

Application of Energy Concepts to Groundwater Flow: Adaptive Modeling of a Leaky Aquifer

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The net work and energy flux at the boundaries of an aquifer change its internal energy and overcome its resistance to flow. In saturated porous media, the change in internal (strain) energy is stored in the elastic soil matrix and in pore water compression. In unsaturated media, an additional term accounts for changes in gravitational potential energy. The energy approach complements conventional insight by allowing spatially distributed processes to be integrated into energy and work terms which characterize a system's response to a set of excitations. Specifically, a technique is developed in this paper to interpret the dynamic behavior of a one-dimensional leaky aquifer in terms of its composite energy functions. In particular, the work interaction at the leaky boundary is used as an index of the significance of the leakage: when the work parameter indicates a relatively small leakage, the flow components of the multiaquifer can be isolated and modeled separately with a controllable loss of accuracy.

INTRODUCTION

Transient flow conditions in a porous medium can be described in terms of composite energy functions. The underlying concept is that disturbances in equilibrium flow conditions—whether brought about by work at a system's boundaries, by heat exchange, or by direct matter/energy transfer—are associated with changes in a system's energy. Shifts in the relative magnitude of the various energy norms thus indicate the dominance of the different physical phenomena in the system as well as the importance of phenomena at its boundaries. Unlike the traditional approach of calculating the flow velocity and piezometric head at a point or a set of points, the energy method directly provides an integrated view of the transient response of the entire system.

In a previous paper, *Karney and Seneviratne* [1991] applied the energy concept to assess transient conditions in confined aquifers. In particular, the rate of change of internal energy of the porous medium was shown to be a natural index of the unsteadiness of the system, an insight which led to an adaptive algorithm for adjusting the time step in a transient flow model. In addition, the energy principle was used to compute the sensitivity of different aquifer regions to data acquisition errors.

In the current paper, energy expressions are presented for both confined and unconfined aquifers and their work interaction through a leaky boundary. As subsequent developments show, the rate work is done at a leaky boundary provides a robust indication of the significance of the leakage. In fact, when the work across the leaky boundary is relatively small, the leaky system can be replaced by a simpler confined flow model, subject to a controllable loss of accuracy. This is important, for the assumption that confining formations are impervious is seldom satisfied. Because of subsurface irregularities, confinements tend to vary from less to highly pervious, thus causing complex groundwater

flow patterns. Hence the importance of leakage between adjacent aquifers/aquitards forms a continuum of flow interactions, ranging from strong to weak, for which the traditional designations of "confined" and "leaky" are special cases. It has been typical in groundwater modeling to treat leaky and confined cases as discrete options rather than as a continuum of approximations.

In modeling groundwater flow, "leakage" strictly refers to flow interactions which take place through top and/or bottom confining layers. Yet such systems often consist of three or more individual flow components. For example, a confined aquifer overlain by an unconfined aquifer could have an intervening silt layer which is semipervious causing a flow interaction. Only rarely can the dynamic response of one component in such a multiaquifer be isolated from another a priori. Usually, the system needs to be modeled simultaneously, which requires both extensive computational effort and considerable hydrogeologic data, particularly under transient conditions. However, if a procedure can be found which demonstrates that the flow interactions between the two aquifers is indeed negligible, then it may be possible to analyze each component separately, thus reducing data and computational time requirements.

The energy approach presented in this paper provides a basis for switching from a leaky model to a confined model by calculating the work term at the leaky boundary. The importance of the leakage is evaluated on the relative magnitude of the flow work occurring during the period of simulation. Specifically, the relative magnitude of the leakage is taken as the ratio of the magnitude of the work done at the leaky boundary at any given time to the magnitude of the maximum work done at that boundary during the period of simulation. When the work interaction at the leaky boundary is less than a user-specified threshold value, the model switches from the leaky to a confined model. In order to place these developments in perspective, a brief review of the flow equations and energy relations is first presented.

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SIMPLIFIED MULTIAQUIFER FLOW EQUATIONS

The present study considers a relatively simple two-aquifer system—a confined aquifer overlain by an uncon-

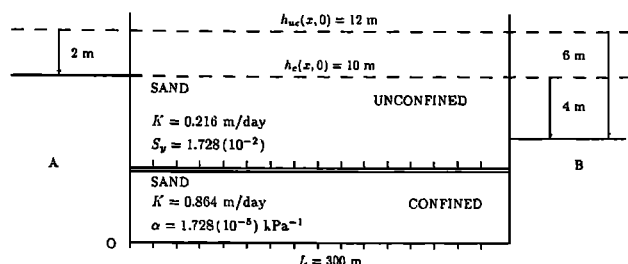


Fig. 1. Multiaquifer bounded by two constant head reservoirs.

finer aquifer with vertical leakage across the intervening aquitard, such as the one shown in Figure 1. This system consists of three hydraulically connected flow components: the confined aquifer, the unconfined aquifer, and the aquitard. The material properties and hydraulic head differential governs the flow in the aquitard. When the compressibility effects in the aquitard are insignificant, the flux through it can be represented by Darcy's law, without recourse to a full transient analysis. Neuman and Witherspoon [1969] state that storage effects can be neglected if the conductivities of the aquifers are at least 2 orders of magnitude greater than that of the aquitard and that associated errors in the computed piezometric head in this case would be less than 5%. Since the flow parameters selected in this study satisfy this condition, the flow in the aquitard is assumed to be vertical with no time lag.

Assuming one-dimensional (nearly horizontal) flow in the leaky-confined aquifer, mass conservation requires that

$$\frac{\partial(n\rho)}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} - \rho \left[\frac{h_{uc} - h}{B\sigma_c} \right] = 0 \quad (1)$$

in which n is the average porosity, ρ is the fluid density, v_x is the horizontal flow velocity, h is the piezometric head in the confined aquifer, h_{uc} is the saturated thickness of the unconfined aquifer, B is the thickness of the confined aquifer, and σ_c is the coefficient of leakage across the confining stratum [Bear, 1979]. To simplify the equations, the head terms h and h_{uc} are often evaluated at the previous time step; these "linearized" variables will be written with an overbar as \bar{h} and \bar{h}_{uc} . Finally, if Darcy's law

$$v_x = -K_x \frac{\partial h}{\partial x} \quad (2)$$

is substituted into the simplified continuity equation, the resulting equation can be rearranged as

$$\frac{\partial \left(T_x \frac{\partial h}{\partial x} \right)}{\partial x} = S \frac{\partial h}{\partial t} - \left[\frac{h_{uc} - \bar{h}}{\sigma_c} \right] \quad (3)$$

in which T_x is the transmissivity and S is the storage coefficient of the confined aquifer.

When both the soil matrix and water compressibility effects are included, the simplified continuity equation (5) for the leaky unconfined aquifer can be written as [Bear, 1979]

$$\frac{\partial(h_{uc} v_x)}{\partial x} + [S_y + \gamma(\alpha + n\beta)] \frac{\partial h_{uc}}{\partial t} + \left[\frac{h_{uc} - \bar{h}}{\sigma_c} \right] = 0. \quad (4)$$

where S_y is the specific yield of the unconfined aquifer and γ is the unit weight of water. Note that it is implicitly assumed in this equation and those that follow that the datum for the head is set at the base of the unconfined aquifer. Thus, using Darcy's law in this equation and neglecting the compressibility terms produces the quasi-linear flow equation for the leaky-unconfined aquifer,

$$\frac{\partial \left(K_x \frac{\partial h_{uc}}{\partial x} \right)}{\partial x} = \frac{S_y}{h_{uc}} \frac{\partial h_{uc}}{\partial t} + \left[\frac{h_{uc} - \bar{h}}{h_{uc} \sigma_c} \right]. \quad (5)$$

where K_x is the hydraulic conductivity of the unconfined aquifer.

Equations (3) and (5) are solved simultaneously for the piezometric head distribution and the saturated thickness. In this paper, the piezometric head distribution is obtained using a Galerkin's finite element program [e.g., Huyakorn and Pinder, 1983; Pinder and Gray, 1977; Istock, 1989]. The results of the finite element code were compared with standard closed-form solutions for simple flow situations and agreement between the solutions was very good [Karney and Seneviratne, 1991; Seneviratne, 1991]. Once the piezometric head is obtained using the finite element code, the energy terms discussed in the next section are computed in a "postprocessor" manner for each time step.

ONE-DIMENSIONAL ENERGY EQUATION

One virtue of the energy method is the ease with which the state of a system can be assessed. For example, when written in terms of mechanical energy in a saturated porous medium, conservation of energy requires that the network and energy flux into a unit volume of a system be partitioned between changes in internal energy and frictional dissipation. In this section, more complete statements of mechanical energy are derived by carefully combining the expressions of mass conservation with Darcy's law for both the confined and unconfined case. Additional examples of this process are described by Karney and Seneviratne [1991] and Seneviratne [1991].

Leaky Confined Aquifer

The confined aquifer is considered to be heterogeneous and of constant thickness. Multiplying equation (1) by h gives

$$h \frac{\partial(n\rho)}{\partial t} + h \frac{\partial(\rho v_x)}{\partial x} - h\rho \left[\frac{h_{uc} - \bar{h}}{B\sigma_c} \right] = 0. \quad (6)$$

Further, Darcy's law may be multiplied by v_x/K_x to give

$$\frac{v_x^2}{K_x} + v_x \frac{\partial h}{\partial x} = 0. \quad (7)$$

Summing equations (6) and (7) produces

$$h \frac{\partial(\rho v_x)}{\partial x} + \rho v_x \frac{\partial h}{\partial x} + \rho \frac{v_x^2}{K_x} + h \frac{\partial[n\rho]}{\partial t} - h\rho \left[\frac{h_{uc} - \bar{h}}{B\sigma_c} \right] = 0 \quad (8)$$

This equation when multiplied by gdx and integrated produces,

$$g[\rho v_x h]_0^L + g \int_0^L \rho \frac{v_x^2}{K_x} dx + g \int_0^L h \frac{\partial(n\rho)}{\partial t} dx - g \int_0^L h \rho \left[\frac{h_{uc}^- - \bar{h}}{B\sigma_c} \right] dx = 0. \quad (9)$$

Assuming that the spatial variations in density are negligible compared to the corresponding temporal variations, equation (9) can be written as,

$$\gamma \int_0^L \frac{v_x^2}{K_x} dx + \frac{\gamma^2}{2} [\alpha + n\beta] \int_0^L \frac{\partial[h^2]}{\partial t} dx - \gamma \int_0^L h \left[\frac{h_{uc}^- - \bar{h}}{B\sigma_c} \right] dx = -\gamma[v_x h]_0^L \quad (10)$$

The first term on the left-hand side of equation (10) depicts the domain-integrated dissipation, the second term represents the strain energy associated with the compressibility of the soil matrix and the pore water, while the work interactions at the leaky boundary are indicated by the third term. The right-hand side represents the work done at the Dirichlet boundaries.

Leaky Unconfined Aquifer

The energy equation for the unconfined aquifer is derived in a manner similar to the derivation of equation (10). Specifically, equation 4 can be multiplied by h_{uc} , added to an appropriate form of equation (7), and the resulting sum multiplied by $g dx$ and integrated over the domain. The final result is

$$\gamma \int_0^L \frac{v_x^2}{K_x} dx + \frac{\gamma^2}{3} \frac{[\alpha + n\beta]}{h_{uc}^-} \int_0^L \frac{\partial[h_{uc}^3]}{\partial t} dx + \frac{\gamma}{2} \frac{S_y}{h_{uc}^-} \int_0^L \frac{\partial[h_{uc}^2]}{\partial t} dx - \gamma \int_0^L h_{uc} \left[\frac{h_{uc}^- - \bar{h}}{h_{uc}^- \sigma_c} \right] dx = -\gamma[v_x h_{uc}]_0^L$$

and can be interpreted as the energy equation for the unconfined aquifer. Specifically, the first term on the left-hand side of equation (11) depicts the domain-integrated dissipation, the second term represents the strain energy associated with the compressibility of the soil matrix, the third term indicates the potential energy associated with the dewatering of the soil pores, and the pore water and the work interactions at the leaky boundary are indicated by the fourth term. The right-hand side represents the work done at the Dirichlet boundaries. For the unconfined case, the term h_{uc} appears both as a dimension (element height) and as a pressure elevation; as a result, the energy associated with compressibility is related to the cube of the changes in saturated zone thickness. In the next section, equations (10) and (11) are illustrated by analyzing the energy transformations in the simple two-aquifer system.

PARTITIONING OF ENERGY IN LEAKY AQUIFERS

Figure 1 depicts a leaky system with two aquifers. The confined aquifer material sand, has a hydraulic conductivity of 0.864 m/day and a bulk compressibility of $1.728(10^{-5})$

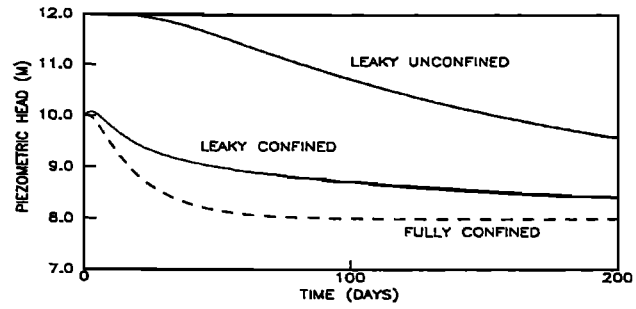


Fig. 2. Piezometric head distribution with a boundary disturbance.

kPa^{-1} . The unconfined aquifer material is silt having a hydraulic conductivity of 0.216 m/day and a specific yield of $1.728(10^{-2})$ and a 0.5-m-thick clay layer with hydraulic conductivity of $3.0(10^{-5})$ m/day separates the two aquifers. The initial saturated thickness in the unconfined aquifer is 12 m and the initial piezometric head in the confined aquifer is 10 m. The steady initial head differential across the aquitard causes steady seepage from the unconfined aquifer (higher head 12 m) to the confined aquifer (lower head 10 m).

The energy transformations in the system are computed for two different scenarios. In the first, the system is disturbed by an instantaneously falling piezometric head level in reservoir B; in the second, the disturbance is caused by groundwater pumping. The piezometric head drawdowns and the energy transformations in the system due to these excitations are analyzed in the following sections. In both cases the flow problem is solved using a finite element problem and the energy components are computed in the postprocessor manner described earlier.

Energy Transformations Due to a Boundary Excitation

The boundary conditions in the two aquifers change as follows: in the unconfined aquifer, the piezometric head at the left boundary at $x = 0$ drops to 10 m and the right boundary at $x = L$, the head drops to 6 m, while in the confined aquifer the head boundary condition at $x = 0$ remains at 10 m and the head at the right boundary drops to 6 m. Hence both aquifers now share common head values at the boundaries, 10 m upstream and 6 m downstream respectively. The resulting piezometric head distribution for both the aquifers are computed at a hypothetical observations well, 150 m from the origin O, and the resulting transient drawdowns are shown in Figure 2.

The confined aquifer was then modeled without leakage, subject to identical boundary conditions and the response is compared with the leaky case in Figure 2. When compared with the fully confined case, the leaky-confined aquifer exhibits a slower drawdown rate due to the recharge from the upper layer. This is an observation specific to this system with given aquifer properties and boundary conditions. However, under different aquifer and flow properties the leaky confined aquifer may discharge into, instead of recharge from the unconfined aquifer. In which case, the leaky confined aquifer may drawdown faster than the corresponding fully confined case.

The boundary drawdown of 6 m creates transient conditions in both the confined and unconfined aquifers. As depicted in Figure 2, the positive head differential toward the

confined aquifer causes leakage from the unconfined to the confined aquifer. The work interactions at the control surface of the confined aquifer arise from three components: the work done on the system at the left boundary ($x = 0$), the work done on the system at the right ($x = L$) boundary and work done by the system at the right ($x = L$) boundary which is considered positive. This is depicted for the confined portion of Figure 3. The negative work at the aquitard indicates work being done on the confined aquifer, while the positive network interactions at the head boundaries feature work done by the system.

The network interactions on the confined aquifer are partitioned between three physical phenomena; the elastic compressibility of the soil matrix, the compressibility of the pore water and the frictional dissipation. The work interaction at the aquitard depicts the magnitude of the leakage at that boundary, as depicted in the lower portion (confined) of Figure 3. The leakage reaches a maximum close to 25 days and then diminishes indicating milder leakage at the aquitard beyond 300 days. It is apparent from Figure 3, that the compressibility effects of the confined aquifer are also negligible beyond 100 days. Quasi-steady state analysis of the entire system cannot be recommended at this stage without a proper knowledge of the transient conditions of the unconfined aquifer.

The unconfined portion of Figure 3 shows that the magni-

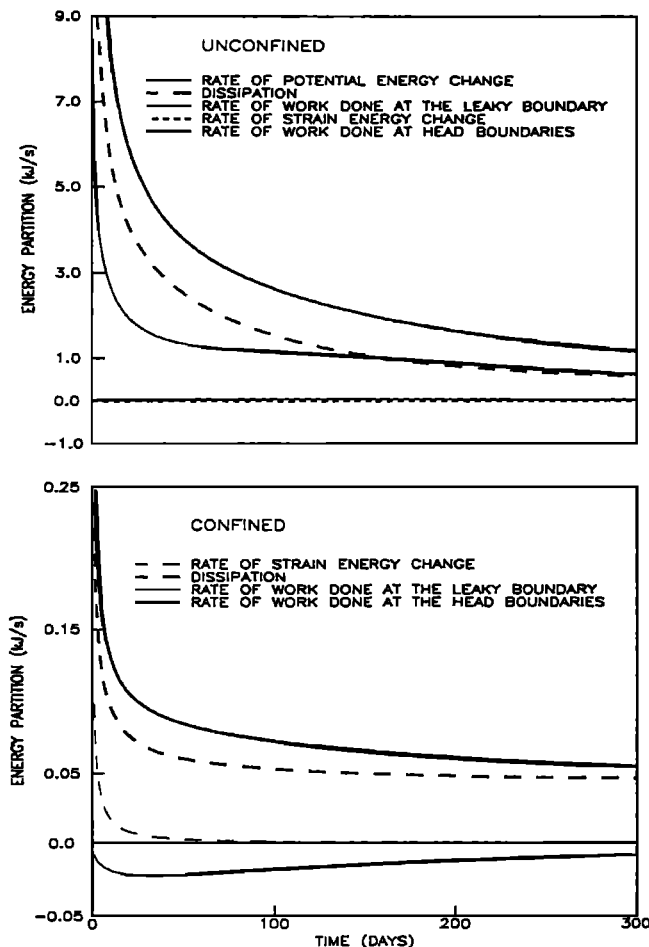


Fig. 3. Energy partitioning in the multiaquifer with boundary excitation.

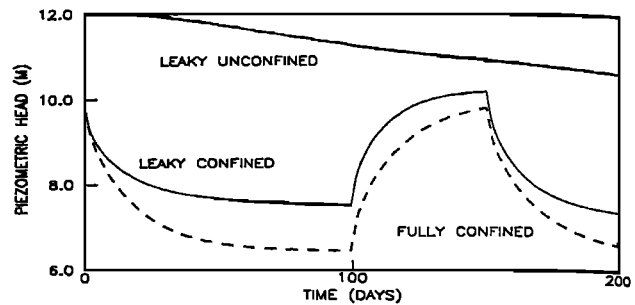


Fig. 4. Piezometric head distribution with pumping.

tudes of both the work done at the aquitard and the strain energy due to compressibility effects are negligible when compared with the rate of frictional dissipation and the rate of potential energy change in the pore water due to dewatering. The work associated with leakage is two orders of magnitude less than the value of net work done by the head boundaries. However, in the confined case, the maximum difference between the two work terms (i.e., the work done at the leaky boundary and the work done at the head boundaries) is only an order of magnitude up to 100 days of simulation.

In the unconfined case, the net work done at the boundaries is partitioned between the frictional dissipation, change in potential energy due to dewatering of soil pores, and the elastic effects of the soil skeleton. Not surprisingly, the greater attenuating influence of fluid storage in the unconfined aquifer produce a more gradual transient than that in the confined case. In the confined aquifer, the compressibility effects are insignificant after 100 days. However, in the unconfined aquifer the transient effects are still present even at 300 days.

Thus energy summaries are a convenient way of characterize the flow conditions in multiaquifers. In particular, they can be used to assess the importance of physical phenomena such as compressibility and the leakage at the pervious boundaries. In the next section, a similar analysis is done to characterize flow conditions in a multiaquifer when groundwater is pumped from the multiaquifer.

Energy Transformations Due to Groundwater Pumping

The objective of this study is to assess the energy transformations in the multiaquifer due to intermittent groundwater pumping from the confined aquifer at the rate of $10 \text{ m}^3 \text{ m}^{-2} \text{ day}^{-1}$ from a centrally located pump. The intermittent pumping schedule is as follows: groundwater is pumped from the confined aquifer in the multiaquifer shown in Figure 1 for 100 days. For the next 50 days, there is no pumping and the system is recharged by the constant head boundaries, increasing the piezometric head. Pumping begins once again at 150 days, and continues up to 200 days. Figure 4 depicts the resulting piezometric head drawdowns at 150 m.

The energy transformations in both the confined and unconfined aquifers are shown in Figure 5. The rate of work done at the leaky boundary of the confined aquifer has approached a steady value of approximately -0.03 kJ/s when the pumping is stopped at 100 days. Subsequently, the confined aquifer is recharged by both the fixed head boundaries and the aquitard until the second pumping event begins

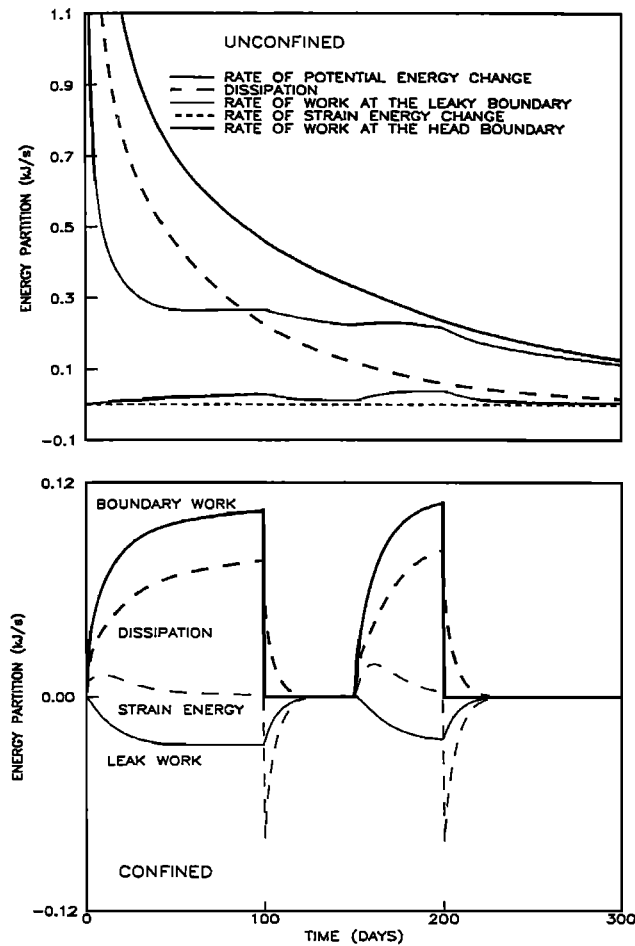


Fig. 5. Energy partitioning in the multiaquifer with pumping.

at 150 days. The effect of recharge from the boundaries gradually reduces the rate of leakage at the pervious boundary as seen from Figure 5. Hence between 100 and 150 days the leaky confined aquifer can be modeled as fully confined aquifer and the effect of the unconfined aquifer can be ignored. On the basis of the rate of leakage, the groundwater flow model switches from a leaky model to a confined model and, hence, substantially improves the overall computational efficiency.

ADAPTIVE MODELING OF THE LEAKY AQUIFER

Transient modeling of multiaquifers is computationally intense due to the simultaneous solution of two or more hydraulically interactive aquifers. The objective of the adaptive concept is to assess the magnitude of the interaction between the adjacent aquifers and if considered to be insignificant based on user specified information, then adopt a simpler model. Figure 6 depicts the adaptive concept when groundwater is pumped from the confined aquifer. The piezometric head distributions shown in the figure are calculated at a distance 150 m from the origin. The degree of numerical approximation of the numerical scheme is determined by evaluating the work at the aquitard; the program "decides" to switch models based on a "numerical threshold" parameter NT determined by

$$NT = \frac{\text{Work Done at the Leaky Boundary at a Given Time}}{\text{Maximum Work Done at the Leaky Boundary}}$$

The NT parameter indicates the prominence of leakage at any given time. When NT is less than a user specified value, the model switches from the detailed leaky model to a simpler confined model. A value of 1.0 indicates the maximum leakage, while 0.0 indicates fully confined conditions. Figure 6 shows the model responses for four different values of NT. For example, if NT is 0.1, the leaky model switches to the confined model when the work associated with leakage at the aquitard is one tenth of the maximum leakage observed occurred up to now. However, when the second pumping event begins at 150 days, the model shifts back to the leaky model and operates until the NT becomes 0.1 once again. Depending on the accuracy required, the user specifies the threshold degree of leakage. However, such a selection can only be made by a prior detailed simulation of the flow situation. Hence the adaptive concept becomes useful in cases where a large number of simulations are required, such as in selecting the best option from a large number of feasible alternatives.

It should be noted that the definition of the adaptive switch in terms of work automatically invokes both head and flow processes, since work is proportional to their product. Thus, the NT parameter is often more sensitive to how flow processes are physically coupled than, say, methods based on a simpler mass balance. For example, a recharge and a discharge well could both penetrate the same aquifer and could produce an overall mass balance for the aquifer, despite the fact that considerable interaction with other systems is taking place. The work parameter tends to be more robust as an indicator under these circumstances.

Figure 6 depicts the transient head response of the multiaquifer at a hypothetical observation well 150 m from the origin due to a boundary disturbance. The model response for four different degrees of leakage is shown in Figure 6. A degree of leakage of 0.3 resulted in a head response which coincided with the response of the detailed leaky model. It is also observed from Figure 6, that this flow case is highly sensitive to the specified NT parameter. Whereas in the pumping case, a change in the specified NT parameter from 0.1 to 0.8 showed smaller variations in the head responses.

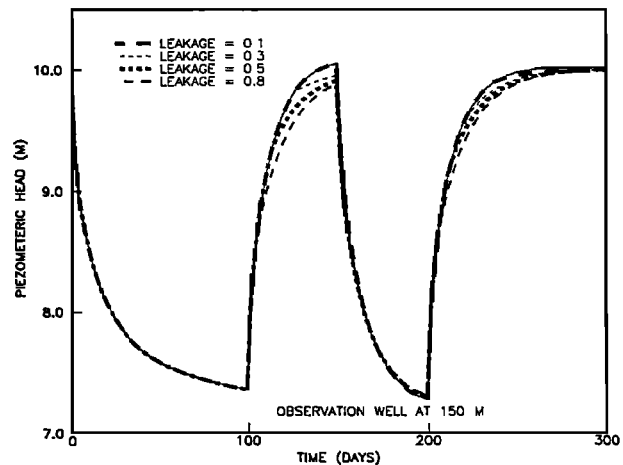


Fig. 6. Adaptive modeling of the multiaquifer with pumping.

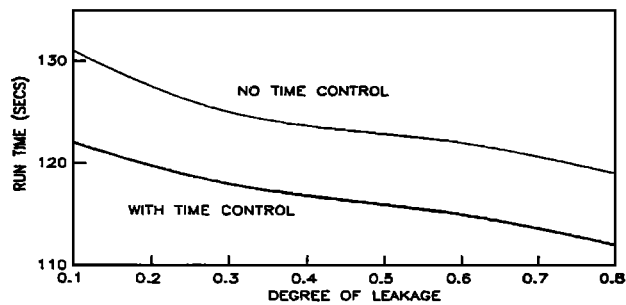


Fig. 7. Comparison of run times with and without time control (with pumping).

The adaptive procedure could be particularly useful in the simulation of systems in which the transient effects diminish slowly, as would be the case in clay materials. As shown in Figure 6, beyond 200 days of the boundary excitation, the system response is slow and rather uneventful. Hence, when the system passes through such a phase, the adaptive procedures can be effectively used, and result in some saving of computational effort (Figure 7).

CONCLUSIONS

The energy method proves to be an effective way of "collapsing" or integrating the dynamic behavior of aquifers into energy parameters which clearly describe the state of the system. By comparing relative magnitudes of the individual terms, it is possible to assess the dominating physical phenomena.

The importance of leakage at the semipervious layer is assessed by the magnitude of the corresponding work term. When the system undergoes rapid changes, the rate of work done at the leaky boundary is also acute; indicating a high leakage flux. However, when the leakage ceases, the work term also gradually diminishes. Hence, the work interaction at the seepage face becomes a natural index of the significance of the seepage. On the basis of this concept, the multiaquifer is modeled as a single confined aquifer, when the leakage is less than a user-specified threshold value. The energy approach improves the computational efficiency of the model subject to a user specified tolerance, and also, becomes particularly useful in the absence of reliable flow parameters of the intervening aquitards.

Although the examples presented in this paper are somewhat artificial, they indicate that even a simple-minded application of the energy concept can lead to beneficial insights. While the computational savings for the adaptive leak model are not dramatic at present, they show promise of

greater savings for more complex systems and thus indicate a possible area for further research.

NOTATION

- B saturated thickness.
- g gravitational acceleration.
- h piezometric head in confined aquifer.
- \bar{h} piezometric head in unconfined aquifer.
- h_{uc} piezometric head in confined aquifer (previous time step).
- h_{uc}^- piezometric head in unconfined aquifer (previous time step).
- K_x heterogeneous hydraulic conductivity.
- L length of the one-dimensional aquifer.
- n porosity.
- p fluid pressure.
- S storage coefficient.
- S_y specific yield in the unconfined aquifer.
- T transmissivity.
- t time.
- v_x flow velocity in the x direction.
- x space dimension.
- α coefficient of bulk compressibility of the soil.
- β coefficient of bulk compressibility of the pore water.
- γ unit weight of water.
- ρ density of water.
- σ_c coefficient of leakage of aquitard.

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