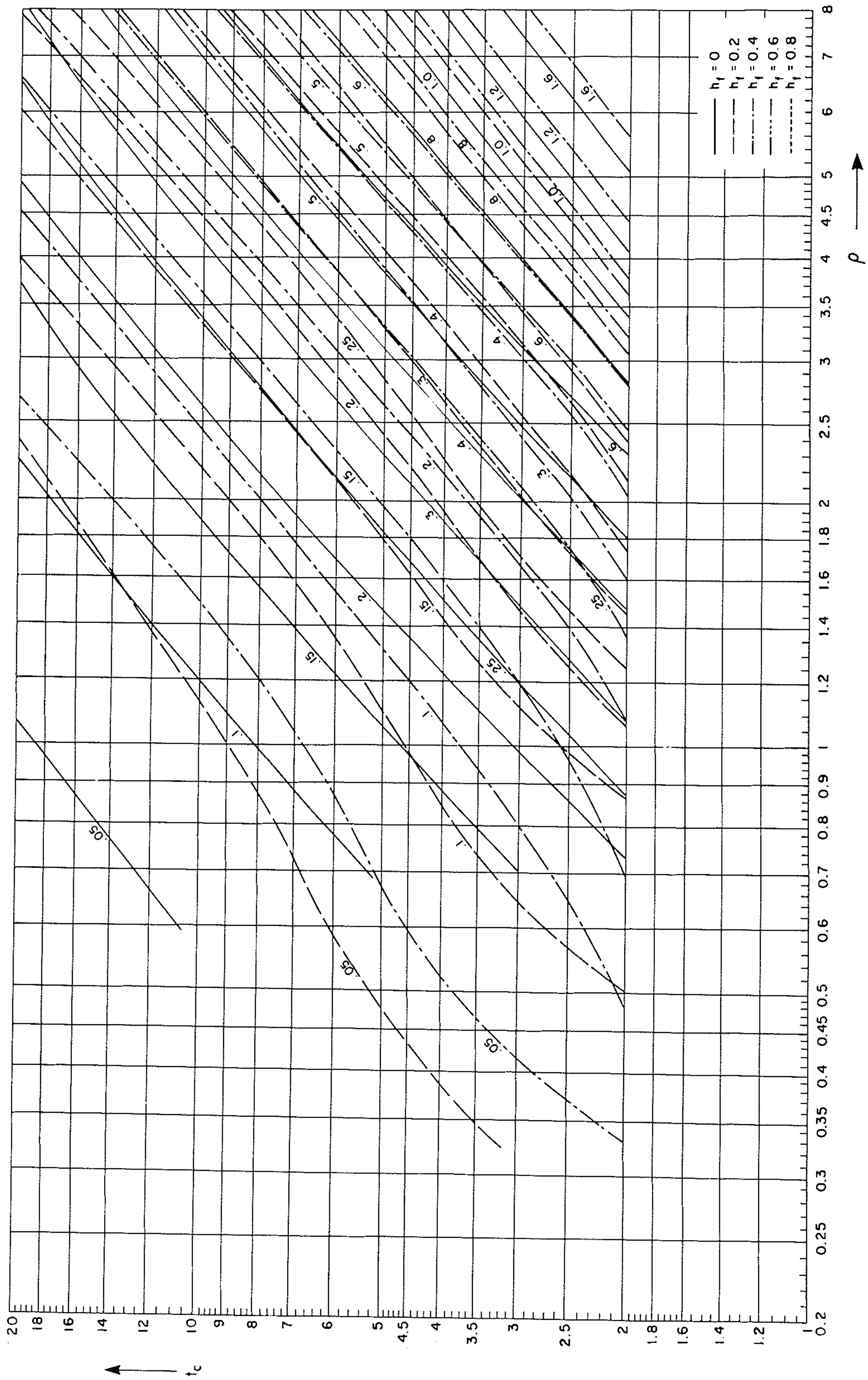

FIG. 14. Maximum pressure head rise at the quarter point: equal-percentage valve closure from full opening only.

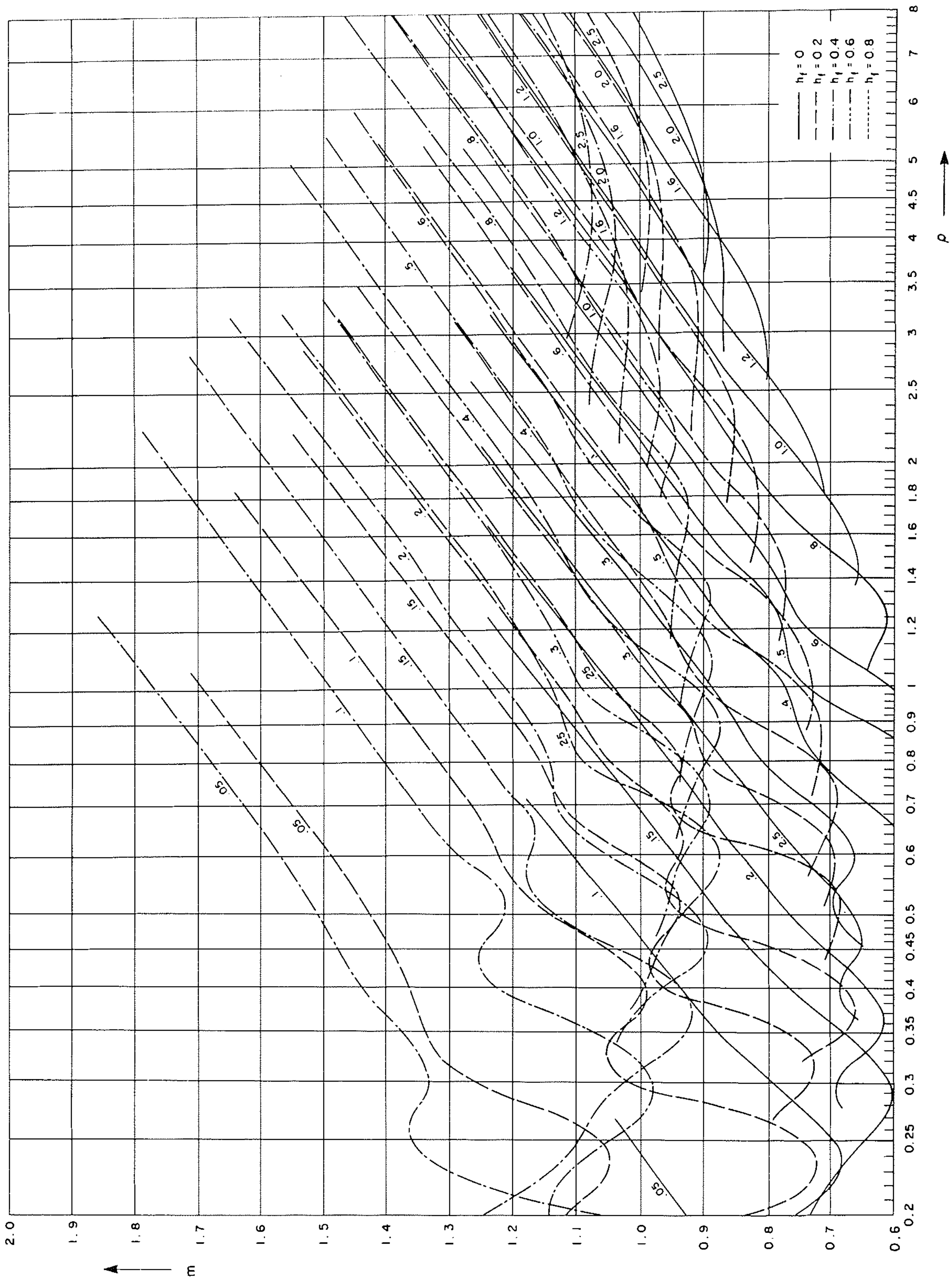

FIG. 15. Exponent m for equal-percentage valve closure from full opening only.

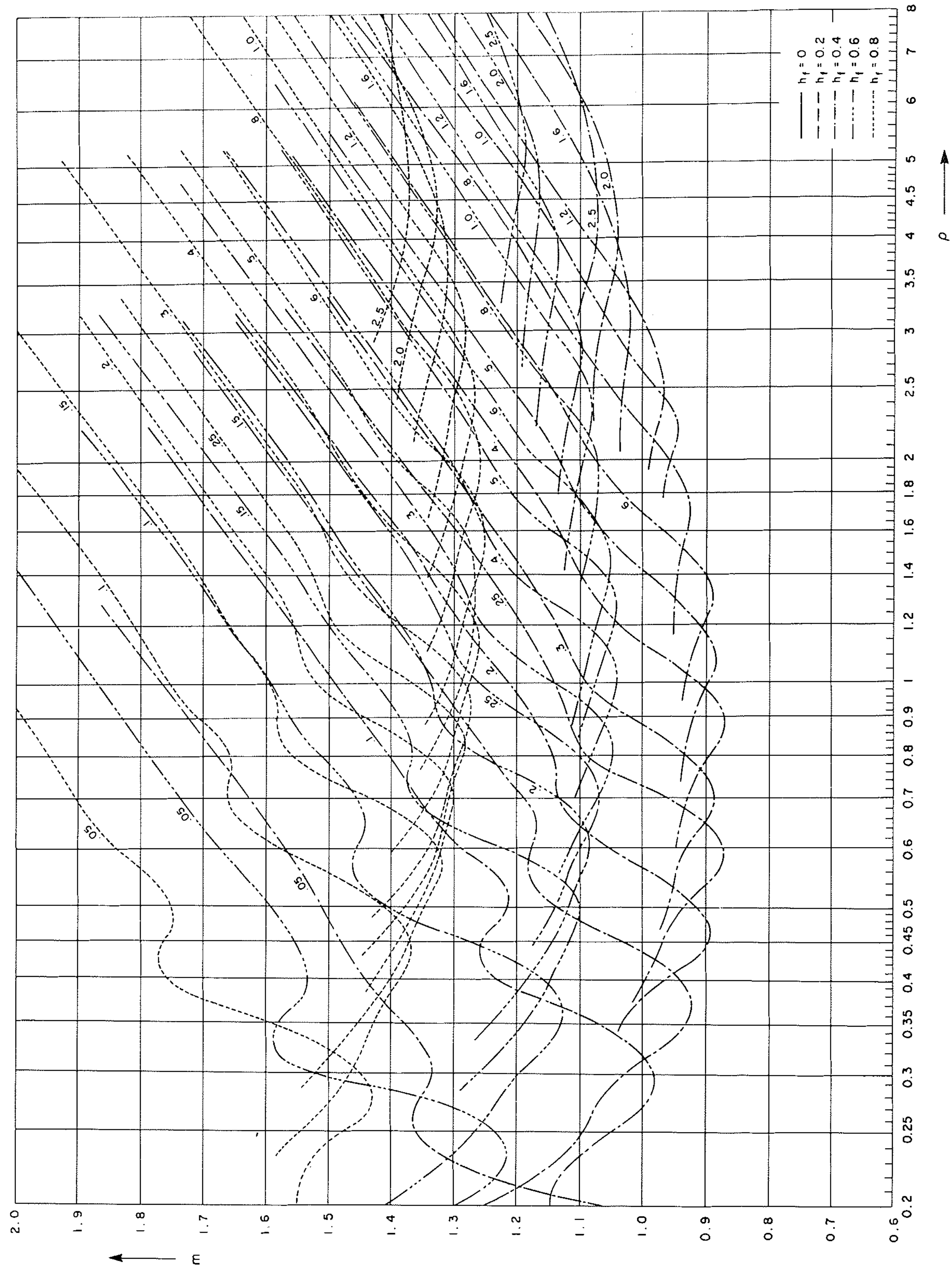

FIG. 16. Exponent m for equal-percentage valve closure from full opening only.

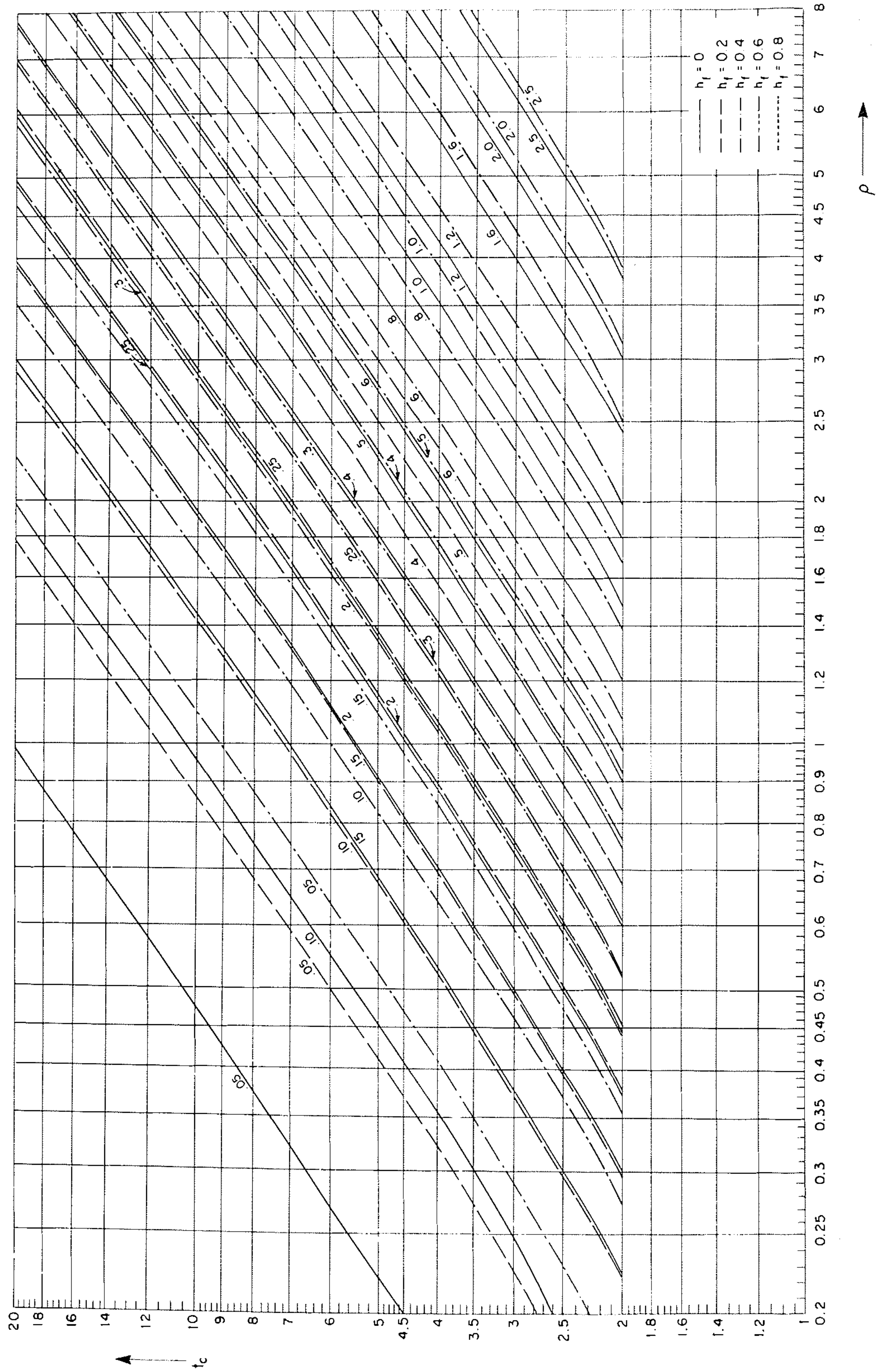

FIG. 17. Maximum pressure head rise at the valve: optimum valve closure from full opening only.

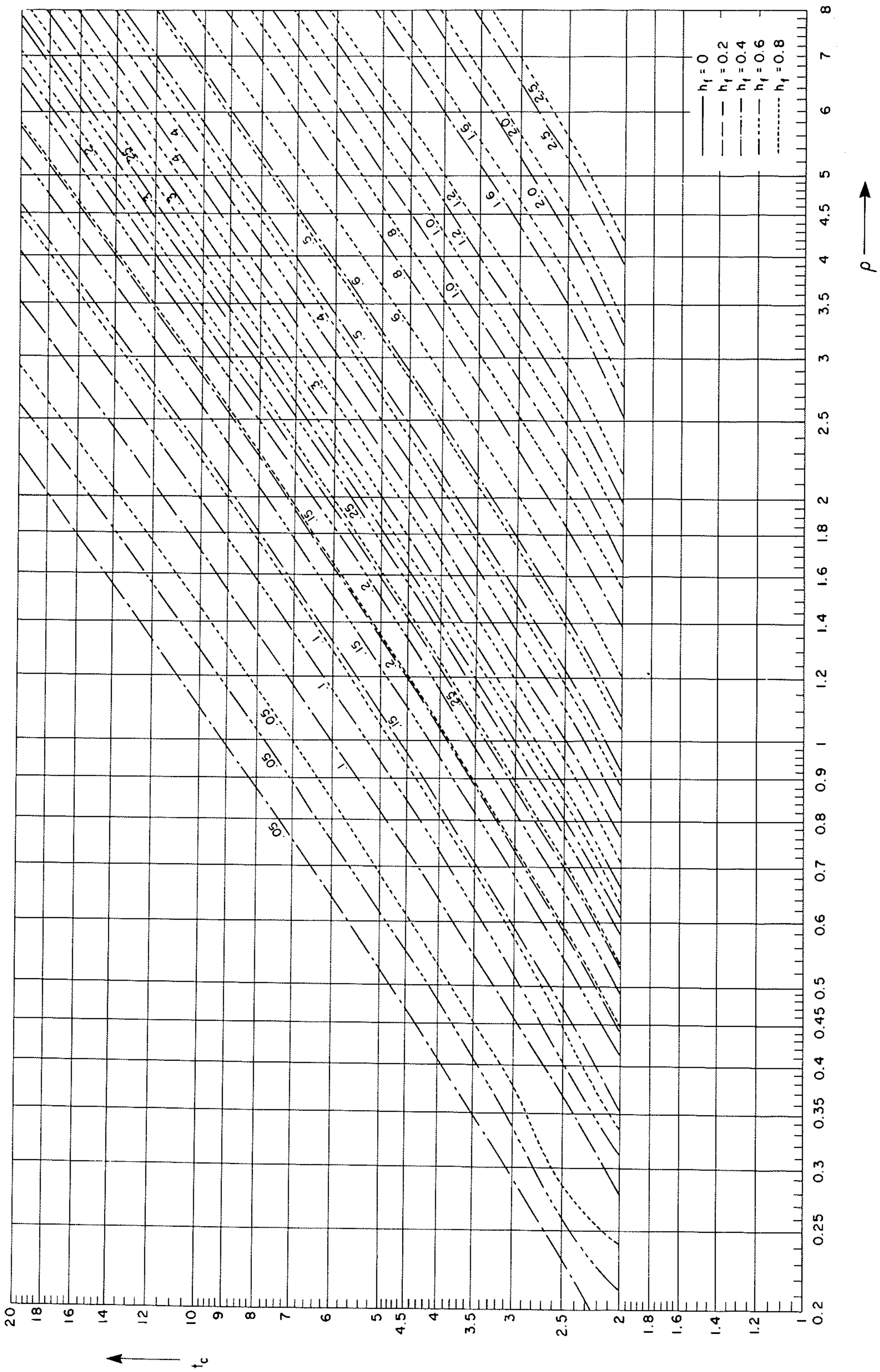

FIG. 18. Maximum pressure head rise at the valve: optimum valve closure from full opening only.

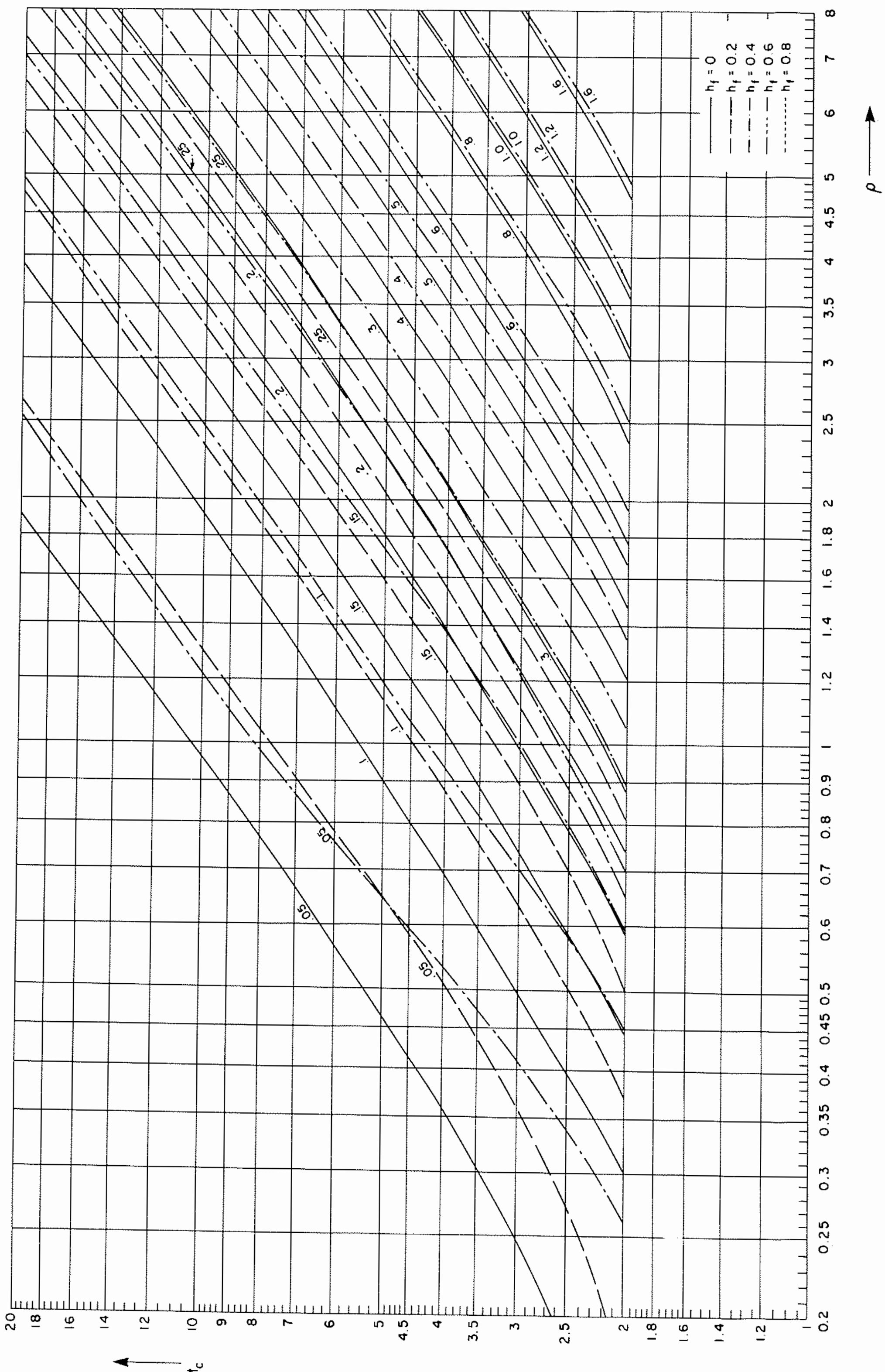

FIG. 19. Maximum pressure head rise at the midpoint: optimum valve closure from full opening only.

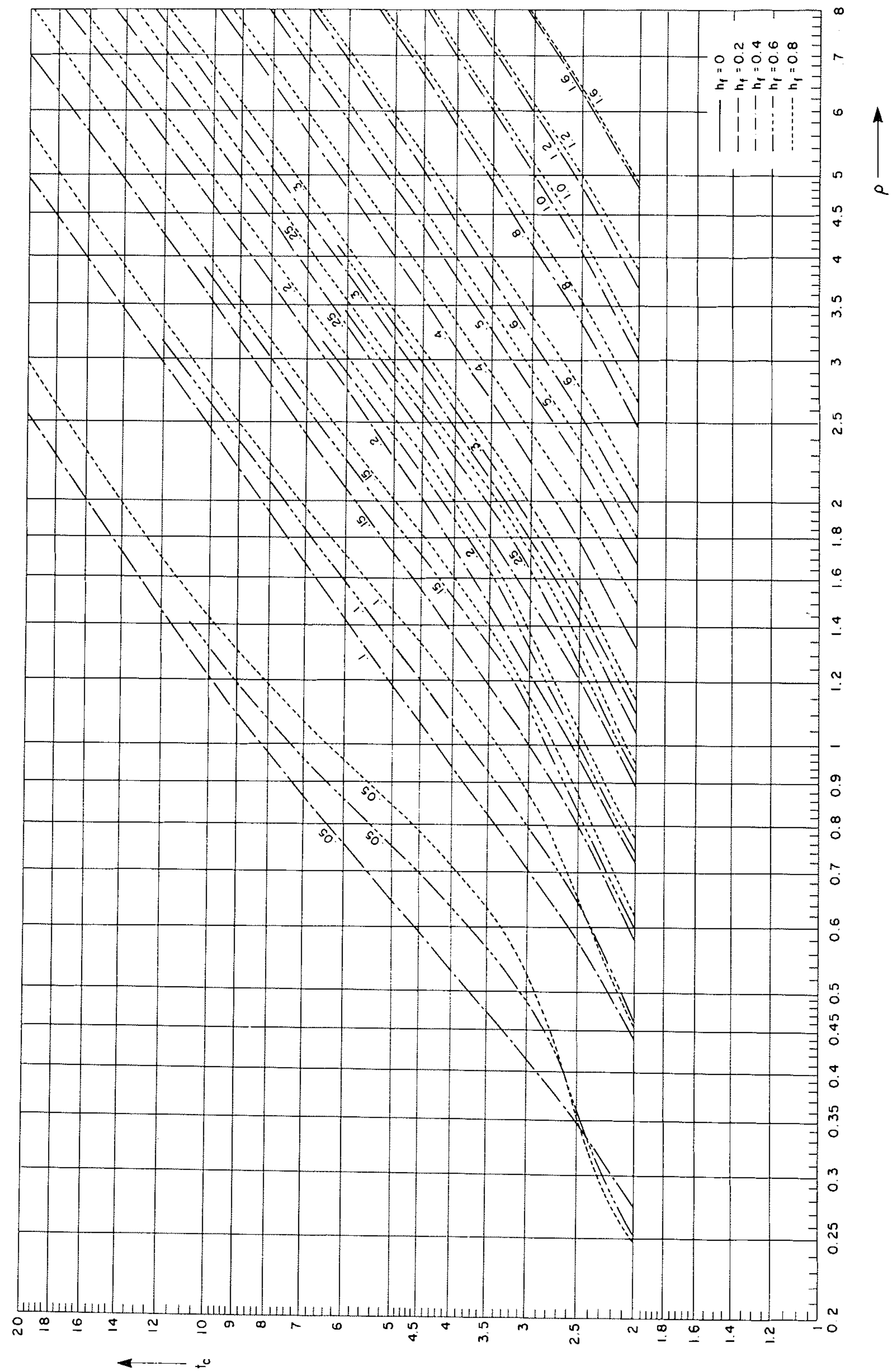

FIG. 20. Maximum pressure head rise at the midpoint: optimum valve closure from full opening only.

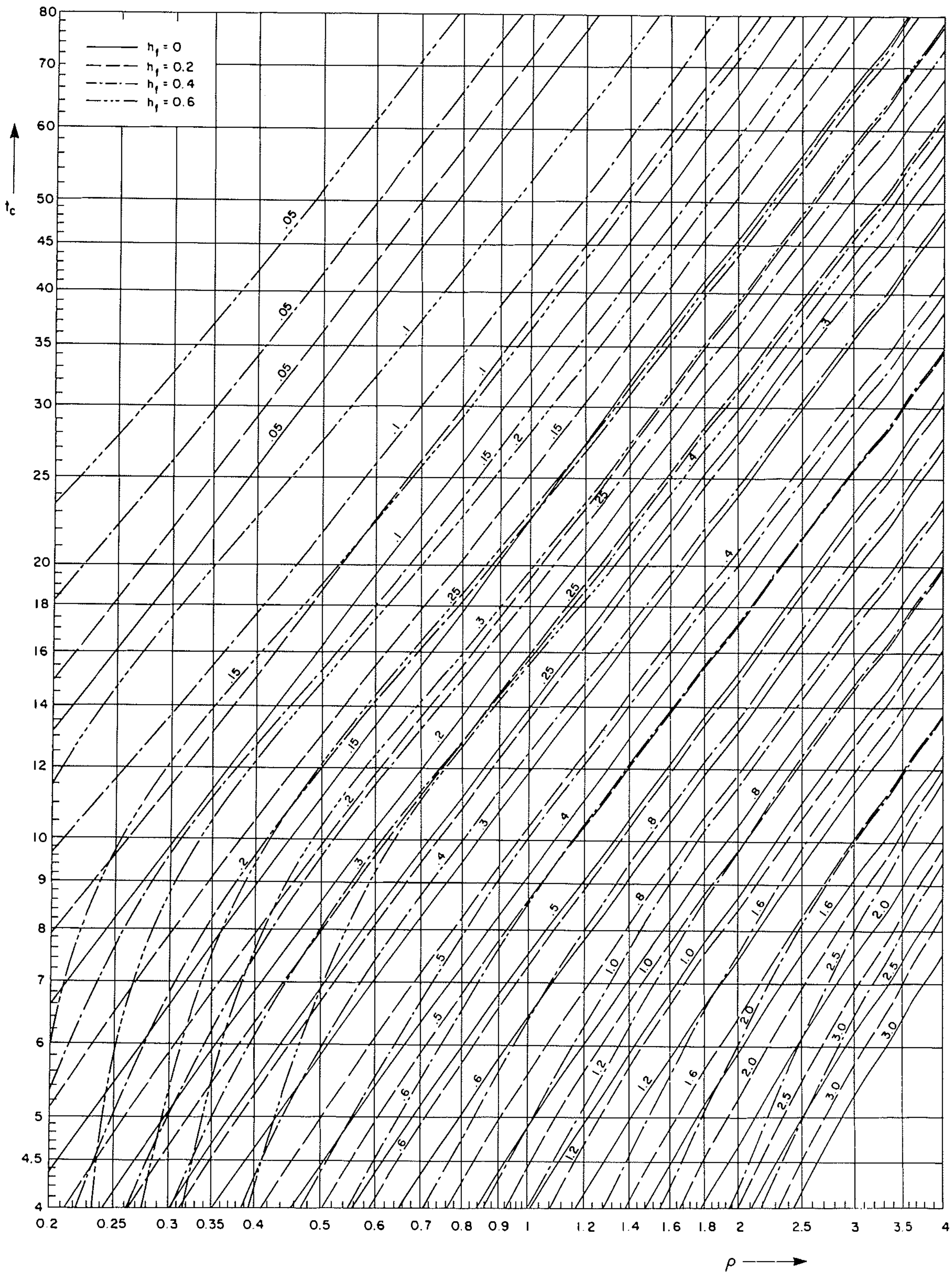

FIG. 21. Maximum pressure head rise at the valve: parabolic valve closure from full opening only.

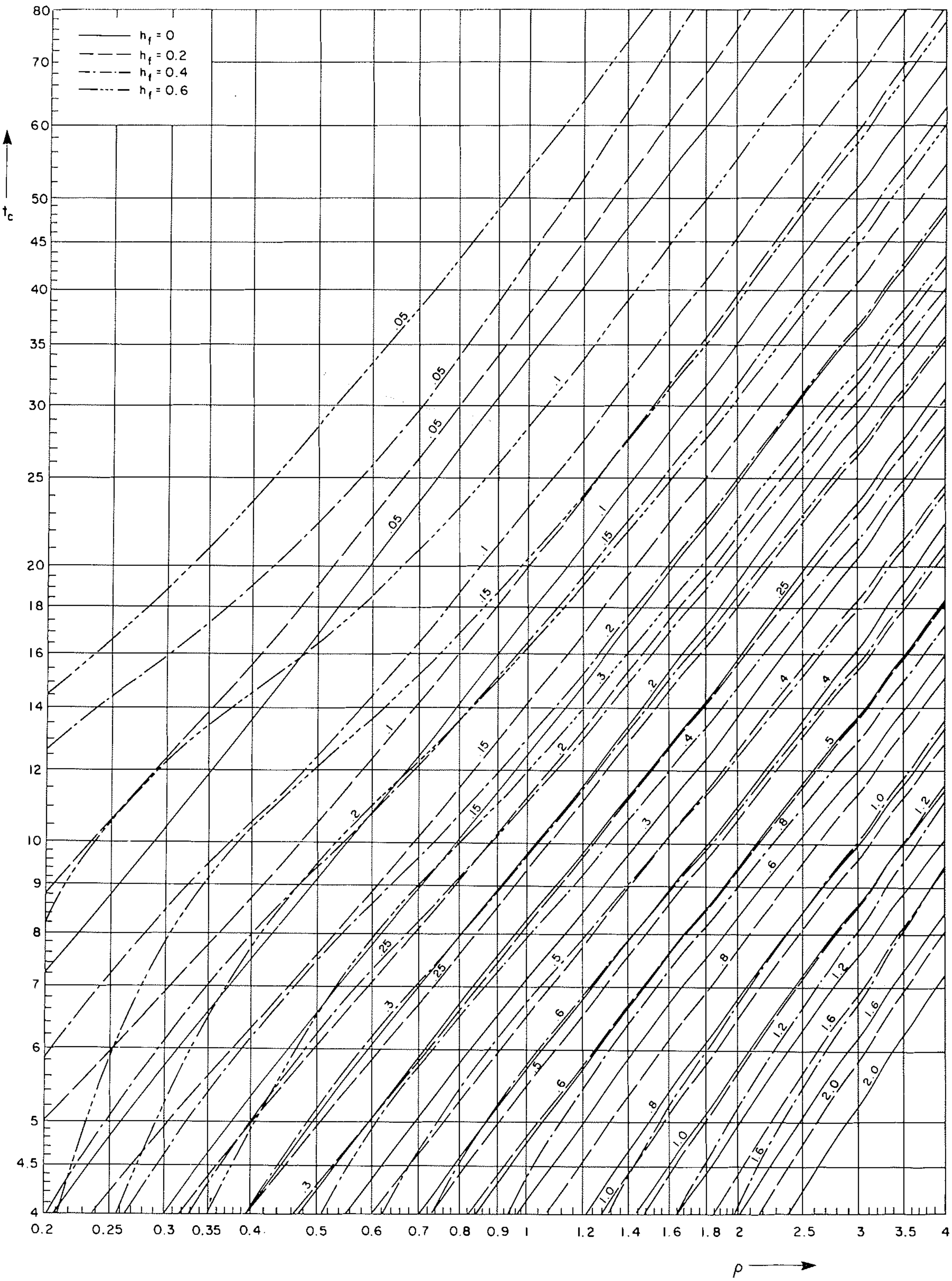

FIG. 22. Maximum pressure head rise at the midpoint: parabolic valve closure from full opening only.

Although over 650 closure curves are presented in these tables for the optimum closure arrangement, interpolations are still needed to draw the curve for the general case. Linear interpolation is satisfactory.

The analysis of the water hammer that results from the uniquely defined uniform and parabolic closure arrangements is straightforward. In this paper the results of the uniform closure are presented mainly for direct comparison with those of the parabolic, equal-percentage, and optimum closures.

The analysis of water hammer for an equal-percentage closure is more complicated than that for a uniform closure. A trial-and-error procedure is needed to arrive at the most suitable exponent m for each individual case. The use of Figs. 15 and 16, however, enables the curve for an equal-percentage closure to be easily constructed. The corresponding maximum water hammer can be read from Figs. 10–14. This maximum may be verified by the graphical method of water hammer analysis by using the constructed closure curve.

The analysis of water hammer for the optimum closure arrangement is not difficult when only the closure from the full gate is considered (Driels 1975; Jones and Wood 1972; Wylie and Streeter 1983). Pipe wall friction complicates both the derivation of the closure curve and the analysis of the resulting maximum water hammer. A trial-and-error procedure is needed for the derivation of the closure curves, with somewhat erratic closure curves being obtained, when the optimum closure is strictly applied. For these applications, this systematic study is most useful in suggesting the shapes of practical closure curves, for which the maximum water hammer can be read from Figs. 17–20. This maximum may be verified by the graphical method, by using the constructed closure curve, thus eliminating a long computer program.

Figures 21 and 22 show the pressure head rise resulting from the parabolic closure for the valve end and for the midpoint of the pipeline.

In general, the graphs permit a quick evaluation of the influence of closure time and water velocity on water hammer, and the influence of each individual closure arrangement is readily observed. This is useful at the preliminary design stage. Moreover, the suitability of the practical closure arrangements could be evaluated from the data presented and the resulting maximum water hammer could be found by graphical means.

Example

Given a pipeline constant $\rho = 2.0$ and a relative pipe wall friction $h_f = 0.2$, find the required relative closure time t_c to limit the maximum pressure head rise at the valve to 40% of the static head H_r. Use the uniform, equal-percentage, optimum, and parabolic closure arrangements.

The graphs in Figs. 6, 10, 17, and 21 can be used to read the required relative closure time (in terms of $2L/a$ s). From these graphs, a relative closure time of $t_c = 6.6, 7.5, 4.8$, and 20.8 is obtained for the uniform, equal-percentage, optimum, and parabolic arrangements respectively.

Conclusions

1. At high ρ and t_c values, the parabolic valve closure always produces a substantially higher pressure head rise than the uniform valve closure, which in turn produces a substantially higher pressure head rise than the equal-percentage closure, which again produces a substantially higher pressure head rise than the optimum closure. The difference decreases toward the low ρ and t_c values; it disappears when t_c approaches unity.
2. For substantial pipe friction ($h_f \geq 0.4$), the pressure head resulting from the parabolic closure can be more than 10 times the pressure head resulting from the corresponding optimum closure.
3. Pressure head rise at the midpoint is always more than one-half of the corresponding rise at the valve end; pressure head at the quarter point is always more than one-quarter of the corresponding rise at the valve end.

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List of symbols

A_g	area of valve opening (m^2)
a	water hammer wave velocity (m/s)
C_d	coefficient of discharge
D	pipe diameter (m)
f	friction factor

g	acceleration of gravity (m/s^2)
H_f	head loss due to viscosity and wall friction (m)
H_{max}	maximum pressure head (m)
H_0	pressure head at valve in steady state (m)
H_r	static pressure head at valve (m)
h_f	relative head loss
Δh_{max}	maximum relative pressure head rise
L	pipe length (m)
m	exponential parameter
s	relative valve travel
T	time (s)
T_c	total time of closure (s)
t_c	relative time of closure
Δt	time interval (s)
V_0	initial steady state velocity (m/s)
Δx	length of reach (m)
ρ	pipeline constant
τ	relative valve opening
NOTE:	subscript $_0$ refers to initial steady state condition

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